

Mathematical Modelling and Numerical Simulation of COVID-19 Transmission Dynamics

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ABSTRACT

The spread of COVID-19 has grown into a pandemic with havoc worldwide desolation, thereby posing a huge threat to future global public health. The administration of all debilitated states has effectuated certain control measures. In the present study, a conceptual mathematical model is propounded where governmental actions are taken into account. Infection dynamics is contemplated and the enactment of viral environmental reservoir in the transmission of the disease is spotlighted in the current model. It is worth taking a look on the detailed analysis of the proposed model. Sensitized parametric variation is portrayed through graphical embodiment. Moreover, it is noteworthy to observe that both local and global stability analysis are performed analytically. Furthermore graphical representations have added the final touch up for clear understanding and visualization of the offshoots of this contemporaneous research. It may be concluded that though COVID-19 will coexist with mankind for long, it will let off its pandemic nature soon with strict administrative measures.

Keywords: COVID-19, Mathematical model, Control measures, Infection dynamics.

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1 Introduction

From the last year, the world has been shivering as an outcome of the pandemic caused by novel coronavirus (COVID-19). It has although started in Wuhan city of China in the month of December, 2019, as an outbreak of pneumonia, the cause was yet to be known. According to the report of World Health Organization (WHO) the real cause of this sudden epidemic is a novel coronavirus (SARS-Cov-2), named by the International Committee on Taxonomy of Viruses (ICTV) on 11 February 2020 and the corresponding disease caused due to it is COVID-19. The number of deaths occurred till date as a result of COVID-19 has surpassed the number of deaths caused by other two previously originated coronaviruses (Severe Acute Respiratory Syndrome Coronavirus, SARS-CoV and Middle East Respiratory Syndrome Coronavirus, MERS-CoV).

Based on genomic sequencing analysis, research and scientific community has concluded that two possible tracks are the most probable origins for SARS-CoV-2. One possibility is that the evolution of virus in the present pathogenic stage is through obvious selection in a non-human host and then to humans after direct contact with animals. This situation may be generally summed up as zoonotic transmission. Another possibility is that a non-pathogenic version of the virus got transmitted from an animal host into humans and then mutated to its present

pathogenic status in humans. There is a controversy regarding the asymptomatic transmission of the disease. As per the report published on January 30, there was an evidence of asymptomatic transmission (Rothe, Schunk, Sothmann, Bretzel, Froeschl and et al., 2020) but ultimately it was found to be fake (Kupferschmidt, 2020).

Typical symptoms and signs of COVID-19 include fever, body ache, dry cough, fatigue, chills, headache, sore throat, loss of appetite, and loss of smell. Sometimes COVID-19 causes acute symptoms like high fever, severe cough along with shortness of breath and bilateral lung infiltration in some people leading to pneumonia. It is noteworthy to mention that these symptoms are quite similar to that of the symptoms of SARS-CoV and MERS-CoV infections (Gralinski and Menachery, 2020). Certain rare symptoms of COVID-19 include nausea, vomiting and diarrhea.

The ongoing outbreak of COVID-19 throughout the world has compelled us to explore the history of various pandemic in past. In such process, the history of 1918 influenza epidemic in United Kingdom is investigated. It is worthy of mentioning that the mild symptoms shown by COVID-19 in most cases resemble with that of pandemic influenza rather than the two previously detected coronaviruses (Zhao, Gao, Zhuang, Chong, Cai and Ran, n.d.). In 1918, a large proportion of people died due to pneumonia, the consequence of influenza infection. Thus it is quite logical to consider the 1918 influenza pandemic model in order to frame the mathematical model of COVID-19.

Various models for COVID-19 have already been proposed by the scientific community in the recent past. A susceptible-exposed-infectious-recovered (SEIR) model has been used by Wu et al. (Wu, Leung and Leung, 2020) for conceptualizing the virus transmission dynamics. Tang et al. (Tang, Wang, Li, Bragazzi, Tang and et al., 2020) has developed a deterministic compartmental model by taking into account the progress of COVID-19 from medical point of view. Roosa et al. (Roosa, Lee, Luo, Kirpich, Rothenberg, Hyman, Yan and Chowell, 2020) has conducted a thorough study where three phenomenological models are proposed and are validated with epidemics other than COVID-19. In a very recent work of Lin et al. (Lin, Zhao, Gao, Lou, Yan, Musa, Wang, Cai, Wang, Yang and He, 93), individual reaction and governmental action effects are taken into consideration. The idea is favoured from the work of He et al. (He, Dushoff, Day, J and Earn, 2013) where individual reaction, holiday effects in addition to weather conditions are effectively conceptualized while modelling influenza-based death in London. Moreover, Gao et al. (Zhu, Gao, Li, C.Wang, Feng and et al., 2020) proposed an algorithm having depth enough to examine the contagiousness of novel coronavirus and thereby estimating its latent carriers. Most of the models developed so far have not considered the intimidating effect of environment in COVID-19 transmission. The cough and sneeze of infected persons spread virus to the environment and the infection may spread to exposed people. Such transmission was mainly prevalent in earlier times when there was no awareness about risk. Furthermore, there is a high possibility of survival of the virus for few days in the environment. Thus, this would increase the risk of contamination through respiratory droplets, surfaces and fomites (Cheng and Shan, 2020), (Sahin, Erdogan, Agaoglu, Dineri, Cakirci and et al., 2020). The existence of virus in the environment is confirmed by some reports (Geller, Varbanov and R. E. Duval, 2012). In a very up-to-date work depending on twenty two categories of coronavirus, it is divulged that coronaviruses such as SARS-CoV, MERS-CoV and endemic human coronaviruses may persist on fomites for nine days (Kampf, Todt, Pfaender and Steinmann, 2020). Quite a large number of mathematical models are being developed worldwide to predict the future of COVID-19. These estimations have wide-ranging effects in

planning governmental strategies to restrain the ongoing pandemic condition prevalent. It is really noteworthy that Yang and Wang (Yang and Wang, 2020) have advocated an updated yet simple mathematical model of COVID-19 by taking into account the major issues of multiple transmission pathways and an environmental compartment representing pathogen concentration.

In the present study, a more realistic and updated mathematical model is proposed where governmental measures in order to curb the spread of COVID-19 is also taken into consideration. In the formulation of the pandemic spread, various model parameters are necessitated. These parameters are sensitive and therefore if they can be controlled, the disease spread can also be taken under control. Considering this point of view, a sensitized parametric variation is conducted and the outcomes are demonstrated through graphical representations. Further, a stability analysis is performed to get some more necessary and important results which would finally benefit in estimating public health measures. It is worth mentioning that in order to delve deep into the analysis of stability of such a higher order system of ordinary differential equations, QSSA method of approximation is used. Some results are shown analytically while the numerical results are characterized through graphical illustrations. Global stability analysis is also performed analytically which adds support to the conjecture that COVID-19 would persist with reduced potential.

2 Formulation of Mathematical Model

It is adopted that the human population is divided into six compartments: the susceptible (denoted by S), the exposed (denoted by E), the infected (denoted by I), the recovered (denoted by R), the quarantined (denoted by Q) and the dead (denoted by D). Susceptible population consists of individuals who are prone to infection. Exposed population is the representation of individuals who come in contact with infection and they are in incubation period. Fully developed disease symptoms are seen in infected class and has the ability of infecting other people. Quarantined population consists of individuals who are infected and who have come in contact with infected people directly or indirectly. The susceptible population comes in contact with virus, with exposed people and with infected people and a fraction of it is now represented as exposed population. The susceptible population gets declined due to natural death (i.e. death which is not related to COVID-19). The instantaneous accumulation of exposed population depends on the interaction of susceptible population with virus, exposed and infected individuals. The decline of exposed population depends on natural death, viral infection and quarantine rate. The instantaneous accumulation of infected population depends on incubation rate, disease induced death rate, quarantine rate, recovery rate and natural death rate. So on and so forth, the instantaneous accumulation of recovered population depends on recovery rates, death rate due to recurring infection and natural death rate. The instantaneous accumulation of quarantined population depends on quarantine rate, disease induced mortality rate, recovery rate and natural death rate. The dead population depends on disease induced death rates and public reaction. Concentration of coronavirus in environmental reservoir depends on respective rates of exposed, infected and quarantined individuals contributing the coronavirus to the environmental reservoir. The detailed process of the spread of COVID-19 along with safety measures is schematically represented in Figure 1. Thus, the dynamics of the COVID-19 can be formulated as follows:

