

Dynamics of Prey- Predator Model with Michaelis-Menten Type Prey Harvesting

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ABSTRACT

In this paper we investigate the Dynamics of Michaelis-Menten Type Prey Harvesting in prey-predator model. The system is described by a system of ordinary differential equations. The boundedness properties, long term behaviour of the system, equilibrium points are identified. Local stability analysis is discussed at each of its equilibrium points. Global stability is studied by constructing suitable Lyapunov's function. We proved that the system is both locally and globally asymptotically stable. Further Numerical simulation is performed and in support of analytical study.

Keywords: prey-predator, local stability, global stability, Simulation.

Mathematics Subject Classification:34DXX

Journal of Economic Literature (JEL):C62, C63

1. INTRODUCTION

Prey-predator model are building blocks of Eco system. It is the significant relationship in ecology. Many researchers' attention has been captured in this interaction. The Dynamics of prey-predator model are explored by Carlos [2], Freedman [4], Kot [6], Lakshmi Narayan [7], Lokta [8], May.R.M [11], Murry [12,13], Lima [10], Ranjith Kumar [15] and Sita Rambabu [18]. The models with different holling time functional responses are also included in the study. Later harvesting of prey-predator models are also explored by [1,3,16]. The harvesting prey-predator models with holling type response of type $1 + kN_i$ is included in the general prey-predator model with harvesting of Prey with different harvesting efforts considered for investigation.

The dynamics of this relationship can help us to protect the diversity of species in large scale. An important component in this relation is functional response of the predator. The classical type of functional responses has the following forms (Holling types I, II, III & IV).

The basic holling types functional response are available in literature [14, 17]. Authors [1,5,9] studied the dynamics of prey-predator model with the holling type functional responses I,II,III and IV with the Following basic model as

$$\begin{aligned}\frac{dx}{dt} &= rx \left(1 - \frac{x}{K}\right) - \varphi(x)y \\ \frac{dy}{dt} &= y(-d + \epsilon\varphi(x))\end{aligned}\quad (1.1)$$

Here can be defined as functional responses mentioned as I, II,III &IV type.

Liu,W[9] studied the Michaelis-Menten type harvesting in prey-predator model and bifurcation analysis. Xiao [19] analysed the prey-predator dynamics with constant harvesting rate. In spite of the above we proposed the Michaelis-Menten type harvesting in prey-predator model with holling type response $1 + kN_i$ and studied the dynamics of the model includes the boundedness properties, long term behaviour of the system, stability analysis at co-existing state. Finally, the analytical results are supported by numerical simulation.

2. FORMATION OF MODEL

The system of equations for the proposed model when Michaelis-Menten type harvesting function is taken for investigation when harvesting on the prey species.

$$\begin{aligned}\frac{dN_1}{dt} &= a_1N_1 - \alpha_{11}N_1^2 - \frac{\alpha_{12}N_1N_2}{1+kN_1} - \frac{qEN_1}{m_1E+m_2N_1} \\ \frac{dN_2}{dt} &= a_2N_2 - \alpha_{22}N_2^2 + \frac{\alpha_{21}N_1N_2}{1+kN_2}\end{aligned}\quad (2.1)$$

$$\text{With initial conditions } N_1(0) = N_{10} > 0 \text{ \& } N_2(0) = N_{20} > 0 \quad (2.2)$$

The following notations are adopted

N_1 & N_2 : Prey and predator population, a_1, a_2 : Growth rates of prey and predator population

α_{11} : Internal competition of prey species, α_{22} : Internal competition among predator species.

α_{12} : The interaction rate of prey on to the predator, α_{21} : The interaction rate of predator on to the prey

q: catch ability coefficient, E: harvesting effort, m_1 & m_2 are constants.

Consider the following non-dimensional scheme to investigate the dynamics of the model (2.1)

$$\text{Let } h = \frac{qE}{m_2} \text{ \& } g = \frac{m_1E}{m_2}$$

Using the above relations, we obtain the following system of equations

$$\begin{aligned}\frac{dN_1}{dt} &= a_1N_1 - \alpha_{11}N_1^2 - \frac{\alpha_{12}N_1N_2}{1+kN_1} - \frac{hN_1}{g+N_1} \\ \frac{dN_2}{dt} &= a_2N_2 - \alpha_{22}N_2^2 + \frac{\alpha_{21}N_1N_2}{1+kN_2}\end{aligned}\quad (2.3)$$

$$\text{With initial conditions } N_1(0) = N_{10} > 0 \text{ \& } N_2(0) = N_{20} > 0 \quad (2.4)$$

2.1. Positivity and Bounded ness of the Solutions.

In this section we prove the positivity and bounded ness of the solutions of system of equations (2.1) along with the initial conditions (2.2). To prove the results, we use the following two lemmas.

