

COVID-19 MODEL WITH S-I FOR BEDDINGTON DE ANGELIS PREY-PREDATOR SPECIES

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ABSTRACT

This paper presents the three species mathematical models in which the interaction of two preys-one predator where the virus will affected the healthy species under some mutual interference with handling time using Beddington De Angelis FR. The motive of this work is to interaction among healthy (susceptible) prey, infected prey and predator to minimize the spread of disease. Boundedness, Equilibrium points, check the periodic solution and analyse the stability with numerical examples using maple software of Rossler type. Finally the result of this model prey predator of the SI type using numerical simulations.

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1 Introduction

This paper describes the concept of an ecosystem is the modeling of interactions between the constituent species. The dynamical relationship b/w preys and predator is an important area of the research in an ecology. Predator-prey interaction is the most common and is well known for generating the oscillatory dynamics. Predator-prey interaction and their competition are often viewed as the double main building blocks in mathematical models. Therefore mathematical modeling plays a key role in ecological research(Upadhyay and Iyengar, 2013). We propose an eco-epidemiological predator-prey system and study the dynamical of this model system depend on the infection rate, attack rate and other parameters(Saikh and Gazi, 2018).

The newly emerging severe acute respiratory syndrome of corona virus 2 also called as covid-19 has been recognized as a global pandemic by the World Health Organization on 11th march 2020(Bani Younes and Hasan, 2020; coayla Teran, 2021). The corona virus that causes covid-19 is mainly transmitted through droplets generated when an infected person cough, sneeze or exhales(Zeb, Alzahrani, Erturk and Zamam, n.d.; CHAKRAVARTY, 2021). The spread of disease in a high population of susceptibles may be thought of as an invasion process via

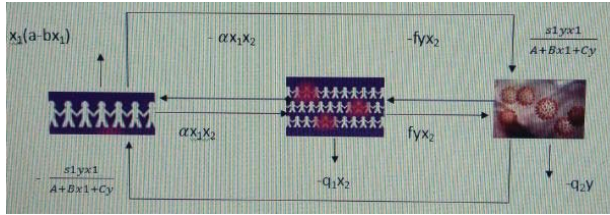


Figure 1: Transfer diagram of eco epidemiological model

independent contacts with a few infectious individuals (Brauer, Castillo-Chavez and Castillo-Chavez, 2012). The deadly covid-19 continue to spread around the globe and mathematical population model can be used to show sus. and inf. as two prey species and covid-19 can be assigned as predator in our competitive prey species of Lotka-Volterra model.

Catering to the needs and satisfy of human beings invariantly robs are the ecological structure of the nature. Often it is possible to prevent such extinction either by force or isolation. One such activity is lockdown that measures can effectively reduce the infection rate (Kar, 2006). This article motivate to give a mathematical model to present the dynamical behavior of covid-19 infections by using B.DeA.Fn.Rp. which is similar to Holling Type II which contained an additional term describing mutual interference by predator between consumers decreases the predator killing efficiency at more predator densities, therefore imposing a non negative feed-back on predator density (Tripathi, Abbas and Thakur, 2015). This model describes a prey-predator with holling type II fn.rp. where harvesting take place of each species into consideration. This was the important factor for the modification of Holling Type-II fn.rp. in the form of B.DeA.Fn.Rp. $f(x_1, x_3) = s x_1 x_3 / (A + B x_1 + C x_3)$ (Garain, Kumar and Mandal, 2021).

By using the valuable methods of ODE, the local stability of the non negative equilibrium points is determined (Xiao, 2007; Hugo, Massawe and Makinde, 2012; Hugo and Simanjilo, 2019). We discuss the boundedness, existence, properties of DE tools with Rossler type solutions.

2 Model Formulation

$$\begin{aligned} \frac{dx_1}{dt} &= a x_1(t) - b(x_1(t))^2 - \alpha x_1(t) x_2(t) - \frac{s x_1(t) x_3(t)}{A + B x_1(t) + C x_3(t)} \\ \frac{dx_2}{dt} &= \alpha x_1(t) x_2(t) - f x_2(t) x_3(t) - g x_2(t) \\ \frac{dx_3}{dt} &= j x_3(t) x_2(t) + \frac{p x_1(t) x_3(t)}{A + B x_1(t) + C x_3(t)} - q x_3(t) \end{aligned} \quad (2.1)$$

The assumptions that we used in formation of predator prey mathematical models involved three sub populations:

$x_1(t)$ is the number of sus. prey population at time t.

$x_2(t)$ is the number of inf. prey population at time t.

$x_3(t)$ is the number of predator population at time t.