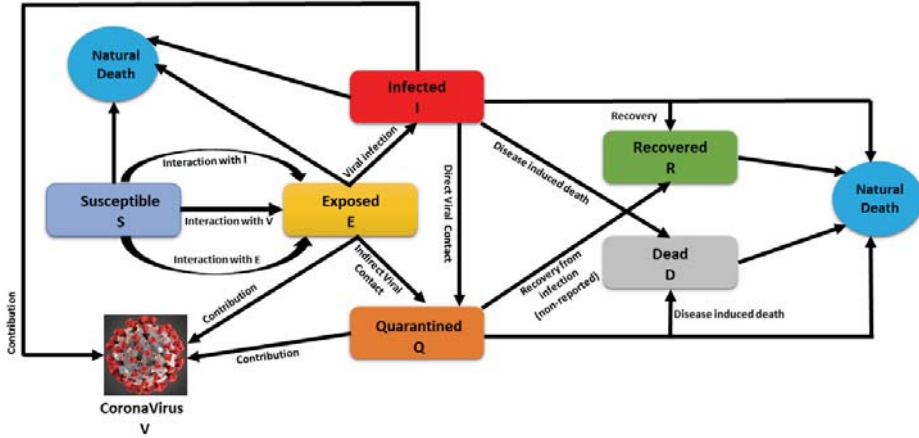


Figure 1: Schematic diagram

$$\frac{dS}{dt} = \Lambda - \beta_E SE - \beta_I SI - \beta_V SV - \mu S \quad (2.1)$$

$$\frac{dE}{dt} = \beta_E SE + \beta_I SI + \beta_V SV - (\alpha + q_E + \mu) E \quad (2.2)$$

$$\frac{dI}{dt} = \alpha E - (\omega + \gamma_I + q_I + \mu) I \quad (2.3)$$

$$\frac{dR}{dt} = \gamma_I I + \gamma_Q Q - \mu R - pR \quad (2.4)$$

$$\frac{dQ}{dt} = q_E E + q_I I - kQ - \mu Q - \gamma_Q Q \quad (2.5)$$

$$\frac{dD}{dt} = kQ + \omega I - \lambda D \quad (2.6)$$

$$\frac{dV}{dt} = \xi_1 E + \xi_2 I + \xi_3 Q - \sigma V \quad (2.7)$$

where S , E , I , R , Q and D represent susceptible population, exposed population, infected population, recovered population, quarantined population and dead population respectively. V represents Coronavirus (SARS-CoV-2) in the environment reservoir. Λ depicts the population influx, μ represents the natural death rate of human hosts. β_E , β_I , β_V are the transmission rates. p stands for the death rate due to recurring infection, ω is the disease-induced death rate. $\frac{1}{\alpha}$ represents incubation period between the infection and the onset of symptoms whereas $\frac{1}{\lambda}$ stands for the mean duration of public reaction. ξ_1 , ξ_2 and ξ_3 are the respective rates of exposed, infected and quarantined individuals contributing the coronavirus to the environmental reservoir. q_E and q_I are the respective quarantine rates of exposed and infected population. γ_I and γ_Q are the respective recovery rates of infected and quarantined population. k is the mortality rate and σ represents the removal rate of virus from the environment. Now, the transmission rates are defined as follows:

$\beta_E = \frac{\beta_{E0}}{1 + cE}$, $\beta_I = \frac{\beta_{I0}}{1 + cI}$ and $\beta_V = \frac{\beta_{V0}}{1 + cV}$, where β_{E0} , β_{I0} and β_{V0} are the maximum values of the respective transmission rates in addition to c depicting the positive coefficient acting as adjustment factor to the respective transmission rates.

3 Local Stability Analysis

For the sake of convenience the set of equations (2.1) - (2.7) is rewritten in the following manner:

$$\frac{dS}{dt} = b_{11} - b_{12} \frac{SE}{1+cE} - b_{13} \frac{SI}{1+cI} - b_{14} \frac{SV}{1+cV} - b_{15}S \quad (3.1)$$

$$\frac{dE}{dt} = b_{22} \frac{SE}{1+cE} + b_{23} \frac{SI}{1+cI} + b_{24} \frac{SV}{1+cV} - b_{21}E \quad (3.2)$$

$$\frac{dI}{dt} = b_{31}E - b_{32}I \quad (3.3)$$

$$\frac{dR}{dt} = b_{41}I + b_{42}Q - b_{43}R \quad (3.4)$$

$$\frac{dQ}{dt} = b_{51}E + b_{52}I - b_{53}Q \quad (3.5)$$

$$\frac{dD}{dt} = b_{61}Q + b_{62}I - b_{63}D \quad (3.6)$$

$$\frac{dV}{dt} = b_{71}E + b_{72}I + b_{73}Q - b_{74}V \quad (3.7)$$

where the values of the parameters are quite easy to figure on. Here, $b_{11} = \Lambda$, $b_{12} = \beta_{E0}$, $b_{13} = \beta_{I0}$, $b_{14} = \beta_{V0}$, $b_{15} = \mu$, $b_{21} = \alpha + \mu + q_E$, $b_{22} = b_{12}$, $b_{23} = b_{13}$, $b_{24} = b_{14}$, $b_{31} = \alpha$, $b_{32} = \omega + \mu + \gamma + q_I$, $b_{41} = \gamma_I$, $b_{42} = \gamma_Q$, $b_{43} = \mu + p$, $b_{51} = q_E$, $b_{52} = q_I$, $b_{53} = \mu + \gamma_Q + k$, $b_{61} = k$, $b_{62} = \omega$, $b_{63} = \lambda$, $b_{71} = \xi_1$, $b_{72} = \xi_2$, $b_{73} = \xi_3$ and lastly $b_{74} = \sigma$.

The following theorem shows that under certain suitable conditions, all the solutions of the system of equations (3.1) - (3.7) are non-negative. A set Γ in $\mathbb{R}_+^7$ may be ascertained such that all solutions starting in Γ remain bounded.

Theorem 3.1. *Let all the parameters of the system of equations (3.1) - (3.7) be positive and Γ be a region in $\mathbb{R}_+^7$ defined as $\Gamma = \{(S, E, I, R, Q, D, V) \in \mathbb{R}_+^7 | 0 \leq S \leq \bar{S}, 0 \leq E \leq \bar{E}, 0 \leq I \leq \bar{I}, 0 \leq R \leq \bar{R}, 0 \leq Q \leq \bar{Q}, 0 \leq D \leq \bar{D}, 0 \leq V \leq \bar{V}\}$. Then Γ is positive invariant and all the solutions starting from Γ are uniformly bounded, the parameters over bar are being the respective upper bounds.*

Proof. First we make an attempt to prove the positive invariant part.

Let $S(0), E(0), I(0), R(0), Q(0), D(0), V(0) \in \Gamma$. If possible, suppose $S(t)$ be non-positive. Then $\exists t_0 > 0$ such that $S(t_0) = 0$ and $S(t) > 0$ for any t satisfying $0 \leq t \leq t_0$. Then necessarily $\left. \frac{dS}{dt} \right|_{t=t_0} \leq 0$.

This is a contradiction, because

$$\begin{aligned} \left. \frac{dS}{dt} \right|_{t=t_0} &= b_{11} - b_{12} \left(\frac{S(t_0)E(t_0)}{1+cE(t_0)} \right) - b_{13} \left(\frac{S(t_0)I(t_0)}{1+cI(t_0)} \right) - b_{14} \left(\frac{S(t_0)V(t_0)}{1+cV(t_0)} \right) - b_{15}S(t_0) \\ &= b_{11} > 0 \end{aligned}$$

where $b_{11} = \Lambda$, $b_{12} = \beta_{E0}$, $b_{13} = \beta_{I0}$, $b_{14} = \beta_{V0}$, $b_{15} = \mu$. Hence $S(t)$ is positive $\forall t \geq 0$.

Next suppose $E(t)$ be non-positive. Then $\exists t_0 > 0$ such that $E(t_0) = 0$ and $E(t) > 0$ for any t satisfying $0 \leq t \leq t_0$. Then necessarily $\left. \frac{dE}{dt} \right|_{t=t_0} \leq 0$. This is a contradiction, because

$$\begin{aligned} \left. \frac{dE}{dt} \right|_{t=t_0} &= -b_{21}E(t_0) + b_{22} \left(\frac{S(t_0)E(t_0)}{1+cE(t_0)} \right) + b_{23} \left(\frac{S(t_0)I(t_0)}{1+cI(t_0)} \right) + b_{24} \left(\frac{S(t_0)V(t_0)}{1+cV(t_0)} \right) \\ &= b_{23} \left(\frac{S(t_0)I(t_0)}{1+cI(t_0)} \right) + b_{24} \left(\frac{S(t_0)V(t_0)}{1+cV(t_0)} \right) > 0 \end{aligned}$$

where $b_{21} = (\alpha + \mu + q_E)$, $b_{22} = \beta_{E0}$, $b_{23} = \beta_{I0}$, $b_{24} = \beta_{V0}$. Hence $E(t)$ is positive $\forall t \geq 0$. Similarly, positivity of the remaining variables, namely $I(t)$, $R(t)$, $Q(t)$, $D(t)$ and $V(t)$ can be shown and hence these proofs are not being provided here.