Lemma1: if $a, b > 0$ and $\frac{dx}{dt} \leq (\geq)x(t)(a - bx(t))$ with $x(0) > 0$ then

$$\log_{t \rightarrow \infty} \sup x(t) \leq \frac{a}{b} \quad (\log_{t \rightarrow \infty} \inf x(t) \geq \frac{a}{b})$$

Lemma2: if $a, b > 0$ and $\frac{dx}{dt} \leq x(t)(a - bx(t))$ with $x(0) > 0$, then for all $t \geq 0$

$$x(t) \leq \frac{a}{b - ce^{-at}} \text{ with } c = b - \frac{a}{x(0)} \quad \text{in particular } x(t) \leq \max\{x(0), \frac{a}{b}\} \text{ for all } t \geq 0$$

Theorem 2.1.1: All the solutions $N_1(t)$ & $N_2(t)$ of the system (2.1) with initial conditions (2.2) are positive i.e., $N_1(t) > 0$ & $N_2(t) > 0$

Proof: From the system of equations (2. 1) the prey equation is given by

$$\begin{aligned} \frac{dN_1}{dt} &= a_1 N_1 - \alpha_{11} N_1^2 - \frac{\alpha_{12} N_1 N_2}{1 + k N_1} - \frac{h N_1}{g + N_1} \quad \text{it follows that } N_1(t) = 0 \text{ is an invariant set. This implies and} \\ N_1(t) &> 0 \text{ for all } t \geq 0. \text{ We apply similar argument for the predator equations } \frac{dN_2}{dt} = a_2 N_2 - \alpha_{22} N_2^2 + \\ \frac{\alpha_{21} N_1 N_2}{1 + k N_2} &N_2(t) = 0 \text{ is an invariant set and hence } N_2(t) > 0 \text{ for all } t \geq 0. \text{ Thus, all the trajectories } R_+^2 \\ &\text{cannot cross the co-ordinate axis. Hence all the solutions } N_1(t) \text{ \& } N_2(t) \text{ are positive.} \end{aligned}$$

Theorem 2.1.2: All the solutions $N_1(t)$ & $N_2(t)$ of the system (2.1) with initial conditions

(2.2) are bounded for all $t \geq 0$.

Proof: From the system of equations (2.1) the prey equation is given by

$$\frac{dN_1}{dt} = a_1 N_1 - \alpha_{11} N_1^2 - \frac{\alpha_{12} N_1 N_2}{1 + k N_1} - \frac{h N_1}{g + N_1} \leq N_1(a_1 - \alpha_{11} N_1) \quad \text{using lemma2 where both } a_1 \text{ \& } \alpha_{11} > 0$$

$$\text{And also } N_1(t) \leq \frac{a_1}{\alpha_{11} - ce^{-a_1 t}} \quad \text{and } c = \alpha_{11} - \frac{a_1}{N_{10}}$$

$$\text{In particular } N_1(t) \leq \max\left\{N_{10}, \frac{a_1}{\alpha_{11}}\right\} = M_1 \text{ for all } t \geq 0. \quad (2.1.2.1)$$

From the predator equation from (2.1) we have equations

$$\frac{dN_2}{dt} = a_2 N_2 - \alpha_{22} N_2^2 + \frac{\alpha_{21} N_1 N_2}{1 + k N_2} \leq N_2 \left(a_2 + \frac{\alpha_{21} N_1}{1 + k N_2} - \alpha_{22} N_2 \right) \quad \text{again, using lemma2}$$

$$\text{Which implies } N_2 \left(a_2 + \frac{\alpha_{21} N_1}{1 + k N_2} - \alpha_{22} N_2 \right) \leq N_2 \left(a_2 + \frac{\alpha_{21} M_1}{1 + k N_2} - \alpha_{22} N_2 \right) \quad \text{from equation (2.1.2.1)}$$

$$\text{In particular } N_2(t) \leq \max \left\{ N_{20}, \frac{a_2(1+kN_{20})+\alpha_{21}M_1}{\alpha_{22}(1+kN_{20})} \right\} = M_2 \text{ for all } t \geq 0. \quad (2.1.2.2)$$

Hence the system (2.1) possesses bounded solutions.

