

Impact of Fear in a Predator-Prey Model with Crowley-Martin Response for Predation

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ABSTRACT

Predation process play the key role in the dynamics of predator-prey model. Fear in prey populations on account of predator's hunting is an important fact in predation process. The aim of this article is to find out the impact of fear among prey populations on the system behavior in a prey dependent predator-prey model of interacting populations. In this model we considered Crowley-Martin response function for predation and intra-specific competition among predator populations. We offer a detailed mathematical analysis of the proposed model to illustrate some of the significant results that has arisen from the interplay of deterministic ecological phenomena and processes. In particular, stability (local and global) and bifurcation (Hopf-Andronov) analysis of this model are conducted for biologically feasible parameters. We attained sufficient conditions to ensure the global stability of the unique positive equilibrium, by using appropriate Lyapunov functions and Lasalle invariance principle. Finally, numerical simulations through computer illustrate the dynamics of the ecological system

Keywords: Predator-prey model, Fear effect, Intra-specific competition, Stability, Bifurcation, Lyapunov function, Crowley-Martin functional response.

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Journal of Economic Literature (JEL) Classification : C63, P28, Q57

1. INTRODUCTION

Ecological systems are characterized by the interactions of different species within their natural environment. Studies of mathematical models are informative in understanding predator-prey interactions in these systems; as a result, predator-prey models have been important in ecological science since the early days of this discipline after the pioneering works of Lotka and Volterra. We start with the modified Lotka-Volterra type predator-prey system

$$\frac{dx}{dt} = rx - d_1x - h_1x^2 - mxy, \quad (1.1a)$$

$$\frac{dy}{dt} = emxy - d_2y, \quad (1.1b)$$

$$x(0) \geq 0, y(0) \geq 0, \quad (1.1c)$$

where $x(t)$ and $y(t)$ are the population densities of prey and predator respectively; r is the birth rate, d_1 is the death rate of prey ($r > d_1$), h_1 is the intra-specific competition among prey populations e ($0 < e < 1$) is the conversion factor, m is the predation rate of predator and d_2 is the death rate of predator. In 1969, Pielou modified the Lotka-Volterra predator-prey model (1.1) by incorporating intra-specific competition in predator population (Pielou, 1969). Bazykin studied the predator-prey model (1.1) with Holling type II functional response for predation process with intra-specific competition among predator (Bazykin, 1998). Haque studied the system (1.1) by incorporating ratio-dependent functional response and intra-specific competition among predator populations (Haque, 2009).

In 2016, (Wang, Zanette & Zou, 2016) first incorporate fear effect predator-prey model (1.1) and conclude that high level of fear can stabilize the system by excluding the existence of periodic solutions. But most of the researchers develop prey-dependent predator-prey model considering fear effect proposed by Wang now [cf. (Zhang, H., Cai, Y., Fu, S. & Wang, W. (2019)), (Panday, P., Samanta, S., Pal, N. & Chattopadhyay, J. (2018), Sarkar, K. & Khajanchi, S. (2020)]. Pal et. al. studied the predator-prey model with fear and hunting cooperation among predator (Pal, 2019). Sk et. al. studied the predator-prey model with fear, refuge and additional food for predator and conclude with local stability and some bifurcation situations (Sk, 2021). Influence of fear effect in ratio-dependent predator-prey model are studied in (Ali, 2022).

Crowley and Martin proposed a functional that can accommodate interference among predators [see Crowley, P.H., Martin, F. K. (1989)]. Functional responses and interference within and between year classes of a dragonfly population, Journal of the North American Benthological Society, 8(3): (211-221). This type of functional response is classified as one of predator-dependent functional response, i.e. they are functions of the abundance of both prey and predator due to predator interference. It is assumed that predator-feeding rate decreases by higher predator density even when prey density is high, and therefore the effects of predator interference on feeding rate remain important all the time whether an individual predator is handling or searching for a prey at a given instant of time. The per capita feeding rate in this formulation is given by

$$p(x, y) = \frac{mx}{1 + ax + by + abxy}$$

Where m , a and b are positive parameters that describe the effects of capture rate, handling time and the magnitude of interference among predators, respectively, on the feeding rate.

The objective of this paper is to analyze prey-predator system with fear effect due to predator's hunting process in a classical prey-predator model. We consider Crowley Martin response function in presence of intra-specific competition among predators.

The rest of this article is organized as follows. In Section 2, we present our mathematical model with Crowley Martin response function for predation. Some preliminary results such as existence and boundedness of the system solution in Section 3. Section 4 presents the behaviour of the system around all positive equilibrium points including local stability, global stability and bifurcation. Direction and stability of periodic orbit generated through hopf-bifurcation is discussed in Section 5. Section 6

supports analytical findings through numerical simulation. The article concludes with a discussion in Section 7.