Next, the part of boundedness may be shown as follows:

From equation 3.1, one may have

$$\begin{aligned}\frac{dS}{dt} &= b_{11} - b_{12}\left(\frac{SE}{1+cE}\right) - b_{13}\left(\frac{SI}{1+cI}\right) - b_{14}\left(\frac{SV}{1+cV}\right) - b_{15}S \\ &\Rightarrow \frac{dS}{dt} + b_{15}S \leq b_{11} \\ &\Rightarrow S(t) \leq \frac{b_{11}}{b_{15}} + K_1 e^{-b_{15}t}\end{aligned}$$

where K_1 is a +ve integration constant. The term $K_1 e^{-b_{15}t}$ vanishes as $t \rightarrow \infty$. So, $S(t) \leq \frac{b_{11}}{b_{15}}$ as $t \rightarrow \infty$, i.e. $S(t)$ remains bounded for all $t \geq 0$.

From equation 3.2, one may have

$$\begin{aligned}\frac{dE}{dt} + b_{21}E &= b_{22}\left(\frac{SE}{1+cE}\right) + b_{23}\left(\frac{SI}{1+cI}\right) + b_{24}\left(\frac{SV}{1+cV}\right) \\ &\Rightarrow \frac{dE}{dt} + b_{21}E \leq B, \text{ where } B = \max\left\{b_{22}\left(\frac{SE}{1+cE}\right), b_{23}\left(\frac{SI}{1+cI}\right), b_{24}\left(\frac{SV}{1+cV}\right)\right\} \\ &\Rightarrow E(t) \leq \frac{B}{b_{21}} + K_2 e^{-b_{21}t}\end{aligned}$$

where K_2 is a +ve integration constant. The term $K_2 e^{-b_{21}t}$ vanishes as $t \rightarrow \infty$. So, $E(t) \leq \frac{B}{b_{21}}$ as $t \rightarrow \infty$, i.e. $E(t)$ remains bounded for all $t \geq 0$.

Similarly, one can prove the boundedness of $I(t)$, $R(t)$, $Q(t)$, $D(t)$ and $V(t)$. $\square$

Theorem 3.2. *The system (3.1) - (3.7) under consideration is locally asymptotically stable at the equilibrium position if $u_{i1} > 0$ for $i = 0, 1, 2, 3, \dots, 6$. The details of $u_{i1} > 0$ for $i = 0, 1, 2, 3, \dots, 6$ are given in the proof.*

Proof. To prove the system (3.1) - (3.7) locally stable, one needs to consider the corresponding linearised system. The Jacobian matrix may be found as follows:

$$J^* = \begin{bmatrix} -j_{11}^* - j_{12}^* - j_{13}^* & 0 & 0 & 0 & -j_{17}^* \\ j_{21}^* - j_{22}^* & j_{23}^* & 0 & 0 & j_{27}^* \\ 0 & b_{31} & -b_{32} & 0 & 0 \\ 0 & 0 & b_{41} & -b_{43} & b_{42} & 0 & 0 \\ 0 & b_{51} & b_{52} & 0 & -b_{53} & 0 & 0 \\ 0 & 0 & b_{62} & 0 & b_{61} & -b_{63} & 0 \\ 0 & b_{71} & b_{72} & 0 & b_{73} & 0 & -b_{74} \end{bmatrix}$$

$$\text{where } j_{11}^* = \frac{b_{12}E^*}{1+cE^*} + \frac{b_{13}I^*}{1+cI^*} + \frac{b_{14}V^*}{1+cV^*} + b_{15},$$

$$j_{12}^* = b_{12}\left(\frac{S^*}{1+cE^*} - \frac{cS^*E^*}{(1+cE^*)^2}\right),$$

$$j_{13}^* = b_{13}\left(\frac{S^*}{1+cI^*} - \frac{cS^*I^*}{(1+cI^*)^2}\right),$$

$$j_{17}^* = b_{14}\left(\frac{S^*}{1+cV^*} - \frac{cS^*V^*}{(1+cV^*)^2}\right),$$

$$j_{21}^* = \frac{b_{12}E^*}{1+cE^*} + \frac{b_{13}I^*}{1+cI^*} + \frac{b_{14}V^*}{1+cV^*},$$

$$j_{22}^* = b_{21} - b_{12}\left(\frac{S^*}{1+cE^*} - \frac{cS^*E^*}{(1+cE^*)^2}\right),$$

$$J_{23}^* = b_{13} \left(\frac{S^*}{1 + cI^*} - \frac{cS^*I^*}{(1 + cI^*)^2} \right),$$

$$J_{27}^* = b_{14} \left(\frac{S^*}{1 + cV^*} - \frac{cS^*V^*}{(1 + cV^*)^2} \right).$$

After evaluating the elements of the Jacobian matrix at the positive equilibrium positions of the system (3.1) - (3.7), one gets the characteristic polynomial at each of these equilibria. As the calculation is quite complex, so here a general approach is provided to prove the stability of the system under study. Let us denote the Jacobian matrix obtained at equilibrium position by J^* . Suppose the characteristic polynomial of J^* may be expressed as $x^7 + c_6x^6 + c_5x^5 + c_4x^4 + c_3x^3 + c_2x^2 + c_1x + c_0 = g(x)$, where $c_k = (-1)^{7-k} \sum_{|D|=7-k} J^*[D]$, i.e. c_k are the sum of all principal minors of order $(7-k)$ of the matrix J^* multiplied by $(-1)^{7-k}$. Here $D \subseteq \{1, 2, \dots, 7\}$ and $J^*[D]$ denotes the principal minors of J^* formed by the columns and rows with indices from D . Let us denote $1 = u_{71}$, $c_5 = u_{72}$, $c_3 = u_{73}$, $c_1 = u_{74}$, $c_6 = u_{61}$, $c_4 = u_{62}$, $c_2 = u_{63}$, $c_0 = u_{64}$. Then Routh table of the polynomial $g(x)$ may be formed as in the following table:

here $u_{ij} = \frac{u_{(i+1)j}u_{(i+2)(j+1)} - u_{(i+1)(j+1)}u_{(i+2)j}}{u_{(i+1)j}}$, for $i = 0, 1, 2, \dots, 7$ and $j = 1, 2, 3, 4$.

x^7	u_{71}	u_{72}	u_{73}	u_{74}
x^6	u_{61}	u_{62}	u_{63}	u_{64}
x^5	u_{51}	u_{52}	u_{53}	0
x^4	u_{41}	u_{42}	u_{43}	0
x^3	u_{31}	u_{32}	0	0
x^2	u_{21}	u_{22}	0	0
x	u_{11}	0	0	0
1	u_{01}	0	0	0

Routh-Hurwitz criterion states that a system is stable if all the terms of the first column are positive and if these conditions are not satisfied then there will be an unstable solution. Hence, the model in (3.1-3.7) is locally asymptotically stable at the positive equilibrium if $u_{i1} > 0$ for $i = 0, 1, 2, \dots, 7$. $\square$

3.1 Reduced Model

In order to understand the dynamics of the fate of COVID-19 in depth, the model is modified in a simple way taking the help of quasi-steady-state approximations (QSSA) (Louzoun, Xue, Lesinski and Friedman, 2014), (Li and Xu, 2016) for the quarantined population (Q), recovered population (R), Dead population (D) and Coronavirus (SARS-CoV-2) concentration (V). In order to smoothen the deal with higher dimensional ordinary differential model, a mathematical perspective, namely, QSSA is used. QSSA is a general method for studying reaction kinetics, where there is a difference in time-scales and different aspects of the model can be characterized as fast or slow process. Moreover, some species which are considered as less significant with respect to others, take part in QSSA. From the timeseries plot it is very clear that recovered population (R) and dead population (D) are meagre with respect to others. Furthermore, quarantined population (Q) is having faster time-scale and Coronavirus (SARS-CoV-2) concentration (V) is relatively less significant with respect to human population facing COVID-19. Using

QSSA, the reduced set of equations are as follows:

$$\frac{dS}{dt} = \Lambda - \beta_E SE - \beta_I SI - \beta_V \frac{S}{\sigma} \left[\left(\xi_1 + \frac{\xi_3 q_E}{k + \mu + \gamma_Q} \right) E + \left(\xi_2 + \frac{\xi_3 q_I}{k + \mu + \gamma_Q} \right) I \right] - \mu S \quad (3.8)$$

$$\frac{dE}{dt} = \beta_E SE + \beta_I SI + \beta_V \frac{S}{\sigma} \left[\left(\xi_1 + \frac{\xi_3 q_E}{k + \mu + \gamma_Q} \right) E + \left(\xi_2 + \frac{\xi_3 q_I}{k + \mu + \gamma_Q} \right) I \right] - (\alpha + \mu + q_E) E \quad (3.9)$$

$$\frac{dI}{dt} = \alpha E - (\omega + \gamma_I + q_I + \mu) I \quad (3.10)$$

The equilibrium points of the reduced system (3.8) - (3.10) are

E_{11} (9.010963455×10^6 , 0, 0) and E_{12} (2.741243068×10^6 , 1158.584186, 752.2244755).