2.2. Permeance:

The long-term behaviour of the dynamical system in particular the positive solutions of the system approach the boundary of the positive orthant. If the system includes two species the positive solutions approach the boundary of positive quadrant (two-dimensional space) and for three species the positive solutions approach the boundary of positive octant (three-dimensional space).

The permeance of the system exist if there exist two positive constants η_1 & η_2 such that each positive solutions of $N_1(t)$ & $N_2(t)$ with initial conditions N_{10} & N_{20} in R_+^2 satisfies,

$$\min\{\lim_{t \rightarrow \infty} \inf N_1(t, N_{10}, N_{20}), \lim_{t \rightarrow \infty} \inf N_2(t, N_{10}, N_{20})\} \geq \eta_1 \text{ and} \\ \max\{\lim_{t \rightarrow \infty} \sup N_1(t, N_{10}, N_{20}), \lim_{t \rightarrow \infty} \sup N_2(t, N_{10}, N_{20})\} \leq \eta_2$$

Theorem 2.2.1. The system (2.1) with initial conditions (2.2) is permanent if $a_2\alpha_{22}(1+kM_1) > \alpha_{12}[a_2(1+kM_1) + \alpha_{12}M_1]$

Proof: From the system of equations (2.1) the prey equation is given by

$$\frac{dN_1}{dt} = a_1N_1 - \alpha_{11}N_1^2 - \frac{\alpha_{12}N_1N_2}{1+kN_1} - \frac{hN_1}{g+N_1} \geq N_1(a_1 - \alpha_{11}N_1 - \alpha_{12}N_2 - h) \geq N_1 \left(a_1 - \alpha_{11}N_1 - \frac{\alpha_{12}\{a_2(1+kM_1) + \alpha_{21}M_1\}}{\alpha_{22}(1+kM_1)} - h \right)$$

using the bounded ness property from equation (2.1.2.2)

$$\text{Hence } \frac{dN_1}{dt} \geq N_1(L_1 - \alpha_{11}N_1) \text{ where } L_1 = a_1 - h - \frac{\alpha_{12}\{a_2(1+kN_{20}) + \alpha_{21}M_1\}}{\alpha_{22}(1+kN_{20})}$$

Using lemma1 $\lim_{t \rightarrow \infty} \inf N_1(t) \geq \frac{L_1}{\alpha_{11}}$ and $\lim_{t \rightarrow \infty} \sup N_1(t) \leq \frac{a_1}{\alpha_{11}}$ from equation (2.1.2.1)

From the system of equations (2.1) the prey equation is given by

$$\frac{dN_2}{dt} = a_2N_2 - \alpha_{22}N_2^2 + \frac{\alpha_{21}N_1N_2}{1+kN_2} \geq N_2(a_2 + \alpha_{21}N_1 - \alpha_{22}N_2) \geq N_2(a_2 + \alpha_{21}M_1 - \alpha_{22}N_2)$$

Using lemma1 $\lim_{t \rightarrow \infty} \inf N_2(t) \geq \frac{L_2}{\alpha_{22}}$ where $L_2 = a_2 + \alpha_{21}M_1$ and

$$\lim_{t \rightarrow \infty} \sup N_2(t) \leq \frac{a_2(1+kN_{20}) + \alpha_{21}M_1}{\alpha_{22}(1+kN_{20})} \text{ from equation (2.1.2.2)}$$

Now choose $\eta_1 = \min \left(\frac{L_1}{\alpha_{11}}, \frac{L_2}{\alpha_{22}} \right)$ and $\eta_2 = \max \left(\frac{a_1}{\alpha_{11}}, \frac{a_2(1+kN_{20}) + \alpha_{21}M_1}{\alpha_{22}(1+kN_{20})} \right)$ we get the permanence of the system (2.1).

2.4 Equilibrium states:

Equating the system of equations (2.1) to zero and solve it, we get the following four equilibrium points .

$$\text{I. The Extinct state } E_1: \bar{N}_1 = 0, \bar{N}_2 = 0 \quad (2.4.1)$$

II. Semi Extinct: The state in which one of one species Extinct and one survive

Case(A) : prey species exist and predator washed out.