The infection occurs because of a corona virus and it spread among prey population according to S-I model. The B.DeA.Fn.Rp. is applied in the sus. prey. The predator population attack both sus. and inf. prey. When the prey population maintains social distance and remedies to take medicine, predator will experience natural death. The parameter description of the model is given in table.1

Table: 1 The parameter of prey-predator mathematical model

parameter	explanation
a	Intrinsic growth rate of prey
b	Competitive prey species
α	Interference coefficient of sus. and inf. prey
s	Predator rate on sus. prey
f	Predator rate on inf. prey
g, q	Natural death for inf. prey and predator respectively
p	Conversion efficiency on sus. prey
j	Conversion efficiency on inf. prey

3 Positivity and Boundedness of theorem

In this above section, we aims to establish the conditions to get non negative as well as bdd. solutions of the system.

Theorem 3.1. *The dynamical system of equations 2.1 is always positive. Then all possible solutions of the system (2.1) are non negative.*

Proof.

Consider the first equations of the system

$$\frac{dx_1}{dt} = ax_1 - b(x_1)^2 - \alpha x_1 x_2 - \frac{sx_1 x_3}{A + Bx_1 + Cx_3}$$

$$\frac{dx_1}{x_1} = [a - bx_1 - \alpha x_2 - \frac{sx_3}{A + Bx_1 + Cx_3}]dt$$

$$\frac{dx_1}{x_1} = \phi(x_1, x_2, x_3)dt$$

where $\phi(x_1, x_2, x_3) = a - b(x_1)^2 - \alpha x_2 - \frac{sx_3}{A + Bx_1 + Cx_3}$ Taking integration in the region $[0, t]$, we get $x_1(t) = x_1(0) \exp \int \phi(x_1, x_2, x_3)dt > 0 \forall t$ as $x_1(0) \geq 0$

next consider the second set of equations

$$\frac{dx_2}{dt} = \alpha x_1 x_2 - f x_2 x_3 - g x_2$$

$$\frac{dx_2}{x_2} = [\alpha x_1 - f x_3 - g]dt$$

$$\frac{dx_2}{x_2} = \varphi(x_1, x_3)dt$$

where $\varphi(x_1, x_3) = [\alpha x_1 - f x_3 - g]$ Taking integration in the region $[0, t]$, we get $x_2(t) = x_2(0) \exp \int \varphi(x_1, x_3) dt > 0 \forall t$ as $x_2(0) \geq 0$
 next consider the third set of equations

$$\frac{dx_3}{dt} = j x_3 x_2 + \frac{p x_1 x_3}{A + B x_1 + C x_3} - q x_3$$

$$\frac{dx_3}{x_3} = [j x_2 + \frac{p x_1}{A + B x_1 + C x_3} - q] dt \quad (3.1)$$

$$\frac{dx_3}{x_3} = \psi(x_1, x_2, x_3) dt \quad (3.2)$$

where $\psi(x_1, x_2, x_3) = [j x_2 + \frac{p x_1}{A + B x_1 + C x_3} - q]$ Taking integration in the region $[0, t]$, we get $x_3(t) = x_3(0) \exp \int \psi(x_1, x_2, x_3) dt > 0 \forall t$ as $x_3(0) \geq 0$. Hence it may be analysed that all the solutions of the system are always positive.

Theorem 3.2. *In this domain $\mathbb{R}_+^3 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 \geq 0, x_2 \geq 0, x_3 \geq 0\}$ all solutions of the system 2.1 are bounded.*

Proof.

We choose $w(t) = x_1 + x_2 + x_3$ then differentiate $w(t)$ with respect to t and we get

$$\frac{dw}{dt} = \frac{dx_1}{dt} + \frac{dx_2}{dt} + \frac{dx_3}{dt}$$

$$\frac{dw}{dt} = (a - b x_1) x_1 - g x_2 - q x_3$$

for any $\chi > 0$ we get

$$\frac{dw}{dt} + \chi w = (a - b x_1) x_1 - g x_2 - q x_3 + \chi x_1 + \chi x_2 + \chi x_3$$

$$= (a + \chi) x_1 + (\chi - g) x_2 + (\chi - q) x_3 - b x_1^2$$

where χ is a non negative constant for $(a + \chi) \geq 0, (\chi - g) \geq 0, (\chi - q) \geq 0$ given $\epsilon > 0$ there exists to such that t on $t \geq t_0$ we get

$$\frac{dw(t)}{dt} + \chi w \leq \frac{(a + \chi)}{b} + (\chi - g) + (\chi - q) = \varsigma + \epsilon$$

To prove this thm., by using the following lemma

Lemma 3.3. *Let ϕ be an absolutely-continuous fn. satisfy the differential inequalities*

$$\frac{d\phi(t)}{dt} + \alpha_1 \phi(t) \leq \alpha_2, t \geq 0,$$

where $(\alpha_1, \alpha_2) \in \mathbb{R}_+^2, \alpha_1 \neq 0$ then $\forall t \geq \bar{T} \geq 0, \phi(t) \leq \frac{\alpha_2}{\alpha_1} - (\frac{\alpha_2}{\alpha_1} - \phi(\bar{T})) e^{-\alpha_1(t - \bar{T})}$.