2. THE MODEL:

According to the above assumptions, the model (1.1) becomes

$$\frac{dx}{dt} = \frac{rx}{1+ky} - d_1x - h_1x^2 - \frac{mxy}{1+ax+by+abxy}, \quad (2.1a)$$

$$\frac{dy}{dt} = \frac{emxy}{1+ax+by+abxy} - d_2y - h_2y^2, \quad (2.1b)$$

$$x(0) \geq 0, y(0) \geq 0 \quad (2.1c)$$

where the system parameters $r, k, d_1, h_1, m, a, e, d_2, h_2$ are all positive constants; $r, k, d_1, h_1, m, e, d_2$ are defined earlier in model (1.1); a is the saturation constant, h_2 is the intra-specific competition among predator populations.

3. PRELIMINARY RESULTS

3.1. Existence and positive invariance

For $t > 0$, letting $X \equiv (x, y)^T, F: R^2 \rightarrow R^2, F = (f_1, f_2)^T$, system (2.2) can be written as $\frac{dX}{dt} = F(X)$. Here $f_i \in C^\infty(R)$ for $i = 1, 2$; where $f_1 = \frac{rx}{1+ky} - d_1x - h_1x^2 - \frac{mxy}{1+ax+by+abxy}, f_2 = \frac{emxy}{1+ax+by+abxy} - d_2y - h_2y^2$.

Since the vector function F is a smooth function of the variables (x, y) in the positive quadrant

$\Omega = \{(x, y): x > 0, y > 0\}$, the local existence of the solution hold.

3.2. Boundedness results

Boundedness implies that the system is biologically well behaved. The following propositions ensure the boundedness of the system (2.1).

3.2.1. The prey population is always bounded from above.

Proof. From the first equation of the system (2.1) it can be shown that

$$\lim_{n \rightarrow \infty} \text{Sup } x(t) \leq \frac{r - d_1}{h_1} = x_0 \text{ (say).}$$

3.2.2. The predator population is bounded.

Proof. From the second equation of the system (2.1)

$$\begin{aligned} \frac{dy}{dt} &= \frac{mxy}{1+ax+by+abxy} - y - h_2y^2 \leq \frac{mxy}{(1+ax)(1+by)} - y \leq \frac{mx_0y}{1+ax_0} - y = \left(\frac{mx_0}{1+ax_0} - 1 \right) y \\ &\Rightarrow y(t) \leq y(0) \exp \left\{ \left(\frac{mx_0}{1+ax_0} - 1 \right) t \right\} \end{aligned}$$

which is finite for $t < \infty$. Hence the claim.

3.3. Equilibria and their feasibility

System (2.1) has the following three positive equilibria $E_i(x_i, y_i), i = 1, 2, 3$. E_0 is the origin, $E_1 \equiv (x_1, 0)$,

$E_2 \equiv (x_2, y_2)$, where $x_1 = \frac{r-d_1}{h_1}$ and x_2, y_2 are positive root of the equations

$$\begin{aligned} \frac{r}{1+ky} - d_1 - h_1x - \frac{mxy}{1+ax+by+abxy} &= 0 \\ \frac{emx}{1+ax+by+abxy} - d_2 - h_2y &= 0. \end{aligned}$$

4. Stability analysis

The Jacobian matrix of the system (2.1) at any arbitrary point is given by

$$J(x, y) = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}, \quad (4.1)$$

Where

$$\begin{aligned} J_{11} &= \frac{r}{1+ky} - d_1 - 2h_1x - \frac{my}{1+ax+by+abxy} + \frac{amxy(1+by)}{(1+ax+by+abxy)^2} \\ J_{12} &= -\frac{rkx}{(1+ky)^2} - \frac{mx}{1+ax+by+abxy} + \frac{bmxy(1+ax)}{(1+ax+by+abxy)^2} \\ J_{21} &= \frac{emy}{1+ax+by+abxy} - \frac{eamxy(1+by)}{(1+ax+by+abxy)^2} \\ J_{22} &= \frac{emx}{1+ax+by+abxy} - \frac{ebmxy(1+ax)}{(1+ax+by+abxy)^2} - d_2 - 2h_2y \end{aligned}$$

4.1. System behaviour near the origin

The Jacobian matrix at the equilibrium E_0 is $J_0 = \begin{bmatrix} r-d_1 & 0 \\ 0 & -d_2 \end{bmatrix}$.

The eigenvalues of the Jacobian matrix J_0 at E_0 are $r-d_1 > 0, -d_2 < 0$. Hence E_0 is unstable in nature.

4.2. System behaviour near the equilibrium $E_1(x_1, 0)$

(i) E_1 is locally asymptotically stable if $emx_1 < d_2(1+ax_1)$.