To solve for the value of $\bar{N}_1$ from equation (2.3) we get the quadratic equation given by

$$\alpha_{11}N_1^2 + N_1(\alpha_{11} - a_1) + (h - a_1g) = 0 \quad (2.4.2)$$

Solving the above equation (2.4.2) we get two roots for $\bar{N}_1$ which is given by

$$\bar{N}_{1L} = \left(\frac{a_1}{2\alpha_{11}} - \frac{1}{2} \right) - \sqrt{(a_1 - \alpha_{11})^2 - 4\alpha_{11}(a_1g - h)} \quad (2.4.3)$$

$$\bar{N}_{1H} = \left(\frac{a_1}{2\alpha_{11}} - \frac{1}{2} \right) + \sqrt{(a_1 - \alpha_{11})^2 - 4\alpha_{11}(a_1g - h)} \quad (2.4.4)$$

The equilibrium point exists only if $(a_1 - \alpha_{11})^2 > 4\alpha_{11}(a_1g - h)$

$$\text{Hence the equilibrium points are } E_2: (\bar{N}_{1L}, 0) \& (\bar{N}_{1H}, 0) \quad (2.4.5)$$

Case(B) : prey species washed out & predator species exist .

$$E_3: \bar{N}_1 = 0, \bar{N}_2 = \frac{a_2}{\alpha_{22}} \quad (2.4.6)$$

III: Two species are survived

Solve the system of equations (2.3) we get the by quadric equation in N_2 is given by

$$a\lambda^4 + b\lambda^3 + c\lambda^2 + d\lambda + e = 0 \quad (2.4.7)$$

Where

$$a = -\alpha_{11}\alpha_{22}k^2, b = -2\alpha_{11}\alpha_{22}k + gk^2\alpha_{11}\alpha_{22} - a_1k^2\alpha_{22}$$

$$, c = 2a_1\alpha_{22}k - \alpha_{11}\alpha_{22} - 2gk\alpha_{11}\alpha_{22} - hk^2\alpha_{22} \quad (2.4.8)$$

$$, d = 2gka_1\alpha_{22} - 2hka_{22} - g\alpha_{11}\alpha_{22} - kga_2\alpha_{12} - (kg + 1)a_2\alpha_{12} - g\alpha_{12}\alpha_{21}$$

$$e = (a_1g - h)\alpha_{22} - ga_2\alpha_{12}$$

On solving equation (2.4.7), four possible roots exist for $\bar{N}_2$. The second equilibrium point $\bar{N}_1$ is obtained from the following equation

$$\bar{N}_1 = \frac{(1+k\bar{N}_2)(\alpha_{22}\bar{N}_2 - a_2)}{\alpha_{21}} \quad (2.4.9)$$

The Four possible equilibria for this case are obtained by solving equation (2.4.7) and for each value of $\bar{N}_2$ and corresponding value $\bar{N}_1$ is obtained from equation (2.4.9).

2.5.Local stability analysis:

Calculate the Jacobean matrix for the system of equations (2.3) is given by

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

where

$$\begin{aligned} J_{11} &= \frac{\partial f_1}{\partial N_1}, J_{12} = \frac{\partial f_1}{\partial N_2}, \\ J_{21} &= \frac{\partial f_2}{\partial N_1}, J_{22} = \frac{\partial f_2}{\partial N_2} \end{aligned} \quad (2.5.1)$$

Here

$$\begin{aligned} f_1(N_1, N_2) &= a_1 N_1 - \alpha_{11} N_1^2 - \frac{\alpha_{12} N_1 N_2}{1 + k N_1} - \frac{h N_1}{g + N_1}, \\ f_2(N_1, N_2) &= a_2 N_2 - \alpha_{22} N_2^2 + \frac{\alpha_{21} N_1 N_2}{1 + k N_2} \end{aligned} \quad (2.5.2)$$

Calculate the Jacobean matrix i.e.

$$J = \begin{bmatrix} -\alpha_{11} N_1 + \frac{k \alpha_{12} N_2}{(1 + k N_1)^2} + \frac{h N_1}{(g + N_1)^2} & -\frac{\alpha_{12} N_1}{1 + k N_1} \\ \frac{\alpha_{21} N_2}{(1 + k N_2)} & -\alpha_{22} N_2 - \frac{k \alpha_{21} N_1}{(1 + k N_2)^2} \end{bmatrix} \quad (2.5.3)$$

The characteristic equation is given by $\det(J - \lambda I) = 0$.

Find the eigen values for system of equation (2.5.3) and analyze the stability of the system at the equilibrium points.