From the above lemma (Aziz-Alaoui and Okiye, 2003), we get $0 \leq w(t) \leq \frac{\varsigma}{\chi} + w(t_0) e^{-\chi(t - t_0)} + (\frac{\varsigma + \epsilon}{\chi})(1 - e^{-\chi(t - t_0)})$ where $t \rightarrow +\infty$ then letting $\epsilon \rightarrow 0$

Therefore $0 \leq w(t) \leq \frac{\varsigma}{\chi}$, the system of equs. are bounded.

4 Nonlinear system

4.1 Critical point

The equilibrium points of the dynamical system are necessary for the purpose of studying the local stability nature of the eco-epidemic prey -predator model. The system of the equation 2.1 under investigation, has the following equilibrium points. Table: 2 Stability analysis

equilibrium point	explanation
$e0 \{x_1 = 0, x_2 = 0, x_3 = 0\}$	trivial
$e1 \left\{x_1 = 0, x_2 = 0, x_3 = -\frac{Aq}{Cq-p}\right\}$	prey washed out
$e2 \left\{x_1 = 0, x_2 = \frac{Afq-Cgq+gp}{j(Af-Cg)}, x_3 = -\frac{g}{f}\right\}$	susceptible prey species washed out
$e3 \left\{x_1 = \frac{a}{b}, x_2 = 0, x_3 = 0\right\}$	infected prey and predator washed out
$e4 \left\{x_1 = \frac{g}{\alpha}, x_2 = \frac{a\alpha-bq}{\alpha^2}, x_3 = 0\right\}$	predator washed out
$e5 \left\{x_1 = \frac{ap-qs}{bp}, x_2 = 0, x_3 = -\frac{q(Abp+Bap-Bqs)}{bp(Cq-p)}\right\}$	infected prey species washed out
$e6(x_1 = x_1^*, x_2 = x_2^*, x_3 = x_3^*)$	coexistence

4.2 Existence and stability analysis of equilibrium points

The jacobian matrix of the system (2.1) at equilibrium point $E=(x_1(t), x_2(t), x_3(t))$ is given by

$$J = \begin{bmatrix} m_{11} & -\alpha x_1 & m_{13} \\ \alpha x_2 & \alpha x_1 - f x_3 - g & -x_2 f \\ -\frac{x_3^2 p B}{(B x_1 + C x_3 + A)^2} & x_3 j & m_{33} \end{bmatrix}$$

where $m_{11} = -\alpha x_2 - b x_1 + a - \frac{s x_3}{B x_1 + C x_3 + A} + x_1 \left(-b + \frac{s x_3 B}{(B x_1 + C x_3 + A)^2}\right)$,

$m_{13} = x_1 \left(-\frac{s}{B x_1 + C x_3 + A} + \frac{s x_3 C}{(B x_1 + C x_3 + A)^2}\right)$,

$m_{33} = j x_2 + \frac{p x_3}{B x_1 + C x_3 + A} - q + x_3 \left(\frac{p}{B x_1 + C x_3 + A} - \frac{p x_3 C}{(B x_1 + C x_3 + A)^2}\right)$ based on the nature of Eigen values, the given system (1) gets stable when all three eigen values are negative in the case of real roots or negative real parts in the case of complex roots of the characteristic equation for the above jacobian matrix. otherwise the system is unstable.

Theorem 4.1. *The given system of (2.1) is stable at a equilibrium pt.*

$\left\{x_1 = 0, x_2 = 0, x_3 = -\frac{Aq}{Cq-p}\right\}$ provided that $\lambda_1 < 0, \lambda_2 < 0$ with the conditions $qs > ap \& cgq > fAq + gp, \lambda_3$ have negative real parts.

proof.

The jacobian matrix is

$$e1 = \begin{bmatrix} a + \frac{sAq}{Cq-p} \left(-\frac{CAq}{Cq-p} + A\right)^{-1} & 0 & 0 \\ 0 & \frac{fAq}{Cq-p} - g & 0 \\ -\frac{A^2 q^2 p B}{(Cq-p)^2} \left(-\frac{CAq}{Cq-p} + A\right)^{-2} & -\frac{Aqj}{Cq-p} & h_{33} \end{bmatrix}$$

where

$$h_{33} = -\frac{pAq}{Cq-p} \left(-\frac{CAq}{Cq-p} + A\right)^{-1} - q - \frac{Aq}{Cq-p} \left(p \left(-\frac{CAq}{Cq-p} + A\right)^{-1} + \frac{pAqC}{Cq-p} \left(-\frac{CAq}{Cq-p} + A\right)^{-2}\right) \text{ Now}$$

the eigenvalues are $\lambda_1 = \frac{ap-qs}{p} < 0$, $\lambda_2 = \frac{fAq-Cgq+gp}{Cq-p} < 0$, $\lambda_3 = -\frac{q(Cq-p)}{p} < 0$

Theorem 4.2. *The given system of (2.1) is locally asymptotically stable at equilibrium point $\{x_1 = 0, x_2 = \frac{fAq-Cgq+gp}{j(fA-Cg)}, x_3 = -\frac{q}{f}\}$ provided that $\lambda_1 < 0$ with the conditions $Afq\alpha + cagj + gp\alpha > aA fj + cgq\alpha + gjs$, λ_2, λ_3 have negative real parts.*

proof.