Theorem 3.3. *The reduced system (3.8) - (3.10) is locally asymptotically stable at E_{12} if $a_1 > 0$, $a_3 > 0$ and $a_1 a_3 - a_2^2 > 0$.*

Proof. To prove the system (3.8) - (3.10) locally stable, one needs to consider the corresponding linearised system. The Jacobian matrix at the point E_{12} may be found as follows:

$$J_2^* = \begin{bmatrix} -A_{11} & -A_{12} & -A_{13} \\ A_{21} & -A_{22} & A_{13} \\ 0 & a_{16} & -a_{17} \end{bmatrix}$$

where $A_{11} = E^* \left(\frac{\beta_{E0}}{1 + c E^*} + \frac{d_1}{1 + d_2 E^* + d_3 I^*} \right) + I^* \left(\frac{\beta_{I0}}{1 + c I^*} + \frac{e_1}{1 + d_2 E^* + d_3 I^*} \right) + a_{14}$,
 $A_{12} = -J_{12}(S^*, E^*, I^*)$, $A_{13} = -J_{13}(S^*, E^*, I^*)$, $A_{21} = J_{21}(S^*, E^*, I^*)$ and
 $A_{22} = J_{12}(S^*, E^*, I^*) + a_{15}$.

Now the intermediate notations are as follows:

$$\begin{aligned} d_1 &= \frac{\beta_{V0}}{\sigma} \left(\xi_1 + \frac{\xi_3 q_E}{k + \mu + \gamma_Q} \right), d_2 = \frac{c}{\sigma} \left(\xi_1 + \frac{\xi_3 q_E}{k + \mu + \gamma_Q} \right), d_3 = \left(\xi_2 + \frac{\xi_3 q_I}{k + \mu + \gamma_Q} \right), a_{14} = \mu, \\ J_{12} &= -S \left[\left(\frac{\beta_{E0}}{1 + c E} + \frac{d_1}{1 + d_2 E + d_3 I} \right) - E \left\{ \frac{c \beta_{E0}}{(1 + c E)^2} + \frac{d_1 d_2}{(1 + d_2 E + d_3 I)^2} \right\} - \frac{e_1 d_2}{(1 + d_2 E + d_3 I)^2} I \right], \\ e_1 &= \frac{\beta_{V0}}{\sigma} \left(\xi_2 + \frac{\xi_3 q_I}{k + \mu + \gamma_Q} \right), a_{15} = \alpha + \mu + q_E, a_{16} = \alpha, a_{17} = \omega + \gamma_I + \mu + q_I, \\ J_{13} &= -S \left[\frac{\beta_{I0}}{1 + c I} - \frac{d_1 d_3 E}{(1 + d_2 E + d_3 I)^2} + \frac{e_1}{1 + d_2 E + d_3 I} - I \left\{ \frac{c \beta_{I0}}{(1 + c I)^2} + \frac{e_1 d_3}{(1 + d_2 E + d_3 I)^2} \right\} \right], \\ J_{21} &= E \left(\frac{\beta_{E0}}{1 + c E} + \frac{d_1}{1 + d_2 E + d_3 I} \right) + I \left(\frac{\beta_{I0}}{1 + c I} + \frac{e_1}{1 + d_2 E + d_3 I} \right). \end{aligned}$$

The characteristic equation is $\lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3 = 0$, λ being the eigen values. The used parameters are defined as follows:

$a_1 = A_{11} + A_{22} + a_{17}$, $a_2 = a_{17} A_{22} + A_{12} A_{21} - a_{16} A_{13} + A_{11} a_{17} + A_{11} A_{22}$ and
 $a_3 = a_{17} A_{11} A_{22} - a_{16} A_{13} A_{11} + a_{17} A_{12} A_{21} + a_{16} A_{21} A_{13}$.

If $a_1 > 0$, $a_3 > 0$ and $a_1 a_3 - a_2^2 > 0$ then the eigen values will be all negative thereby making the reduced system locally asymptotically stable at E_{12} . $\square$

4 Global Stability

Theorem 4.1. *The present model system represented by (2.1) - (2.7) is globally asymptotically stable at the interior equilibrium position $(S^*, E^*, I^*, R^*, Q^*, D^*, V^*)$ if the following conditions hold based on the assumption that $\beta_E(E)$, $\beta_I(I)$ and $\beta_V(V)$ are non decreasing function of the variables E , I and V respectively:*

$b_1 I^* + (1 - b_2) \beta_I^* I^* S^* < b_2 (\xi_1 E^* + \xi_2 I^* + \xi_3 Q^*)$ and $a_2 Q^* < q_E E^* + q_I I^*$,
 where $b_1 = \gamma_I + q_I + \omega + \xi_2$, $b_2 = 1/(\alpha + q_E + \xi_1)$ and $a_2 = k + \gamma_Q + \xi_3$.

Proof. Let us consider the function $L(X) = \int_{X^*}^X \frac{X - X^*}{X} dX$ for $X > 0$, $X^* > 0$ and X can be treated as a representative of the members of all the classes of individuals. Obviously $L(X) \geq 0$ with the equality retaining if and only if $X = X^*$. Differentiating the above function corresponding to all the members characterizing the entire population and making use of the equilibrium relations of the system, one may obtain the following:

$$\begin{aligned} \frac{dL(S)}{dt} &= \left(\frac{S - S^*}{S} \right) \frac{dS}{dt} \\ &\leq \beta_E^* S^* E^* \left(1 - \frac{S^*}{S} - \frac{\beta_E E S}{\beta_E^* E^* S^*} + \frac{\beta_E E}{\beta_E^* E^*} \right) + \beta_I^* S^* I^* \left(1 - \frac{S^*}{S} - \frac{\beta_I I S}{\beta_I^* I^* S^*} + \frac{\beta_I I}{\beta_I^* I^*} \right) \\ &\quad + \beta_V^* S^* V^* \left(1 - \frac{S^*}{S} - \frac{\beta_V V S}{\beta_V^* V^* S^*} + \frac{\beta_V V}{\beta_V^* V^*} \right) \\ \frac{dL(E)}{dt} &= \left(\frac{E - E^*}{E} \right) \frac{dE}{dt} \\ &\leq \beta_E^* S^* E^* \left(1 - \frac{S \beta_E}{S^* \beta_E^*} + \frac{\beta_E E S}{\beta_E^* E^* S^*} - \frac{E}{E^*} \right) + \beta_I^* S^* I^* \left(1 - \frac{E}{E^*} + \frac{\beta_I I S}{\beta_I^* I^* S^*} - \frac{\beta_I I S E^*}{\beta_I^* I^* S^* E} \right) \\ &\quad + \beta_V^* S^* V^* \left(1 - \frac{E}{E^*} + \frac{\beta_V V S}{\beta_V^* V^* S^*} - \frac{\beta_V V S E^*}{\beta_V^* V^* S^* E} \right) \end{aligned}$$

Now

$$\frac{dL(S)}{dt} + \frac{dL(E)}{dt} \leq \beta_I^* S^* I^* \left(\frac{I}{I^*} - \frac{E}{E^*} - \ln \left(\frac{I}{I^*} \right) + \ln \left(\frac{E}{E^*} \right) \right) + \beta_V^* S^* V^* \left(\frac{V}{V^*} - \frac{E}{E^*} + \ln \left(\frac{E}{E^*} \right) - \ln \left(\frac{V}{V^*} \right) \right),$$

where the last inequality follows from the assumption that

$$\left(1 - \frac{\beta_E^*}{\beta_E} \right) \leq 0 \Rightarrow E^* \leq E \Rightarrow \left(\frac{\beta_E E}{\beta_E^* E^*} - 1 \right) \geq 0, \text{ etc.}$$