Case (i) $E_1(0,0)$ is unstable

Case (ii) A: The eigen equation for E_2 is $\lambda^2 + \left(-\alpha_{11} \bar{N}_1 - k \alpha_{21} \bar{N}_1 + \frac{h \bar{N}_1}{(g + \bar{N}_1)^2}\right) \lambda = 0$, where roots are

$\lambda = 0$ & $\lambda = -\left(\alpha_{11} \bar{N}_1 + k \alpha_{21} \bar{N}_1 - \frac{h \bar{N}_1}{(g + \bar{N}_1)^2}\right)$ hence the system neutrally stable if

$$\alpha_{11} \bar{N}_1 + k \alpha_{21} \bar{N}_1 > \frac{h \bar{N}_1}{(g + \bar{N}_1)^2}$$

Case (ii) B: The eigen equation for E_3 is $\lambda^2 + \alpha_{22} \bar{N}_2 \lambda = 0$, where roots are

$\lambda = 0$ & $\lambda = -\alpha_{22} \bar{N}_2$ hence the system neutrally stable.

Case (iii): Co-existing case E_4 :

The characteristic equation is given by $a \lambda^2 + b \lambda + c = 0$ (2.5.4)

where $a=1$,

$$b = \left(\alpha_{11}N_1 - \frac{k\alpha_{12}N_2}{(1+kN_1)^2} - \frac{hN_1}{(g+N_1)^2} + \alpha_{22}N_2 - \frac{k\alpha_{21}N_1}{(1+kN_2)^2} \right),$$

$$c = \left(\frac{\alpha_{12}\alpha_{21}N_1N_2}{(1+kN_1)(1+kN_2)} - \frac{k\alpha_{12}\alpha_{22}N_2^2}{(1+kN_1)^2} + \alpha_{11}\alpha_{22}N_1N_2 - \frac{h\alpha_{22}N_2N_1}{(g+N_1)^2} + \frac{k\alpha_{21}\alpha_{11}N_1^2}{(1+kN_2)^2} - \frac{k^2\alpha_{12}\alpha_{21}N_1N_2}{(1+kN_1)^2(1+kN_2)^2} - \frac{hk\alpha_{21}N_1^2}{(g+N_1)^2(1+kN_2)^2} \right) \quad (2.5.5)$$

The system is stable if the sums of roots are negative and products of roots are positive i.e.

$$\text{If } \alpha_{11}N_1 + \alpha_{22}N_2 < \frac{k\alpha_{12}N_2}{(1+kN_1)^2} + \frac{hN_1}{(g+N_1)^2} + \frac{k\alpha_{21}N_1}{(1+kN_2)^2} \quad \text{and} \quad \frac{\alpha_{12}\alpha_{21}N_1N_2}{(1+kN_1)(1+kN_2)} + \alpha_{11}\alpha_{22}N_1N_2 + \frac{k\alpha_{21}\alpha_{11}N_1^2}{(1+kN_2)^2} >$$

$$\frac{k\alpha_{12}\alpha_{22}N_1N_2^2}{(1+kN_1)^2} + \frac{h\alpha_{22}N_2N_1}{(g+N_1)^2} + \frac{k^2\alpha_{12}\alpha_{21}N_1N_2}{(1+kN_1)^2(1+kN_2)^2} + \frac{hk\alpha_{21}N_1^2}{(g+N_1)^2(1+kN_2)^2} \quad (2.5.6)$$

E_4 is locally asymptotically stable if condition (2.5.6) is satisfied otherwise unstable

2.6. Global stability:

Theorem 2.6.1: The co-existing state $E_4(\overline{N}_1, \overline{N}_2)$ is globally asymptotically stable

Proof: Let the Lyapunov function be

$$V(\overline{N}_1, \overline{N}_2) = l_1(N_1 - \overline{N}_1) - \overline{N}_1 \log\left(\frac{N_1}{\overline{N}_1}\right) + l_2\left[(N_2 - \overline{N}_2) - \overline{N}_2 \log\left(\frac{N_2}{\overline{N}_2}\right)\right] \quad (2.6.1)$$

Find the derivative of equation (2.6.1) is

$$\frac{dV}{dt} = l_1 \frac{dN_1}{dt} \left[1 - \frac{\overline{N}_1}{N_1} \right] + l_2 \frac{dN_2}{dt} \left[1 - \frac{\overline{N}_2}{N_2} \right] \quad (2.6.2)$$

$$= l_1[N_1 - \overline{N}_1] \left[\left(a_1 - \alpha_{11}N_1 - \frac{\alpha_{12}N_2}{1+kN_1} - \frac{h}{g+N_1} \right) \right] +$$

$$l_2[N_2 - \overline{N}_2] \left[a_2 - \alpha_{22}N_2 + \frac{\alpha_{21}N_1}{1+kN_2} \right] \quad (2.6.3)$$