The jacobian matrix is

$$e2 = \begin{bmatrix} a - \frac{\alpha(fAq-Cgq+gp)}{j(fA-Cg)} + \frac{sg}{f} \left(-\frac{Cg}{f} + A\right)^{-1} & 0 & 0 \\ \frac{\alpha(fAq-Cgq+gp)}{j(fA-Cg)} & 0 & -\frac{(fAq-Cgq+gp)f}{j(fA-Cg)} \\ -\frac{g^2pB}{f^2} \left(-\frac{Cg}{f} + A\right)^{-2} & -\frac{gj}{f} & K_{33} \end{bmatrix}$$

where

$$K_{33} = \frac{fAq-Cgq+gp}{fA-Cg} - \frac{gp}{f} \left(-\frac{Cg}{f} + A\right)^{-1} - q - \frac{g}{f} \left(p \left(-\frac{Cg}{f} + A\right)^{-1} + \frac{gpC}{f} \left(-\frac{Cg}{f} + A\right)^{-2}\right)$$

Now the eigenvalues are

$$\lambda_1 = \frac{Aafj-A\alpha fq-Cagj+C\alpha gq-\alpha gp+gjs}{j(Af-Cg)},$$

$$\lambda_2 = -1/2 \frac{gp f A - \sqrt{k_1}}{A^2 f^2 - 2 AC fg + C^2 g^2},$$

$$\lambda_3 = -1/2 \frac{gp f A + \sqrt{k_2}}{A^2 f^2 - 2 AC fg + C^2 g^2}$$

where $k_1 = 4 A^4 f^4 gq - 16 A^3 C f^3 g^2 q + 24 A^2 C^2 f^2 g^3 q - 16 AC^3 f g^4 q + 4 C^4 g^5 q + 4 A^3 f^3 g^2 p - 12 A^2 C f^2 g^3 p + 12 AC^2 f g^4 p - 4 C^3 g^5 p + A^2 f^2 g^2 p^2$,
 $k_2 = 4 A^4 f^4 gq - 16 A^3 C f^3 g^2 q + 24 A^2 C^2 f^2 g^3 q - 16 AC^3 f g^4 q + 4 C^4 g^5 q + 4 A^3 f^3 g^2 p - 12 A^2 C f^2 g^3 p + 12 AC^2 f g^4 p - 4 C^3 g^5 p + A^2 f^2 g^2 p^2$

Theorem 4.3. *The dynamical system (2.1) is stable at equilibrium point $\{x_1 = \frac{a}{b}, x_2 = 0, x_3 = 0\}$ only if $a > 0$, $bg > a\alpha$, $q > 0$*

proof.

The jacobian matrix is

$$e3 = \begin{bmatrix} -a & -\frac{\alpha a}{b} & -\frac{as}{b} \left(\frac{Ba}{b} + A\right)^{-1} \\ 0 & \frac{\alpha a}{b} - g & 0 \\ 0 & 0 & -q \end{bmatrix}$$

Now the eigenvalues are

$$\lambda_1 = -a < 0,$$

$$\lambda_2 = \frac{\alpha a - bg}{b} < 0,$$

$$\lambda_3 = -q < 0$$

Theorem 4.4. *The given system of (2.1) is locally asymptotically stable at equilibrium pt. $\{x_1 = \frac{g}{\alpha}, x_2 = \frac{a\alpha-bg}{\alpha^2}, x_3 = 0\}$ provided that $\lambda_1 < 0$ with the conditions $\alpha^2 q + bgj > aj\alpha$ and λ_2, λ_3 have negative real parts.*

proof.

The jacobian matrix is

$$e4 = \begin{bmatrix} a - 2 \frac{bg}{\alpha} - \frac{\alpha a - bg}{\alpha} & -g & -\frac{gs}{\alpha} \left(\frac{Bg}{\alpha} + A \right)^{-1} \\ \frac{\alpha a - bg}{\alpha} & 0 & -\frac{(\alpha a - bg)f}{\alpha^2} \\ 0 & 0 & \frac{j(\alpha a - bg)}{\alpha^2} - q \end{bmatrix}$$

Now the eigenvalues are

$$\lambda_1 = \frac{a\alpha j - \alpha^2 q - bgj}{\alpha^2} < 0,$$

$$\lambda_2 = 1/2 \frac{-bg + \sqrt{-4a\alpha^2 g + 4\alpha bg^2 + b^2 g^2}}{\alpha} < 0,$$

$$\lambda_3 = -1/2 \frac{bg + \sqrt{-4a\alpha^2 g + 4\alpha bg^2 + b^2 g^2}}{\alpha} < 0$$

Theorem 4.5. *The given system (2.1) is locally asymptotically stable at equilibrium point $\{x_1 = \frac{ap - qs}{bp}, x_2 = 0, x_3 = -\frac{q(Abp + Bap - Bqs)}{bp(Cq - p)}\}$ provided that $\lambda_1 < 0$ with the conditions $Bfq^2s + ap^2\alpha + cbgpg + cq^2\alpha > pq\alpha + bgp^2 + capq\alpha + Abfpq + Bafpq$, λ_2, λ_3 have negative real parts.*

proof.