Proof: (i) The Jacobian matrix at the equilibrium E_1 is $J_1 = \begin{bmatrix} -r+d_1 & rkx_1 - \frac{mx_1}{(1+ax_1)} \\ 0 & \frac{emx_1}{(1+ax_1)} - d_2 \end{bmatrix}$.

The eigenvalues of the Jacobian matrix J_1 at E_1 are $-r+d_1, \frac{emx_1}{(1+ax_1)} - d_2$. Hence E_1 is locally asymptotically stable if the condition $emx_1 < d_2(1+ax_1)$ is satisfied.

4.3. System behaviour near the equilibrium $E_2(x_2, y_2)$

(i) E_2 is locally asymptotically stable if $amy_2 < h_1(1+ax_2)^2(1+by_2)$.

(ii) The system experiences Hopf-bifurcation around E_2 for $m = m^{[HB]}$, where

$$m^{[HB]} = \frac{(h_1x_2+h_2y_2)(1+ax_2)^2(1+by_2)^2}{x_2y_2\{a(1+by_2)-eb(1+ax_2)\}}.$$

(iii) Local stability of E_2 ensures its global stability if $amy_2 < h_1(1+ax_2)(1+by_2)$.

Proof: (i) The Jacobian matrix J_2 evaluated at E_2 is given by $J_2 = (c_{ij})_{2 \times 2}$, where

$$\begin{aligned} c_{11} &= -h_1x_2 + \frac{amx_2y_2}{(1+ax_2)^2(1+by_2)}, c_{12} = -\frac{rkx_2}{(1+ky_2)^2} - \frac{mx_2}{(1+ax_2)(1+by_2)^2} < 0, c_{21} = \frac{emy_2}{(1+ax_2)^2(1+by_2)} > 0, c_{22} = \\ &= -\frac{ebmx_2y_2}{(1+ax_2)(1+by_2)^2} - h_2y_2 < 0. \text{ Its eigenvalues are} \end{aligned}$$

$$\lambda_{1,2} = \frac{1}{2} \left[(c_{11} + c_{22}) \pm \sqrt{(c_{11} + c_{22})^2 - 4(c_{11}c_{22} - c_{12}c_{21})} \right]. \quad (4.2)$$

If we assume $c_{11} < 0$ then $\lambda_{1,2}$ both are either negative or complex numbers with negative real parts.

Hence, E_2 is locally asymptotically stable if $c_{11} < 0$ that is, $amy_2 < h_1(1+ax_2)^2(1+by_2)$.

(ii) From (4.2), we see that λ_1 and λ_2 are purely imaginary if and only if there is a $m = m^{[HB]}$ such that

$$m^{[HB]} = \frac{(h_1x_2 + h_2y_2)(1+ax_2)^2(1+by_2)^2}{x_2y_2\{a(1+by_2) - eb(1+ax_2)\}}.$$

But for $i = 1, 2$; $Re\left(\frac{d\lambda_i}{dm}\right)_{at[m=m^{[HB]}]} \neq 0$.

Therefore, the system enters into Hopf-bifurcation at E_2 for $m = m^{[HB]}$.

(iii) Let $R^2_+ = \{(x, y) \in R^2_+, x > 0, y > 0\}$ and consider the scalar function $L: R^2 \rightarrow R$ defined by

$$L = \left\{ (x - x_2) - x_2 \ln \frac{x}{x_2} \right\} + P \left\{ (y - y_2) - y_2 \ln \frac{y}{y_2} \right\}, \quad (4.3)$$

where P is a positive constant determined latter. The derivative of the above equation (4.3) along the solution of the system (2.1) is given by

$$\begin{aligned} \frac{dL}{dt} &= \left(1 - \frac{x_2}{x}\right) \frac{dx}{dt} + P \left(1 - \frac{y_2}{y}\right) \frac{dy}{dt} = \left(1 - \frac{x_2}{x}\right) \left[\frac{rx}{(1+ky)} - d_1x - h_1x^2 - \frac{mxy}{(1+ax)(1+by)} \right] + P \left(1 - \frac{y_2}{y}\right) \left[\frac{emy}{(1+ax)(1+by)} - d_2y - h_2y^2 \right] \\ &= (x - x_2) \left[\frac{r}{(1+ky)} - d_1 - h_1x - \frac{my}{(1+ax)(1+by)} \right] + P(y - y_2) \left[\frac{emx}{(1+ax)(1+by)} - d_2 - h_2y \right] \end{aligned} \quad (4.4)$$