Similarly one can verify that

$$\begin{aligned} \frac{dL(I)}{dt} &= \left(\frac{I - I^*}{I} \right) \frac{dI}{dt} \leq \alpha E^* \left[\frac{E}{E^*} - \frac{I}{I^*} + \ln \left(\frac{I}{I^*} \right) - \ln \left(\frac{E}{E^*} \right) \right], \\ \frac{dL(R)}{dt} &= \left(\frac{R - R^*}{R} \right) \frac{dR}{dt} \leq \gamma_I I^* \left[\frac{I}{I^*} - \frac{R}{R^*} + \ln \left(\frac{R}{R^*} \right) - \ln \left(\frac{I}{I^*} \right) \right] + \gamma_Q Q^* \left[\frac{Q}{Q^*} - \frac{R}{R^*} + \ln \left(\frac{R}{R^*} \right) - \ln \left(\frac{Q}{Q^*} \right) \right], \\ \frac{dL(Q)}{dt} &= \left(\frac{Q - Q^*}{Q} \right) \frac{dQ}{dt} \leq q_E E^* \left[\frac{E}{E^*} - \frac{Q}{Q^*} + \ln \left(\frac{Q}{Q^*} \right) - \ln \left(\frac{E}{E^*} \right) \right] + q_I I^* \left[\frac{I}{I^*} - \frac{Q}{Q^*} + \ln \left(\frac{Q}{Q^*} \right) - \ln \left(\frac{I}{I^*} \right) \right], \\ \frac{dL(D)}{dt} &= \left(\frac{D - D^*}{D} \right) \frac{dD}{dt} \leq k Q^* \left[\frac{Q}{Q^*} - \frac{D}{D^*} + \ln \left(\frac{D}{D^*} \right) - \ln \left(\frac{Q}{Q^*} \right) \right] + \omega I^* \left[\frac{I}{I^*} - \frac{D}{D^*} + \ln \left(\frac{D}{D^*} \right) - \ln \left(\frac{I}{I^*} \right) \right], \\ \frac{dL(V)}{dt} &= \left(\frac{V - V^*}{V} \right) \frac{dV}{dt} \leq \xi_1 E^* \left[\frac{E}{E^*} - \frac{V}{V^*} + \ln \left(\frac{V}{V^*} \right) - \ln \left(\frac{E}{E^*} \right) \right] + \xi_2 I^* \left[\frac{I}{I^*} - \frac{V}{V^*} + \ln \left(\frac{V}{V^*} \right) - \ln \left(\frac{I}{I^*} \right) \right] \\ &\quad + \xi_3 Q^* \left[\frac{Q}{Q^*} - \frac{V}{V^*} + \ln \left(\frac{V}{V^*} \right) - \ln \left(\frac{Q}{Q^*} \right) \right], \text{ so that} \\ \frac{dL}{dt} &\leq \frac{dS}{dt} + \frac{dE}{dt} + \frac{dI}{dt} + \frac{dR}{dt} + \frac{dQ}{dt} + \frac{dD}{dt} + \frac{dV}{dt} \\ &\leq \left[\beta_I^* I^* S^* - \alpha E^* + \left(\gamma_I + q_I + \omega + \xi_2 \right) I^* \right] \left(\frac{I}{I^*} - \ln \frac{I}{I^*} \right) \\ &\quad + \left[\beta_I^* I^* S^* - \left(\alpha + q_E + \xi_1 \right) E^* + \beta_V^* S^* V^* \right] \left(\ln \frac{E}{E^*} - \frac{E}{E^*} \right) \\ &\quad + \left[\beta_V^* S^* V^* - \xi_1 E^* - \xi_2 I^* - \xi_3 Q^* \right] \left(\frac{V}{V^*} - \ln \frac{V}{V^*} \right) + \left(\gamma_I I^* + \gamma_Q Q^* \right) \left(\ln \frac{R}{R^*} - \frac{R}{R^*} \right) \\ &\quad + \left[\left(k + \gamma_Q + \xi_3 \right) Q^* - q_E E^* - q_I I^* \right] \left(\frac{Q}{Q^*} - \ln \frac{Q}{Q^*} \right) + \left(\omega I^* + k Q^* \right) \left(\ln \frac{D}{D^*} - \frac{D}{D^*} \right) \\ &= 0. \end{aligned}$$

with the equality holding if and only if $(S, E, I, R, Q, D, V) \equiv (S^*, E^*, I^*, R^*, Q^*, D^*, V^*)$. We have $f(X) = \ln X = (X - 1) - \frac{1}{2}(X - 1)^2 + \frac{1}{3}(X - 1)^3 - \dots$, $0 < X < 1$.

Therefore, $\ln X - X \approx X - 1 - X = -1 < 0$. Hence $\frac{dL}{dt} \leq 0$ under the following conditions

$$(b_1 + \beta_I^* S^*) I^* < \alpha E^*, \quad b_2 S^* (\beta_I^* I^* + \beta_V^* V^*) - E^* > 0,$$

$$\beta_V^* S^* V^* < \xi_1 E^* + \xi_2 I^* + \xi_3 Q^*, \quad a_2 Q^* < q_E E^* + q_I I^*.$$

Combining these inequalities, we finally have the conditions for global stability as

$$b_1 I^* + (1 - b_2) \beta_I^* I^* S^* < b_2 (\xi_1 E^* + \xi_2 I^* + \xi_3 Q^*) \text{ and } a_2 Q^* < q_E E^* + q_I I^*, \text{ where}$$

$$b_1 = \gamma + q_I + \omega + \xi_2, \quad b_2 = \frac{1}{\alpha + q_E + \xi_1} \text{ and } a_2 = k + \gamma_Q + \xi_3.$$

□

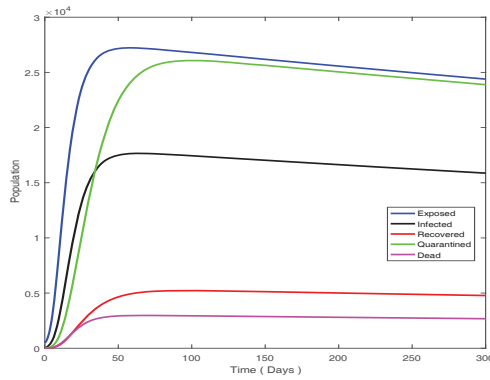


Figure 2: Time series representation of various categories of population involved in the dynamics of pandemic.

5 Numerical Results and Discussion

In view of substantiating the theoretical investigation, it is worthwhile to carry out numerical simulation based on the listed data in Table 1 for all the COVID-19 model parameters of the system undertaken just to manifest the agreement of the numerical results with their analytical counterparts. The dynamical response of the nonlinear model system of concern is examined in-depth numerically. Numerical integration for the present model system is carried out with the choice of relevant epidemiological parameter values supplemented by the set of initial population density. The initial value problem for the present system of equations subject to the set of initial conditions is solved numerically using Runge-Kutta 4th order method with desired degree of accuracy and a befitting MATLAB code is developed to obtain several interesting phenomena. Using the entire set of model parametric values enlisted in Table 1, the following two positive equilibria are obtained:

$$E_1 = (9.010963455 \times 10^6, 0, 0, 0, 0, 0, 0),$$

$$E_2 = (2.741243068 \times 10^6, 1158.584186, 752.2244755, 2250.988763, 1125.089537, 126.6833504, 4579.668863).$$

All the eigenvalues of the Jacobian matrix of the system (3.1) - (3.7) at these equilibria are obtained numerically and examined their stability. It appears from the nature of the eigenvalues that the axial equilibrium position E_1 is unstable while the positive interior equilibrium E_2 is stable. This feature of stability of equilibrium ensures the stability of the entire model system under contemplation in the environment of present scenario of epidemics in COVID-19.

The time-series delineations for diverse categories of population participating in the dynamics of epidemics arising from novel coronavirus are illustrated in Fig. 2 spanned over a period of 300 days. One may observe from the general characteristics of the representative curves that all the categories of population start growing at the onset, rise to certain peaks and thereafter become almost flat with further advancement of time. The flatness of the curves corresponding to exposed, quarantined and infected population subsides gradually and the decaying trend is revealed for rest of the time. Unlike them, the recovered and the dead individuals remain flat for the entire period of time with their respective peak values. The increasing rates of the exposed, quarantined and infected population are recorded to be noticeably higher than those for the recovered and dead ones towards the onset of the dynamics of epidemiology. As the disease outbreaks, more and more people get exposed, many are quarantined and some are infected leaving behind an appreciable number of both recovery and death with the advancement of time.

It is worth mentioning that the numerical simulation of the model system such as the present one bears the potential for its usage in the realm of epidemiology in terms of its credibility and acceptability to humanity at large. For this purpose one needs to explore the variability of some key model parameters on the dynamic response of the system and to estimate their influence on the population of all designations thereby.

Figure 3 exhibits five different panels (a) - (e) to demonstrate how the entire population comprising assorted classes of individuals get influenced by the variation of natural mortality rate μ . The mortality rate is varied by increasing / decreasing its value from its base value for certain comprehensible situations. It is interesting to note that in the event of increasing mortality, the population declines rapidly irrespective of the class they belong to. On the contrary, all the categories of population remain unperturbed however in the case of decreasing mortality in the environment of novel coronavirus. On the other hand, the consequence of disease induced death rate ω on the entire population is meticulously captured through Fig. 4 comprising five distinct frameworks (a) - (e) according to various classes of individuals. These results reveal that the moment the disease-linked death rate is allowed to rise, all the classified population excepting the dead ones are reduced substantially as anticipated. Obviously the dead people continue to enhance largely with the escalation of disease induced death rate. A reverse phenomenon may be observed when the system experiences lower disease induced death rate.