The jacobian matrix is $e5 = \begin{bmatrix} n_{11} & -\frac{\alpha(ap - qs)}{bp} & -\frac{(Cq - p)^2 s(ap - qs)}{p^2(Abp + Bap - Bqs)} \\ 0 & n_{22} & 0 \\ -\frac{Bg^2}{p} & -\frac{q(Abp + Bap - Bqs)j}{bp(Cq - p)} & -\frac{q(Cq - p)}{p} \end{bmatrix}$ where

$$n_{11} = -\frac{BCapq^2s - BCq^3s^2 + Aabp^3 - Abp^2qs + Ba^2p^3 - 3Bap^2qs + 2Bpq^2s^2}{p^2(Abp + Bap - Bqs)}$$

$$n_{22} = \frac{Abfpq + Bafpq - Bfq^2s + Ca\alpha pq - C\alpha q^2s - Cbgpg - a\alpha p^2 + \alpha pq + bgp^2}{bp(Cq - p)}$$

now the eigenvalues are $\lambda_1 = \frac{Abfpq + Bafpq - Bfq^2s + Ca\alpha pq - C\alpha q^2s - Cbgpg - a\alpha p^2 + \alpha pq + bgp^2}{bp(Cq - p)}$

$$\lambda_2 = -1/2 \frac{l_{11}}{p^2(Abp + Bap - Bqs)}, \lambda_3 = -1/2 \frac{l_{11}}{p^2(Abp + Bap - Bqs)}, \text{ where } l_{11} = ACbp^2q^2 + BCap^2q^2 + BCapq^2s - BCpq^3s - BCq^3s^2 + Aabp^3 - Abp^3q - Abp^2qs + Ba^2p^3 - Bap^3q - 3Bap^2qs + Bp^2q^2s + 2Bpq^2s^2 - \sqrt{l_1}, l_1 = A^2C^2b^2p^4q^4 + 2ABC^2abp^4q^4 + 2ABC^2abp^3q^4s - 2ABC^2bp^3q^5s - 2ABC^2bp^2q^5s^2 + B^2C^2a^2p^4q^4 + 2B^2C^2a^2p^3q^4s + B^2C^2a^2p^2q^4s^2 - 2B^2C^2ap^3q^5s - 4B^2C^2ap^2q^5s^2 - 2B^2C^2apq^5s^3 + B^2C^2p^2q^6s^2 + 2B^2C^2pq^6s^3 + B^2C^2q^6s^4 - 2A^2Cab^2p^5q^2 - 2A^2Cb^2p^5q^3 + 2A^2Cb^2p^4q^3s - 4ABCa^2bp^5q^2 + 2ABCa^2bp^4q^2s - 4ABCabp^5q^3 + 4ABCabp^4q^3s - 4ABCabp^3q^3s^2 + 4ABCbp^4q^4s + 2ABCbp^2q^4s^3 - 2B^2Ca^3p^5q^2 + 2B^2Ca^3p^4q^2s - 2B^2Ca^2p^5q^3 + 2B^2Ca^2p^4q^3s - 8B^2Ca^2p^3q^3s^2 + 4B^2Cap^4q^4s + 2B^2Cap^3q^4s^2 + 10B^2Cap^2q^4s^3 - 2B^2Cp^3q^5s^2 - 2B^2Cp^2q^5s^3 - 4B^2Cpq^5s^4 + A^2a^2b^2p^6 + 2A^2ab^2p^6q - 2A^2ab^2p^5qs + A^2b^2p^6q^2 - 2A^2b^2p^5q^2s + A^2b^2p^4q^2s^2 + 2ABa^3bp^6 + 4ABa^2bp^6q - 8ABa^2bp^5qs + 2ABabp^6q^2 - 6ABabp^5q^2s + 10ABabp^4q^2s^2 - 2ABbp^5q^3s + 2ABbp^4q^3s^2 - 4ABbp^3q^3s^3 + B^2a^4p^6 + 2B^2a^3p^6q - 6B^2a^3p^5qs + B^2a^2p^6q^2 - 4B^2a^2p^5q^2s + 13B^2a^2p^4q^2s^2 - 2B^2ap^5q^3s + 2B^2ap^4q^3s^2 - 12B^2ap^3q^3s^3 + B^2p^4q^4s^2 + 4B^2p^2q^4s^4$$

Theorem 4.6. *The interior equilibrium point $(x_1 = x_1^*, x_2 = x_2^*, x_3 = x_3^*)$ is locally asymptotically stable.*

Proof.