At the equilibrium point E_2 of the system (2.1), we have

$$\frac{r}{(1+ky_2)} - d_1 - h_1x_2 - \frac{my_2}{(1+ax_2)(1+by_2)} = 0, \quad \frac{emx_2}{(1+ax_2)(1+by_2)} - d_2 - hy_2 = 0. \quad (4.5)$$

Using (4.5) the time derivative of L_2 becomes

$$\begin{aligned} \frac{dL}{dt} &= (x - x_2) \left[\frac{r}{(1+ky)} - \frac{r}{(1+ky_2)} - h_1(x - x_2) + \frac{my_2}{(1+ax_2)(1+by_2)} - \frac{my}{(1+ax)(1+by)} \right] + P(y - y_2) \left[\frac{emx}{(1+ax)(1+by)} - \frac{emx_2}{(1+ax_2)(1+by_2)} - h_2(y - y_2) \right] \\ &= (x - x_2)^2 \left[-h_1 + \frac{amy_2}{(1+ax)(1+ax_2)(1+by_2)} \right] \\ &\quad + (x - x_2)(y - y_2) \left[\frac{Pem}{(1+ax)(1+by)(1+ax_2)} - \frac{m}{(1+ax)(1+by)(1+by_2)} - \frac{rk}{(1+ky)(1+ky_2)} \right] + P(y - y_2)^2 \left[-h_2 - \frac{ebmx_2}{(1+by)(1+ax_2)(1+by_2)} \right] \\ &\leq (x - x_2)^2 \left[-h_1 + \frac{amy_2}{(1+ax_2)(1+by_2)} \right] + (x - x_2)(y - y_2) \left[\frac{Pem}{(1+ax_2)} - \frac{m}{(1+by_2)} - \frac{rk}{(1+ky_2)} \right] \\ &\quad + P(y - y_2)^2 \left[-h_2 - \frac{ebmx_2}{(1+ax_2)(1+by_2)} \right]. \end{aligned}$$

Now, we take the co-efficient of $(x - x_2) = 0$, that is, $\frac{Pem}{1+ax_2} - \frac{m}{1+by_2} - \frac{rk}{1+ky_2} = 0$, that is, $P =$

$$\frac{(1+ax_2)}{em} \left[\frac{m}{1+by_2} + \frac{rk}{1+ky_2} \right] > 0.$$

$$\begin{aligned} \frac{dL}{dt} &\leq (x - x_2)^2 \left[-h_1 + \frac{amy_2}{(1+ax_2)(1+by_2)} \right] - P(y - y_2)^2 \left[h_2 + \frac{ebmx_2}{(1+ax_2)(1+by_2)} \right] \\ &= (x - x_2)^2 \left[-h_1 + \frac{amy_2}{(1+ax_2)(1+by_2)} \right] - P(y - y_2)^2 \left[h_2 + \frac{ebmx_2}{(1+ax_2)(1+by_2)} \right] \\ &\leq -[a_{11}(x - x_2)^2 + a_{22}(y - y_2)^2] \end{aligned}$$

$$\leq 0 \text{ if } a_{11} > 0, \quad (4.6)$$

$$\text{where } a_{11} = \left[h_1 - \frac{amy_2}{(1+ax_2)(1+by_2)} \right], a_{22} = P \left[h_2 + \frac{ebmx_2}{(1+ax_2)(1+by_2)} \right],$$

and $\frac{dL}{dt} = 0$ when $(x, y) = (x_2, y_2)$. The proof follows from (4.6) and Lyapunov-Lasale invariance principle (Hale, 1969).

5. Stability and direction of Hopf-bifurcation

In this section, our attention is focused on investigation of the stability of the periodic solution bifurcating from a stable equilibrium $E_2(x_2, y_2)$. For this, we consider $u = x - x_2, v = y - y_2$. Then the system (2.1) reduces to

$$\begin{aligned} \frac{du}{dt} &= b_1u + b_2v + \sum b_{ij}u^i v^j, \\ \frac{dv}{dt} &= c_1u + c_2v + \sum c_{ij}u^i v^j, \end{aligned} \quad (5.1)$$