By proper choice of

$$a_1 = \alpha_{11}\overline{N}_1 + \frac{\alpha_{12}\overline{N}_2}{1+k\overline{N}_1} + \frac{h}{g+\overline{N}_1}, a_2 = \alpha_{22}\overline{N}_2 - \frac{\alpha_{21}\overline{N}_1}{1+k\overline{N}_2} \& l_1 = \frac{1+k\overline{N}_1}{\alpha_{12}}, l_2 = \frac{1+k\overline{N}_2}{\alpha_{21}}$$

$$\text{We get } \frac{dV}{dt} = -\alpha_{11} \frac{1+k\overline{N}_1}{\alpha_{12}} (N_1 - \overline{N}_1)^2 - \frac{\alpha_{22}(1+k\overline{N}_2)}{\alpha_{21}} (N_2 - \overline{N}_2)^2 \quad (2.6.4)$$

Clearly $\frac{dV}{dt} < 0$.

Therefore, the system (2.1) is globally stable at $E_4(\overline{N}_1, \overline{N}_2)$

3. NUMERICAL EXAMPLE:

Example 3.1: Let $a_1=1$, $a_2=1$, $k=0.5$, $\alpha_{11}=0.1$, $\alpha_{12}=0.1$, $\alpha_{21}=0.05$, $\alpha_{22}=0.5$, $m_1=1$, $m_2=2$, $N_1=25$, $N_2=10$.

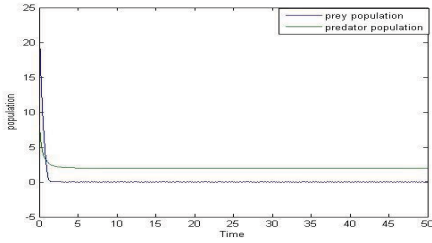


Fig 3.1.1(A)

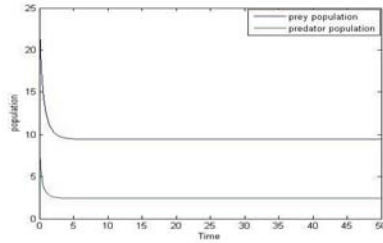


Fig 3.1.1(B)

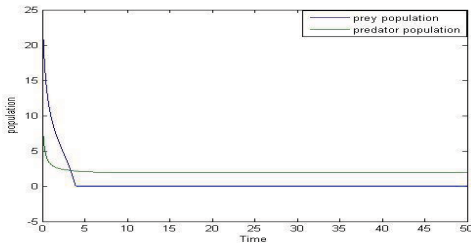


Fig 3.1.1(C)

Fig 3.1.1(A): with (i) $q=10$, $E=5$. (ii) $q=10$, $E=10$. (iii) $q=5$, $E=10$ (iv) $10 \leq q, E \leq 50$.

Stable solutions converging to fixed equilibrium point $E(0,2)$

The range of $10 \leq q, e \leq 50$, $0.5 < k < 500$, $0.1 < m_1, m_2 < 100$ the prey & predator population range varies from $0 \leq N_1 \leq 10$ & $2 \leq N_2 \leq 3$.

Fig 3.1.1(B): with (i) $q=0.5$, $E=0.5$. (ii) $q=0.01$, $e=0.01$. (iii) $q=0.05$, $e=0.05$ (iv) $q=0.1$, $e=0.1$

Stable solutions converging to fixed equilibrium point $E(10,2)$

The range of $0.5 < k < 500$, $0.1 < m_1, m_2 < 100$ the prey & predator population range varies from $0 \leq N_1 \leq 10$ & $2 \leq N_2 \leq 3$.

Fig 3.1.1(C): with (i) $q=10$, $e=0.1$. (ii) $q=10$, $e=0.5$ (iii) $q=1$, $e=1$

Stable solutions converging to fixed equilibrium point $E(0,2)$

The range of $0.5 < k < 500$, $0.1 < m_1, m_2 < 100$ the prey & predator population range varies from $0 \leq N_1 \leq 10$ & $2 \leq N_2 \leq 3$

Example 3.2: Let $a_1=1.5$, $a_2=2$, $k=5$, $\alpha_{11}=0.1$, $\alpha_{12}=0.2$, $\alpha_{21}=0.03$, $\alpha_{22}=0.1$, $m_1=1$, $m_2=2$, $N_1=25$, $N_2=10$

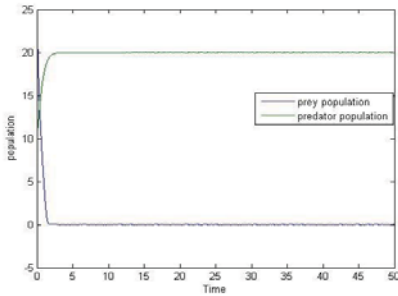


Fig 3.2.1(A)