The Variation of the Jacobian matrix is

$$J_6 = \begin{bmatrix} m_{11} & -\alpha x_1^* & m_{13} \\ \alpha x_2^* & \alpha x_1^* - fx_3^* - g & -x_2^*f \\ m_{31} & x_3^*j & m_{33} \end{bmatrix}$$

$$\text{where } m_{11} = -\alpha x_2^* - bx_1^* + a - \frac{sx_3^*}{Bx_1^* + Cx_3^* + A} + x_1^* \left(-b + \frac{sx_3^* B}{(Bx_1^* + Cx_3^* + A)^2} \right),$$

$$m_{13} = x_1^* \left(-\frac{s}{Bx_1^* + Cx_3^* + A} + \frac{sx_3^* C}{(Bx_1^* + Cx_3^* + A)^2} \right), m_{31} = -\frac{x_3^{*2} p B}{(Bx_1^* + Cx_3^* + A)^2},$$

$$m_{33} = jx_2^* + \frac{px_3^*}{Bx_1^* + Cx_3^* + A} - q + x_3^* \left(\frac{p}{Bx_1^* + Cx_3^* + A} - \frac{px_3^* C}{(Bx_1^* + Cx_3^* + A)^2} \right)$$

The characteristic equn. is $\Lambda(\lambda) = \lambda^3 + A_1\lambda^2 + A_2\lambda + A_3$

$$\text{where } A_1 = -(m_{11} + m_{22} + m_{33}) = 3I + fx_3^* + g - x_1^*\alpha + m_{33} - m_{11},$$

$$= 3I + L$$

$$A_2 = (m_{11}m_{22} + m_{11}m_{33} + m_{12}m_{21} - m_{13}m_{31} + m_{22}m_{33} + m_{23}m_{32}) = 3I^2 - (2m_{11} + 2x_1^*\alpha - 2fx_3^* - 2g + 2m_{33})I + m_{11}x_1^*\alpha - m_{11}fx_3^* - m_{11}g + x_1^*x_2^*\alpha^2 + m_{11}m_{33} - m_{31}m_{13} + m_{33}x_1^*\alpha - m_{33}fx_3^* - gm_{33} + x_3^*x_2^*fj,$$

$$= 3I^2 + 2LI + M$$

$$A_3 = m_{11}m_{32}m_{23} - m_{11}m_{22}m_{33} + m_{12}m_{21}m_{33} - m_{12}m_{23}m_{31} - m_{13}m_{21}m_{32} + m_{13}m_{22}m_{31} =$$

$$I^3 - (g + fx_3^* - x_1^*\alpha - m_{11} - m_{33})I^2 + (x_3^*x_2^*fj - m_{33}fx_3^* + m_{11}m_{33} + m_{33}x_1^*\alpha - m_{33}g + m_{11}x_1^*\alpha - m_{11}fx_3^* - m_{11}g + x_1^*x_2^*\alpha^2 - m_{13}m_{31})I - m_{11}x_3^*x_2^*fj - m_{11}m_{23}x_1^*\alpha + m_{11}m_{33}x_3^*f + m_{11}gm_{33} - m_{33}x_1^*x_2^*\alpha^2 - fm_{31}x_1^*x_2^*\alpha + m_{13}m_{31}x_1^*\alpha - fm_{13}m_{31}x_3^* - m_{13}m_{31}g - m_{13}x_2^*x_3^*j\alpha$$

$$= I^3 - LI^2 + MI - N$$

By Routh Hurwitz criterion, all the eigenvalue of j_6 have non positive real parts if i) $A_1 > 0$, ii) $A_2 > 0$, iii) $A_3 > 0$, iv) $A_1A_2 > A_3$

$$\text{where } A_1A_2 = (3I + L)(3I^2 + 2LI + M)$$

$$= 9I^3 + 9LI^2 + (3M + 2L^2)I + ML$$

$$> I^3 - LI^2 + MI - N = A_3$$

Hence we proved $A_1A_2 > A_3$

Therefore the given system of the nonlinear differential equn. (2.1) is locally asymptotically stable around non-trivial equilibrium pt. $x_1 = x_1^*, y_1 = y_1^*, z_1 = z_1^*$ if the conditions mentioned in the theorem holds.

5 Numerical solution

The given dynamical system of the nonlinear differential equn. (2.1) for the numerical solution.

(1)First we take the parameter of the system as $(a, b, \alpha, A, B, g, q, C, p, j, s, f) =$

$(0.236, 1.05, 1.25, 0.23, 0.545, 0.005, 1.236, 1.450, 0.212, 1.024, 2.05, 2.11)$ at the population

$(x_1, x_2, x_3) = (0.235, 1.46, 1.056)$.