$$i \geq 0, j \geq 0, i + j \geq 2,$$

where

$$\begin{aligned} b_1 &= \frac{r}{1+ky_2} - d_1 - 2h_1x_2 - \frac{my_2}{(1+by_2)(1+ax_2)^2}, b_2 = -\frac{mx_2}{(1+ax_2)(1+by_2)^2}, \\ c_1 &= \frac{my_2}{(1+by_2)(1+ax_2)^2}, c_2 = \frac{mx_2}{(1+ax_2)(1+by_2)^2} - d_2 - 2h_2y_2, \\ b_{20} &= -h_1 + \frac{amy_2}{(1+by_2)(1+ax_2)^3}, b_{11} = -\frac{mx_2}{(1+ax_2)^2(1+by_2)^2}, b_{02} = \frac{bm x_2}{(1+ax_2)(1+by_2)^3}, \\ c_{20} &= -\frac{amy_2}{(1+by_2)(1+ax_2)^3}, c_{11} = \frac{m}{(1+by_2)(1+ax_2)}, c_{02} = -\frac{bm}{(1+ax_2)(1+by_2)^3} - h_2, \\ b_{30} &= -\frac{a^2my_2}{(1+by_2)(1+ax_2)^4}, b_{21} = \frac{am}{(1+ax_2)^3(1+by_2)^2}, b_{12} = \frac{bm}{(1+ax_2)^2(1+by_2)^3}, b_{03} = -\frac{b^2mx_2}{(1+by_2)^3}, \\ c_{30} &= \frac{a^2my_2}{(1+by_2)(1+ax_2)^4}, c_{21} = -\frac{am}{(1+ax_2)^3(1+by_2)^2}, c_{12} = -\frac{bm}{(1+ax_2)^2(1+by_2)^3}, c_{03} = \frac{b^2mx_2}{(1+by_2)^3}. \end{aligned}$$

Following the book of (Perko, 2006), one can obtain the Liyapunov number σ as follows:

$$\begin{aligned} \sigma &= -\frac{3\pi}{2b_2\Delta^{3/2}} \left[\{b_1c_1(b_{11}^2 + b_{11}c_{02} + b_{02}c_{11}) + b_1b_2(c_{11}^2 + b_{20}c_{11} + b_{11}c_{02}) + c_1^2(b_{11}b_{02} + 2b_{02}c_{02}) - \right. \\ &2b_1c_1(c_{02}^2 - b_{20}b_{02}) - 2b_1b_2(b_{20}^2 - c_{20}c_{02}) - b_2^2(2b_{20}c_{20} + c_{11}c_{20}) + (b_2c_1 - 2b_1^2)(c_{11}c_{02} - b_{11}b_{20})\} - \\ &\left. (b_1^2 + b_2c_1)\{3(c_1b_{03} - b_2b_{30}) + 2b_1(b_{21} + c_{12}) + (c_1b_{12} - b_2c_{21})\} \right] \quad (5.2) \end{aligned}$$

The bifurcating periodic solutions are stable (unstable) if $\sigma > 0$ ($\sigma < 0$). Hence, Hopf-bifurcation is supercritical (subcritical) if $\sigma > 0$ ($\sigma < 0$).

6. Numerical analysis

Theoretical analysis cannot be completed without numerical validation. In this article, we perform numerical simulation of the system (2.1) using Runge-Kutta 4th order method by Matlab R2010a and Mapple 16. Stable behaviour of the system around the equilibrium point E_1 is plotted in Figure 1 for the

set of parameter values $r = 5.0, k = 1.0, m = 4.0, e = 0.8, a = 1.5, b = 1.0, d_1 = 1.0, d_2 = 2.0, h_1 = 1.0, h_2 = 0.1$ where the stability condition $emx_1 = 12.8 < d_2(1 + ax_1) = 14$ is satisfied. Figure 2 represents the occurrence of hopf-bifurcation around the equilibrium point E_2 for the set of parameter values $r = 5.0, k = 0.9, m = 2.4, e = 0.8, a = 1.3, b = 0.3, d_1 = 0.3, d_2 = 0.3, h_1 = 0.1, h_2 = 0.1$. As the value of Liyapunov number $\sigma = 69.87799688 > 0$, the bifurcating periodic orbits are stable and the Hopf bifurcation is supercritical. Figure 3 shows the global stability of the system (2.1) around the equilibrium point E_2 for the set of parameter values $r = 5.0, k = 0.9, m = 3.5, e = 0.8, a = 1.5, b = 1.0, d_1 = 0.3, d_2 = 0.3, h_1 = 1.0, h_2 = 0.8$ where the stability conditions $amy_2 = 3.689049634 < h_1(1 + ax_2)^2(1 + by_2) = 21.91783300$ and $amy_2 = 3.689049634 < h_1(1 + ax_2)(1 + by_2) = 4.681648534$ are satisfied. Impact of fear on the system dynamics are plotted in Figure 4 which indicates that the system changes its behaviour from stable steady state to limit cycle, that is, significant level of fear has the destabilizing potential. Intra-specific competition among predators bear the stabilizing potential which facts are presented in Figure 5. Figure 6 shows that prey and predator population sizes are gradually decreasing with respect to the fear parameter k .