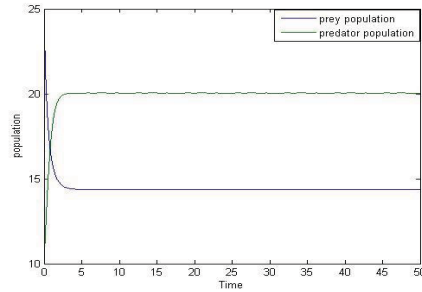


Fig 3.2.1(B)

Fig 3.2.1(A): with (i) $q=10$, $E=5$. $q=10$, $E=0.5$ (ii) $q=10$, $E=10$. (iii) $q=5$, $E=10$ (iv) $10 \leq q, e \leq 50$.

Stable solutions converging to fixed equilibrium point (0,20)

The range of $10 \leq q, e \leq 50$, $0.5 < k < 500$, $0.1 < m_1, m_2 < 100$ the prey & predator population range varies from $0 \leq N_1 \leq 15$ & $19 \leq N_2 \leq 20$.

Fig 3.2.1(B): with (i) $q=0.5$, $E=0.5$ (ii) $q=0.01$, $E=0.01$. (iii) $q=0.05$, $E=0.05$ (iv) $q=0.1$, $E=0.1$ (v) $q=10$, $e=0.1$. (vi) $q=1$, $e=1$

Stable solutions converging to fixed equilibrium point (14,20)

$0.5 < k < 500$, $0.1 < m_1, m_2 < 100$ the prey & predator population range varies from $13 \leq N_1 \leq 15$ & $19 \leq N_2 \leq 20$.

4. CONCLUSION

We consider a two species ecological model based on prey- predators' interactions with prey harvesting of Michaelis-menten type model taken for investigation. The mathematical model with prey predator dynamics was studied and prove that the eco-system is stable. The properties of the model were studied like positivity, boundedness and permealce of the system. The stability analysis of the model was discussed at possible equilibrium points. The global stability analysis of co-existing state is also addressed by choosing proper Lyapunov's function. Numerical simulation is performed in support of analytical results. The stability analysis at four equilibrium points and its nature with three different harvesting efforts are placed below

Equilibrium points	Nature of the system with Mechaelin-Menten type harvesting
$E_1: \bar{N}_1 = 0, \bar{N}_2 = 0$	Un stable
$E_2: \bar{N}_1 \neq 0, \bar{N}_2 = 0$	Neutrally stable
$E_3: \bar{N}_1 = 0, \bar{N}_2 \neq 0$	Neutrally stable if $\alpha_{11}\bar{N}_1 + k\alpha_{21}\bar{N}_1 > \frac{h\bar{N}_1}{(g+\bar{N}_1)^2}$

$E_4: \bar{N}_1 \neq 0, \bar{N}_2 \neq 0$	Asymptotically stable if $\alpha_{11}N_1 + \alpha_{22}N_2 < \frac{k\alpha_{12}N_2}{(1+kN_1)^2} + \frac{hN_1}{(g+N_1)^2} + \frac{k\alpha_{21}N_1}{(1+kN_2)^2} \quad \text{and} \quad \frac{\alpha_{12}\alpha_{21}N_1N_2}{(1+kN_1)(1+kN_2)} +$ $\alpha_{11}\alpha_{22}N_1N_2 + \frac{k\alpha_{21}\alpha_{11}N_1^2}{(1+kN_2)^2} > \frac{k\alpha_{12}\alpha_{22}N_1N_2^2}{(1+kN_1)^2} + \frac{h\alpha_{22}N_2N_1}{(g+N_1)^2} + \frac{k^2\alpha_{12}\alpha_{21}N_1N_2}{(1+kN_1)^2(1+kN_2)^2} +$ $\frac{hk\alpha_{21}N_1^2}{(g+N_1)^2(1+kN_2)^2}$
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Table 4.1

The global stability analysis of co-existing state is also addressed by choosing proper Lyapunov's function and prove that the system is globally asymptotically stable. Further Numerical simulation is performed in support of analytical results. The harvesting effort results are placed below.