All the panels (a) - (e) of Fig. 5 register the impact of the quarantine rate of the exposed population q_E on the characteristics of the entire population affected by pandemic. The usual respective individual attribute gets perturbed to a greater extent in the event of increasing quarantine rate and to a smaller range in the issue of decreasing rate from its threshold level. With the advancement of the quarantine rate of the exposed people, the dead, the exposed and the infected population are dramatically reduced unlike those of quarantined and recovered individuals for obvious reasons. However, a reverse trend is prevailed in the affected community for the entire span of time subject to the reduction of the quarantine rate from its base value. Now in order to address the obvious question of dependency of the quarantine rate of the infected people on the dynamics of the present model system, relevant results (a) - (e) are displayed in Fig. 6. Here too, one may note that increasing quarantine rate q_I of the infected people causes considerable reduction of the dead, exposed and infected population but enhancement of both quarantined and recovered individuals a reverse nature is followed in the case of reduced quarantine rate of infected individuals. Thus the parameters signifying quarantine rates for both the exposed and the infected population have their own importance in the dynamical behaviour of epidemiology concerning novel coronavirus.

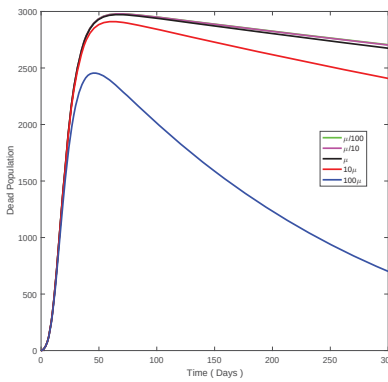
The incubation period $1/\alpha$ is another important model parameter that needs utmost attention for the purpose of better understanding the dynamics of the system under consideration. The results of Fig. 7 include the upshot of varied incubation period on the exposed, infected, quarantined and recovered population presented through four distinct panels (a) - (d). It appears from the outcome of these frameworks that the shortening of the incubation period results in drastic reduction of all the specified population over the entire span of time but they are recorded to enhance as long as the incubation period is extended to some extent due to the development of the disease towards maturity. One may also put on record that although all the branded population are reduced eventually for shorter incubation period, they all get enhanced towards the early period of time.

Fig. 8 illustrates the influence of variation of transmission rate in the exposed class of individuals on the outbreak of exposed, infected and susceptible population presented through three distinct panels (a) - (c). Rapid transmission rate in the exposed category gives rise to both the exposed and the infected population and to some extent decline the susceptible individuals sharply. This observation is quite feasible indeed in the sense that as the transmission rate in the exposed class increases, more and more susceptible population get converted into exposed and infected individuals while on the contrary, the susceptible population remains almost unaltered for smaller transmission rate leading to appreciable reduction of the exposed and infected population. Similar observation from Fig. 9 may be recorded in order to estimate the effect of varied transmission rate on the exposed, infected and susceptible population interacting in the dynamics of pandemic. Besides these, the results of Fig. 10 depict the efficacy of virus transmission rate on the four exponent classes of individuals, namely, the exposed, the infected, the susceptible and the virus concentration as portrayed through (a) - (d). In the consequence of increasing virus transmission, more and more people in the exposed and infected categories get affected resulting in attaining higher virus concentration and a sharp drop in susceptible population. Thus the community is totally engulfed by the epidemic all of a sudden with increasing virus transmission. However, if the virus transmission is reduced significantly by means of some measure of control, then such community transmission may be avoided in order to uphold the susceptible population in tact from any further dropouts.

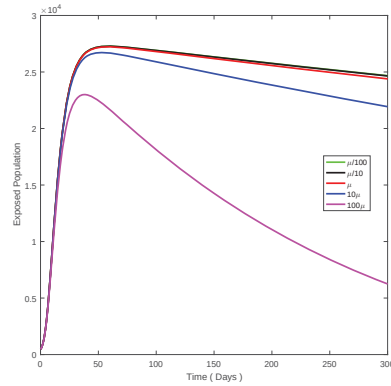
For the purpose of the complexity of conducting the large model system under consideration, it may be reduced to smaller one without any loss of general objectives of the pursuit in order to carry out an in-depth study to explore all possible plights by analyzing the dynamical response. The QSSA technique is employed for this intention to reduce the present larger model to a smaller one consisting of only three major groups of individuals, namely, susceptible, exposed and infected population. The reduced model possesses the equilibrium positions with exactly same numerical values as the original model system based on the same set of model parameter values. This does confirm the consistency of the reduced model with quasi steady state theory. Of the two equilibrium positions $(9.010963455 \times 10^6, 0, 0)$ and $(2.741243068 \times 10^6, 1158.584186, 752.2244755)$ for the reduced model, the interior one is found to be asymptotically stable as in the original model system. Fig. 11 exhibits that the interior equilibrium position is locally asymptotically stable with the choice of five initial spots. All the trajectories get underway from their respective starting positions and eventually approach the equilibrium point following the arrowed flow directions irrespective of the choice of their source locales and hence the equilibrium position becomes asymptotically stable.

Finally, all possible projections of two dimensional phase-portraits corresponding to the reduced model are displayed in the concluding graphical representations through Figs. 12 - 14 as an additional measure of stability of the present reduced model. Effort is also made to plot the phase planes corresponding to (S, E) , (S, I) and (E, I) subspaces in the respective figures. All these three panels have a common feature that is the trajectories of the panel bear a tendency to move along the directed pathway towards the equilibrium point irrespective of their

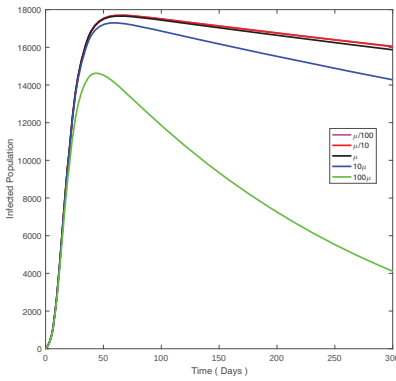
onset positions. This observation further ensures the stability of the present updated model system in epidemiology.



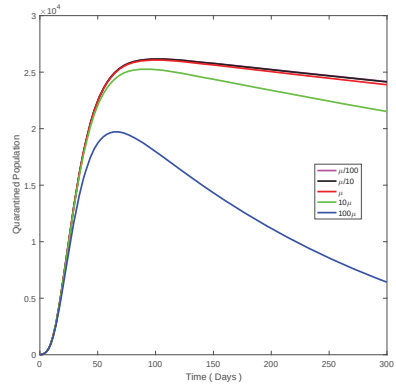
(a)



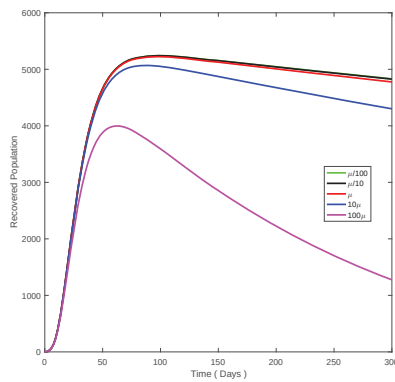
(b)



(c)

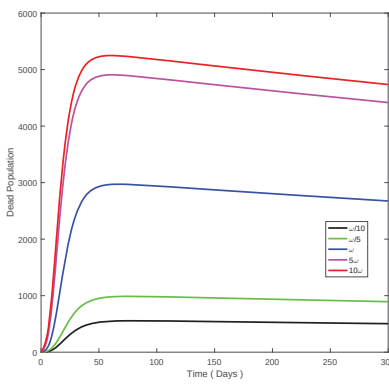


(d)

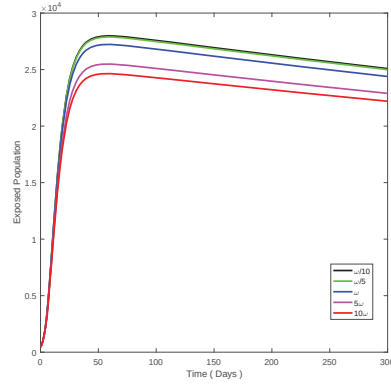


(e)

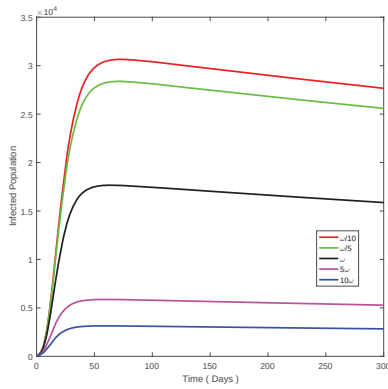
Figure 3: Influence of the variation of natural mortality rate μ on various classes of population [(a) - (e)].