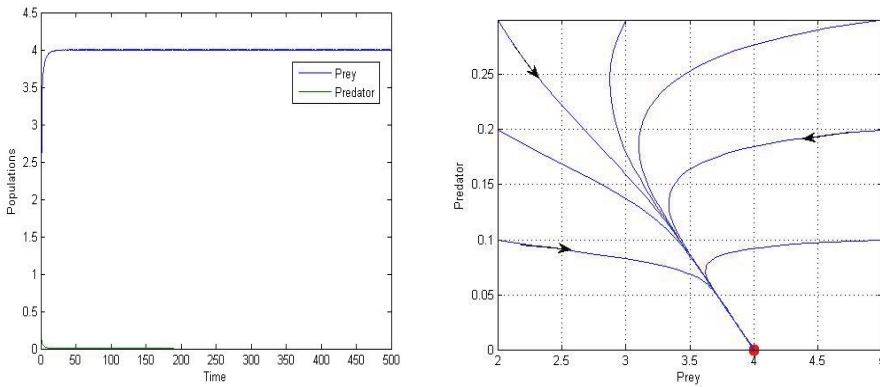


Figure 1: Stability behaviour around axial equilibrium position E_1 of the system (2.1) for the set of parameter values $r = 5.0, k = 1.0, m = 4.0, e = 0.8, a = 1.5, b = 1.0, d_1 = 1.0, d_2 = 2.0, h_1 = 1.0, h_2 = 0.1$.

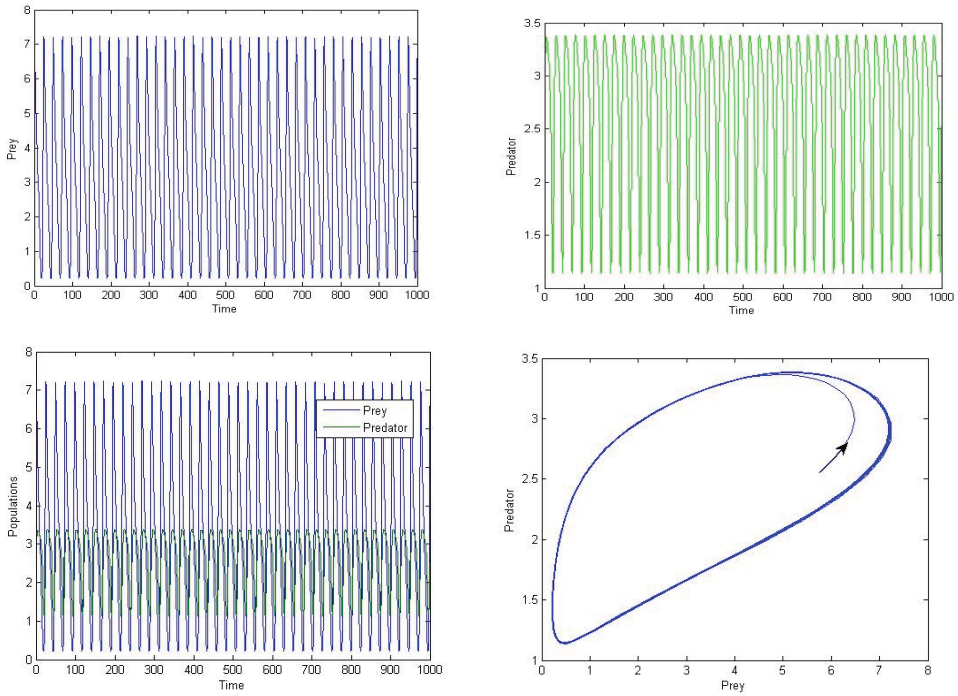


Figure 2: Hopf-bifurcation around interior equilibrium position E_2 of the system (2.1) for the set of parameter values $r = 5.0, k = 0.9, m = 2.4, e = 0.8, a = 1.3, b = 0.3, d_1 = 0.3, d_2 = 0.3, h_1 = 0.1, h_2 = 0.1$.