Examples	Results with Michaelin-Menten type harvesting
Example 3.1	with (i) $q = 0.5, E = 0.5$. (ii) $q = 0.01, e = 0.01$. (iii) $q = 0.05, e = 0.05$ (iv) $q = 0.1, e = 0.1$ Stable solutions converging to fixed equilibrium point $E(10, 2)$ prey population is extinct with (i) $q = 10, E = 5$. (ii) $q = 10, E = 10$. (iii) $q = 5, E = 10$ (iv) $10 \leq q, E \leq 50$. Stable solutions converging to fixed equilibrium point $E(0, 2)$.
Example 3.2	with (i) $q = 0.5, E = 0.5$ (ii) $q = 0.01, E = 0.01$. (iii) $q = 0.05, E = 0.05$ (iv) $q = 0.1, E = 0.1$ (v) $q = 10, e = 0.1$. (vi) $q = 1, e = 1$ Stable solutions converging to fixed equilibrium point $(14, 20)$ prey population is extinct with (i) $q = 10, E = 5$. $q = 10, E = 0.5$ (ii) $q = 10, E = 10$. (iii) $q = 5, E = 10$ (iv) $10 \leq q, e \leq 50$. Stable solutions converging to fixed equilibrium point $(0, 20)$

Table 4.2

The system is not admitting unstable nature for different harvesting efforts. The harvesting effort Michaelin-Menten type harvesting the predator species is exist in absence of prey species and system becomes neutrally stable nature. Hence Michaelin-Menten type harvesting is more realistic in biological considerations.

5. REFERENCES

1. Chen, H., Zhang, C., 2022, Analysis of the dynamics of a predator-prey model with holling functional response. J. Nonl. Mod. Anal. 4, 310–324
2. Carlos Chavez, C., 2012, Mathematical models in population biology and epidemiology, Second Edition, Springer.
3. Edward A. Bender, 1978 Introduction to Mathematical Modelling, John Wiley & Sons,

4. Freedman.H.I.,1980, Deterministic mathematical models in population ecology, Marcel-Decker, New York.
5. Gupta, R.P., Chandra, P.,2013, Bifurcation analysis of modified Leslie-Gower predator-prey model with Michaelis-Menten type prey harvesting. J. Math. Anal. Appl. 398(1), 278–295.
6. Kot, M., 2001,Elements of Mathematical Ecology, Cambridge University press, Cambridge
7. Lakshmi Naryan.K., and Pattabhi Ramacharyulu.N.ch.,2007,A Prey - Predator Modelwith cover for Prey and an Alternate Food for the Predator, and Time Delay., International Journal of Scientific Computing, Vol.1 No.1 pp 7-14 .
8. Lotka. A.J.,1925, Elements of physical biology, Williams and Wilkins, Baltimore.
9. Liu, W., Jiang, Y.L.,2018, Bifurcation of a delayed gause predator-prey model with Michaelis Menten type harvesting. J. Theor. Biol. 438, 116–132.
10. Lima, S.L.,1998, Nonlethal effects in the ecology of predator-prey interactions - What are the ecological effects of anti-predator decision-making? Bioscience 48(1), 25–34.
11. May, R.M.,1973, Stability and complexity in model Eco-Systems, Princeton University press, Princeton.
12. Murray, J.D.,1989,Mathematical Biology, Biomathematics 19, Springer-Verlag, Berlin-Heidelberg-New York.
13. Murray, J.D., Mathematical Biology-I.,2002,an Introduction, Third edition, Springer.
14. Ranjith Kumar Upadhyay, Satteluri R. K. Iyengar.,2014, Introduction to Mathematical Modeling and Chaotic Dynamics, A Chapman & Hall Book, CRC Press.
15. Ranjith Kumar G, Kalyan Das, Lakshmi Narayan Ravindra Reddy B., 2019,Crowding effects and depletion mechanisms for population regulation in prey-predator intra-specific competition model. Computational Ecology &software ISSN 2220-721Xvol 9 no 1 pp 19-36.
16. Seo, G., DeAngelis, D.L.,2011, A predator prey model with a Holling type I functional response including a predator mutual interference. J. Nonlinear Sci. 21, 811–833
17. SreeHariRao.V., and Raja SekharaRao.P.,2009, Dynamic Models and Control of Biological Systems, Springer Dordrecht Heidelberg London New York.
18. SitaRambabu. B., Lakshmi Narayan k., 2019, Mathematical Study of Prey-Predator Model with Infection Predator and Intra-specific Competition, International Journal of Ecology & Development ISSN: 0973-7308vol 34 , issue 3(1) pp 11-21 .
19. Xiao, D.M., Jennings, L.S.,2005, Bifurcations of a ratio-dependent predator-prey system with constant rate harvesting. SIAM J. Appl. Math. 65(3), 737–753.