(2)If we take the parameter of the system as mentioned above in point(1) then the initial condition satisfies with $(x_1, x_2, x_3) = (0, 0, 1.07)$ the predator population (see Fig.2).

(3)If we take the parameter of the system as mentioned above in point(1) then the initial condition satisfies with $(x_1, x_2, x_3) = ((0, 0.6, 0.7), (0.0.236, 1.07))$ the inf. prey and predator population (see Fig.3).

(4)If we take the parameter of the system as mentioned above in point(1) then the initial condition satisfies with $(x_1, x_2, x_3) = (1.23, 0, 0)$ the susceptible prey population (see Fig.4).

(5)If we take the parameter of the system as mentioned above in point(1) then the initial condition satisfies with $(x_1, x_2, x_3) = ((0.4, 1.23, 0), (0.2, 2.0, 0))$ the S-I prey population (see Fig.5).

(6)If we take the parameter of the system as mentioned above in point(1) then the initial condi-

tion satisfies with $(x_1, x_2, x_3) = ((0.4, 0, 1.23), (2.5, 0, 0.45))$ the sus. prey and predator population (see Fig.6).

(7) If we take the parameter of the system as mentioned above in point(1) then the initial condition satisfies with $(x_1, x_2, x_3) = ((0.4, 1.0, 1.23), (0.2, 2.0, 1.0), (2.5, 1.01, 0.45))$ the prey-predator population (see Fig.7).