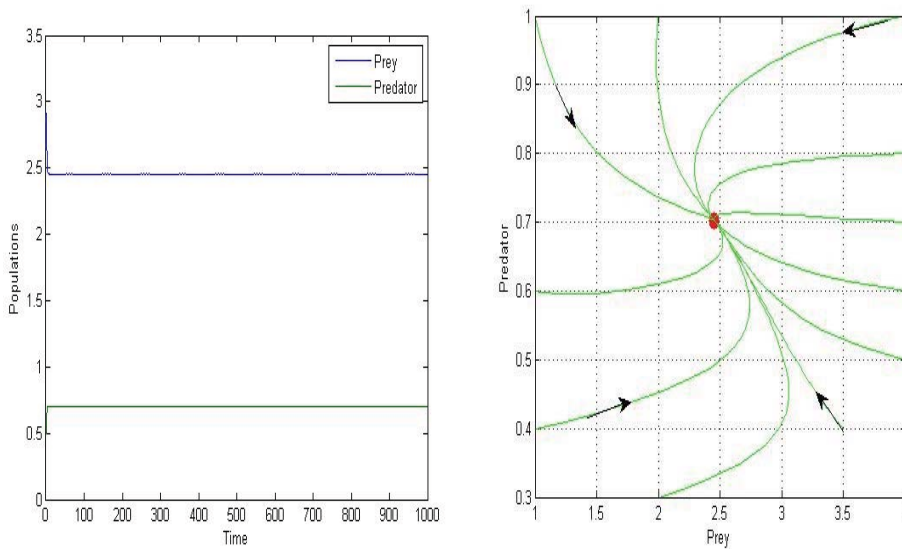


Figure 3: Global stability around interior equilibrium position E_2 of the system (2.1) for the set of parameter $r = 5.0, k = 0.9, m = 3.5, e = 0.8, a = 1.5, b = 1.0, d_1 = 0.3, d_2 = 0.3, h_1 = 1.0, h_2 = 0.8$.

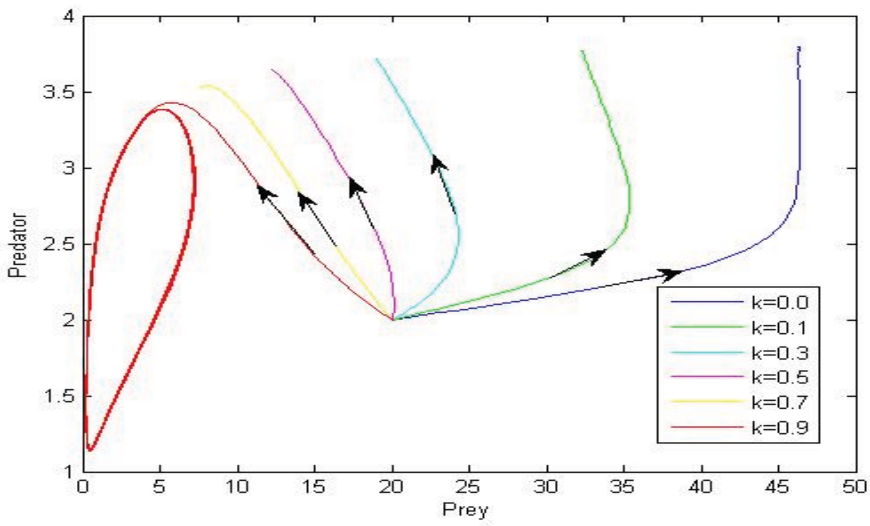


Figure 4: Impact of fear on the system behaviour around interior equilibrium position E_2 of the system (2.1) for different values of k . The other parameters are $r = 5.0$, $m = 3.5$, $e = 0.8$, $a = 1.5$, $b = 1.0$, $d_1 = 0.3$, $d_2 = 0.3$, $h_1 = 0.1$, $h_2 = 0.1$.

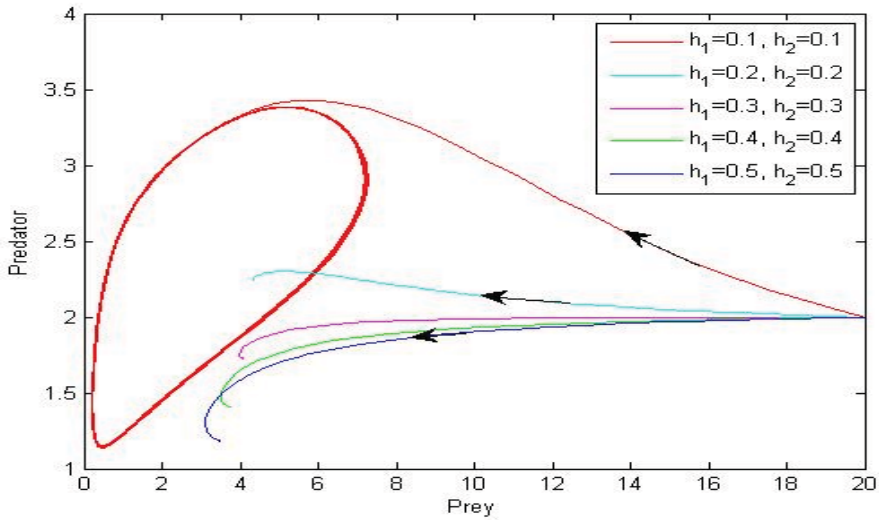


Figure 5: Impact of intra-specific competition among predators on the system behaviour around interior equilibrium position E_2 of the system (2.1) for different values of h_1 , h_2 . The other parameters are $r = 5.0$, $k = 0.9$, $m = 2.4$, $e = 0.8$, $a = 1.3$, $b = 0.3$, $d_1 = 0.3$, $d_2 = 0.3$.