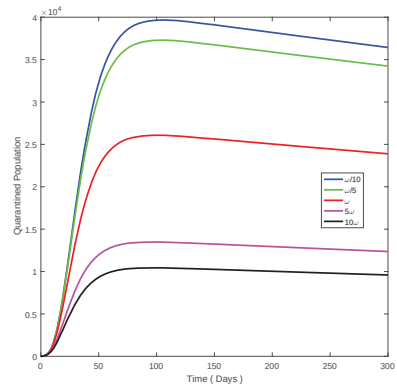
(a)



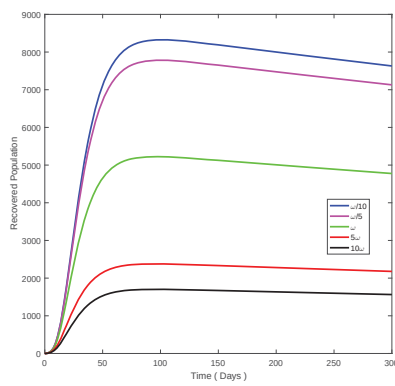
(b)



(c)



(d)

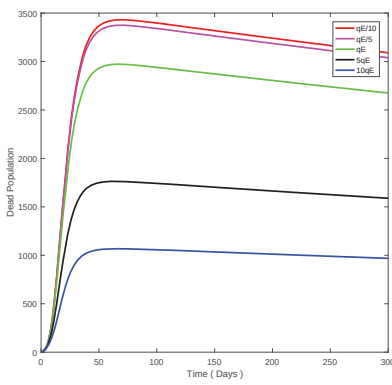


(e)

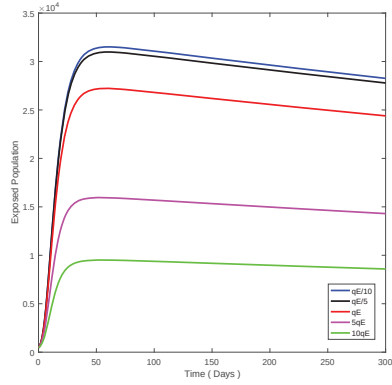
Figure 4: Influence of the variation of disease induced death rate ω on participating population in pandemic [(a) - (e)].

Table 1: Parameter Values

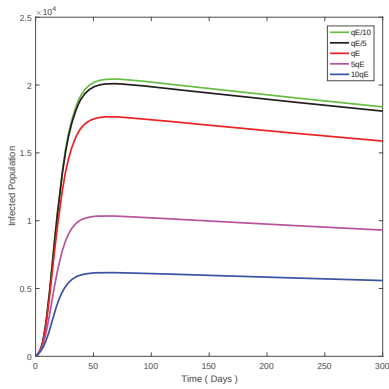
Parameters	Description	Values	References
Λ	Population influx	$271.23(day)^{-1}$	(The government of Wuhan homepage. A, n.d.)
β_{E0}	Maximum value of β_E	$3.11 \times 10^{-8}(person)^{-1}(day)^{-1}$	(Tang et al., 2020)
β_{I0}	Maximum value of β_I	$0.62 \times 10^{-8}(person)^{-1}(day)^{-1}$	(Tang et al., 2020)
β_{V0}	Maximum value of β_V	$1.03 \times 10^{-8}(person)^{-1}(day)^{-1}$	(Yang and Wang, 2020)
c	Positive coefficient	1.01×10^{-4}	(Yang and Wang, 2020)
μ	Natural death rate	$3.01 \times 10^{-5}(day)^{-1}$	(The government of Wuhan homepage. A, n.d.)
α	Reciprocal of incubation period	$1/7(day)^{-1}$	(Spencer, Shutt, Moser, Clegg, Wearing and H. Mukundan, 2020)
λ	Mean duration of public reaction	$0.6(day)^{-1}$	Present Study
ω	Disease induced death rate	$0.1(day)^{-1}$	(Lopez and Rodo, 2020)(Estimated)
γ_Q	Recovery rate of Q	$1/15(day)^{-1}$	(Spencer et al., 2020)(Estimated)
γ_I	Recovery rate of I	$1/20(day)^{-1}$	(Spencer et al., 2020)(Estimated)
ξ_1	Virus shedding rate of E	$2.30(person)^{-1}(day)^{-1}(ml)^{-1}$	(Yang and Wang, 2020)
ξ_2	Virus shedding rate of I	$1.05(person)^{-1}(day)^{-1}(ml)^{-1}$	(Yang and Wang, 2020)(Estimated)
ξ_3	Virus shedding rate of Q	$1.0(person)^{-1}(day)^{-1}(ml)^{-1}$	(Yang and Wang, 2020)(Estimated)
σ	Removal rate of virus	$1.0(day)^{-1}$	(Geller et al., 2012)
q_E	Quarantine rate of E	$0.02(day)^{-1}$	(Lopez and Rodo, 2020)(Estimated)
q_I	Quarantine rate of I	$0.07(day)^{-1}$	(Lopez and Rodo, 2020)
p	Death rate of R	$0.5(day)^{-1}$	Present Study
k_0	Fitted coefficient	0.0007	(Lopez and Rodo, 2020)
k_1	Fitted coefficient	0	(Lopez and Rodo, 2020)



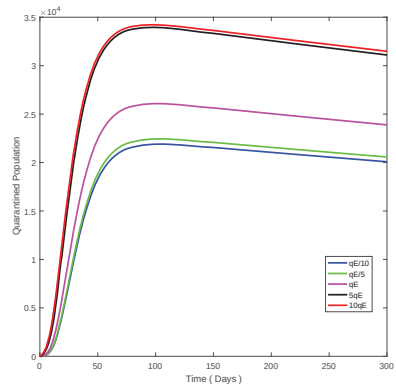
(a)



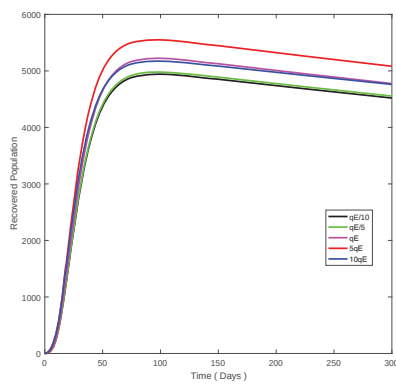
(b)



(c)

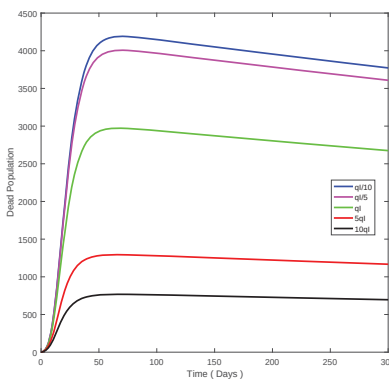


(d)

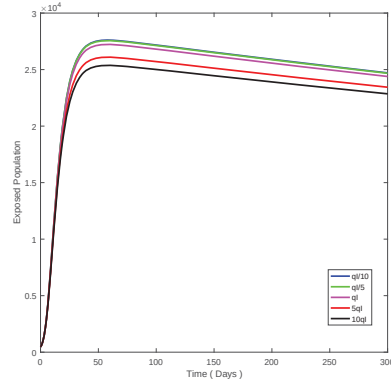


(e)

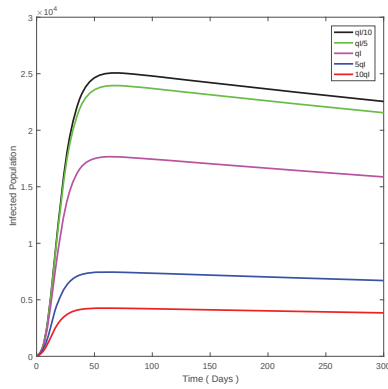
Figure 5: Influence of the variation of quarantine rate of exposed population on the dynamics of pandemic [(a) - (e)].



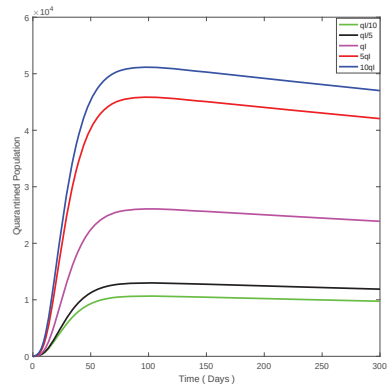
(a)



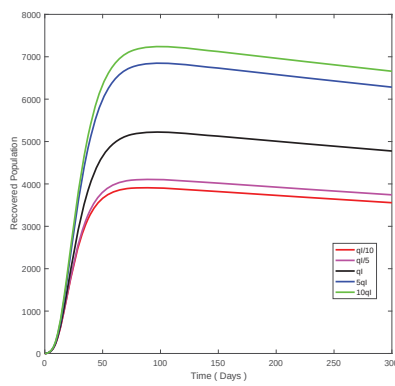
(b)



(c)



(d)



(e)

Figure 6: Influence of the variation of quarantine rate of infected population on the dynamics of pandemic [(a) - (e)].