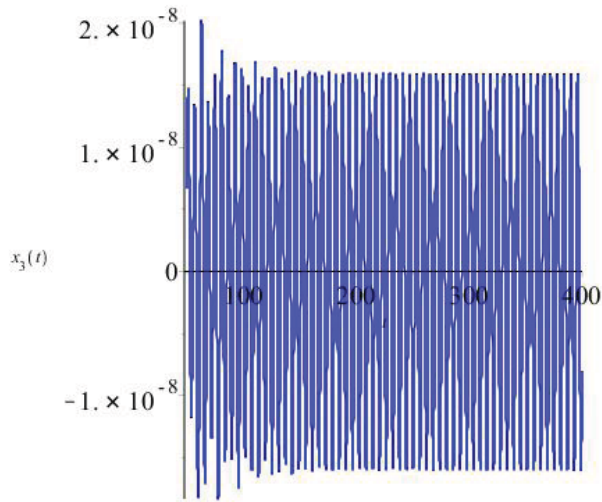


Figure 2: Rossler type of predator interaction

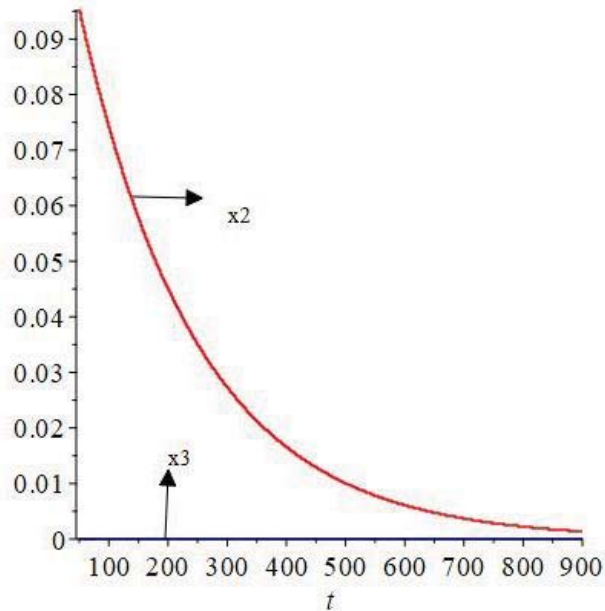


Figure 3: Analysis of Infected prey and predator interaction

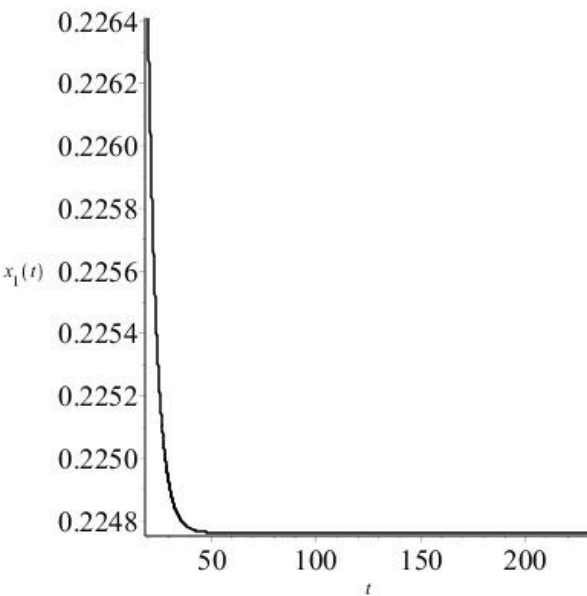


Figure 4: Rossler type of prey interaction

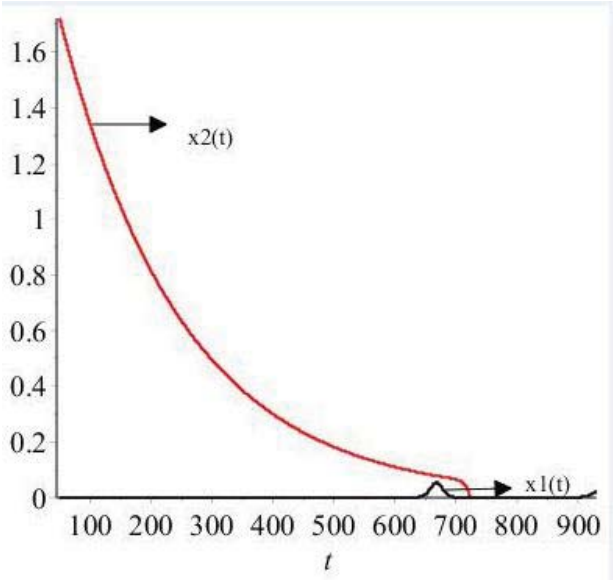


Figure 5: Analysis of susceptible-infected prey interaction

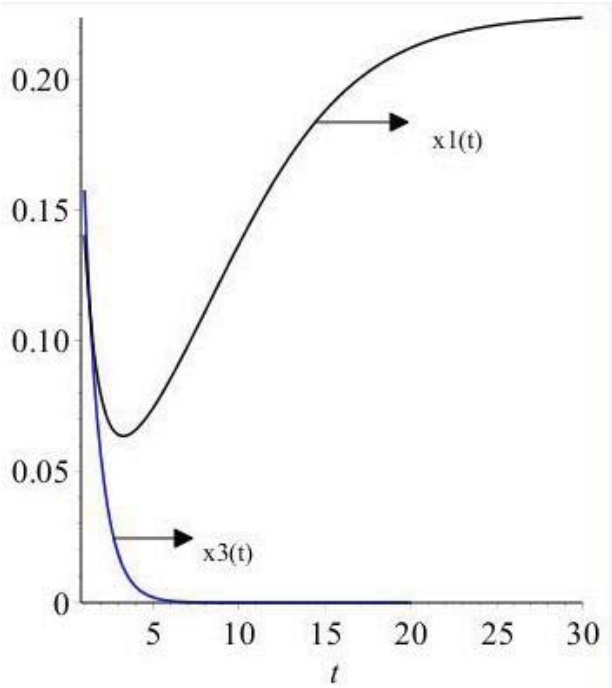


Figure 6: Analysis of susceptible prey and predator interaction

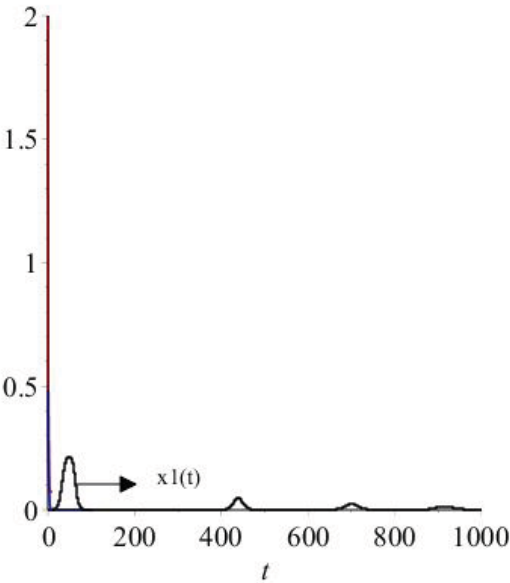


Figure 7: Analysis of prey-predator interaction

6 Discussions and Conclusions

An eco-epidemiological consisting of prey-predator model with S-I type of covid-19 disease in the S-I prey and predator population was proposed and analysed. It is observed that the system is positive and bounded has at most seven trivial, washed out, non trivial equilibrium points. The system of equations may exhibit chaotic oscillation and chaotic attractor that may arise due to Rossler system.

We further analysed that covid-19 model with stability analysis and following conclusion (see Table:2),

→the predator population only take place and it attains oscillatory dynamics (see Fig.2).

→the infected prey is decreasing and predator tends to zero value (see Fig.3).

→the susceptible prey is decreasing (see Fig.4).

→the susceptible prey tends to zero equilibrium value and infected prey tends to be decreasing (see Fig.5).

→the susceptible prey is increasing and predator range is decreasing (see Fig.6).

→In the interior points the susceptible prey is in fluctuations, the infected prey and predator are completely reduced to zero equilibrium value (see Fig.7).

Hence we analysed that the susceptible prey is decreasing when the predator acts more, if we isolate the sus. prey the predator and infected prey is totally reduced and sus. prey attains normal growth. We recommend the inf. population and predator should be separated from susceptible population to avoid the disease spread.

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