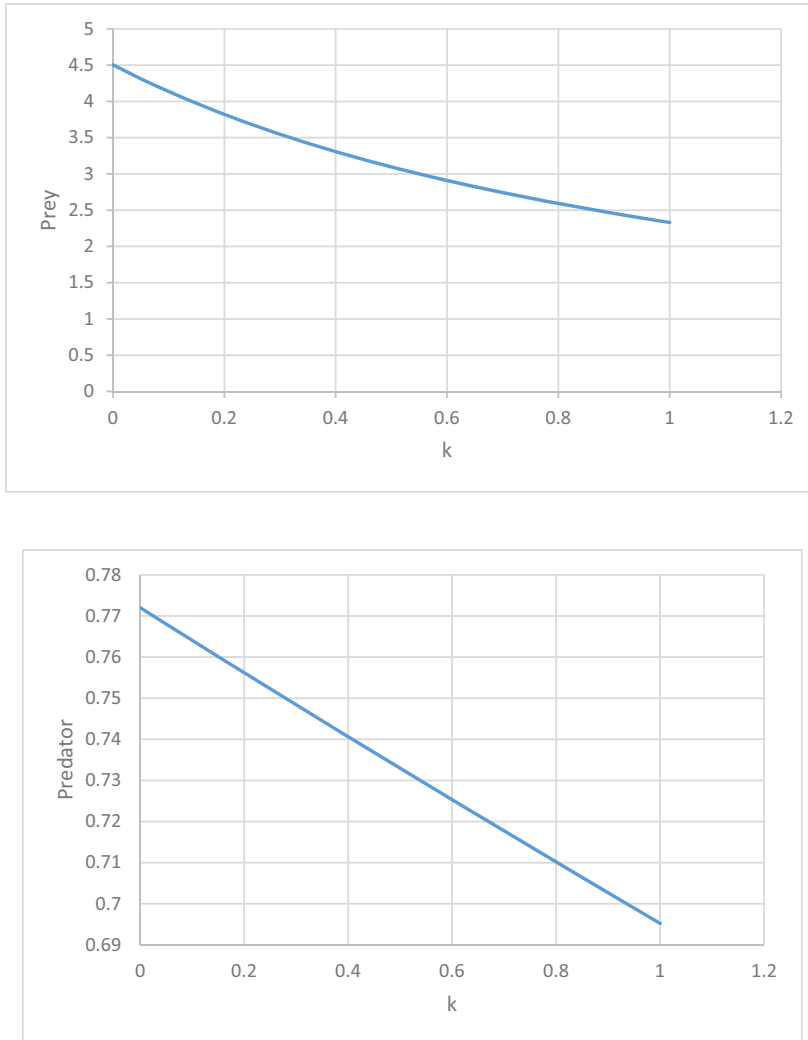


Figure 6: Influence of fear effect on the behaviour of the system (2.1) for the set of parameter values $r = 5.0$, $m = 3.5$, $e = 0.8$, $a = 1.5$, $b = 1.0$, $d_1 = 0.3$, $d_2 = 0.3$, $h_1 = 1.0$, $h_2 = 0.8$.

7. Discussion and conclusion

In this work, a bidimensional continuous-time differential equations system has analysed which is derived from a Logistic-type predator–prey model by considering a Crowley Martin functional response. Although bifurcations in a predator-prey model with Verhulst’s logistic growth rate have been studied by many researchers (Fan and Kuang, 2004), (Xiao and Ruan, 2001), (Hwang, 2003), (Bohner, Fan and Zhang, 2006), there is no article on the impact of fear effect in the prey’s mind due to predator’s hunting process of predator-prey model with Crowley-Martin functional response for predation process. For this model, we have dealt with local and global stability, stability of Hopf-bifurcation in an orderly manner. A complete classification of the equilibrium points, with respect to the various parameters based on their existence conditions, is provided. We have shown in Section 4 that the local and global

stability properties of the system (2.2) around respective equilibrium points and in Section 5 that the system experiences the Hopf-bifurcation as m crosses its critical value $m^{[HB]}$. The normal form theory and center manifold reduction have been made use of and we have derived the explicit formula which determine the stability property of bifurcating periodic solutions. It is observed that fear cannot make any impact on the system behavior when the birth rate of prey species is substantially low, but the level of fear and intra-specific competition among predators jointly affect the system behavior when the birth rate of prey species is substantially high. It is also noted that intra-specific competition among predators has stabilizing potential and fear has destabilizing potential in this model. At the end, it can be concluded that the observations that have been made in this article will help the experimental setups, and as a result, development of theoretical ecology will be enhanced in some measure in the near future.

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