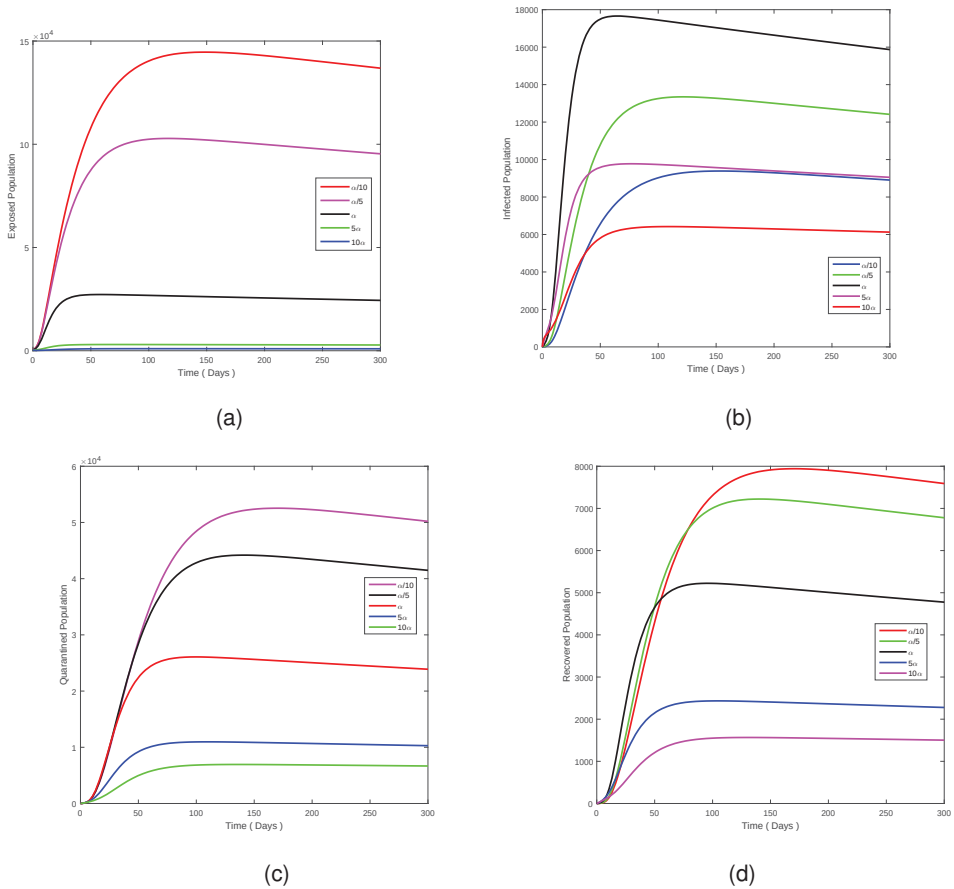
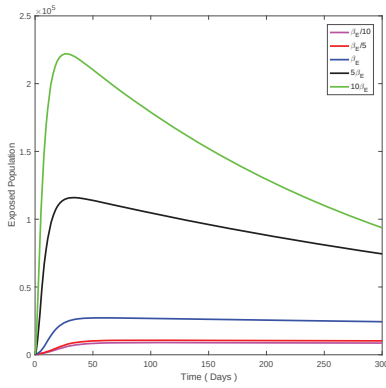
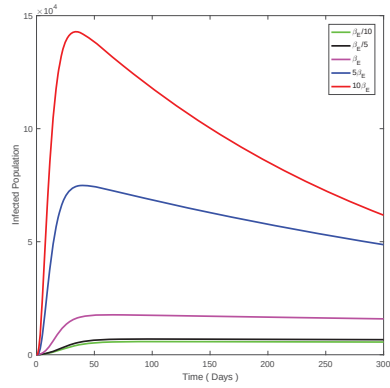


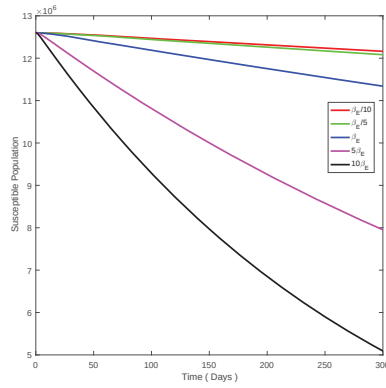
Figure 7: Effect of variation of incubation period on the specified classes of population [(a) - (d)].



(a)



(b)



(c)

Figure 8: Influence of the variation in the transmission rate in the exposed population on the outbreak of exposed, infected and susceptible individuals [(a) - (c)].

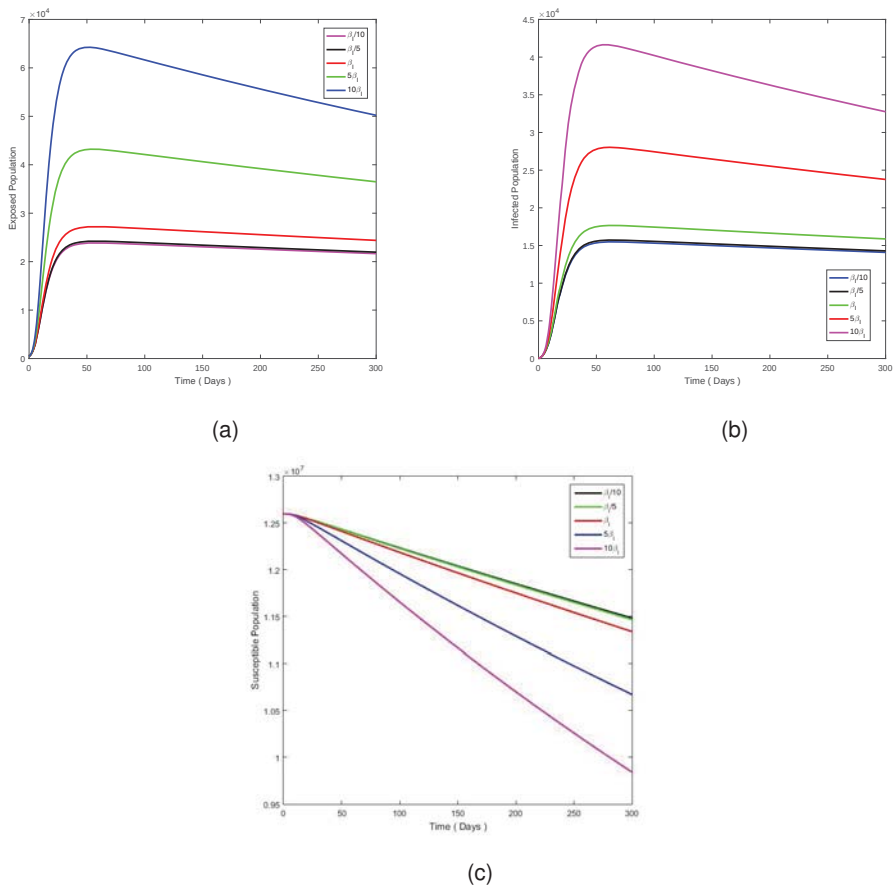
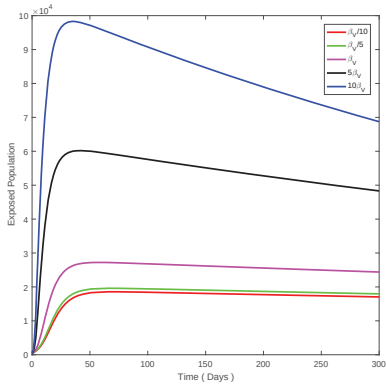
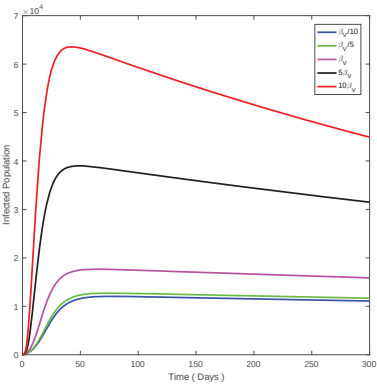


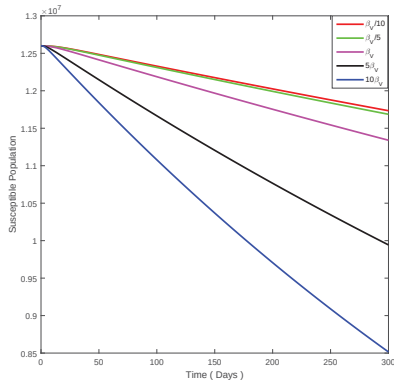
Figure 9: Influence of the variation of transmission rate in the infected category on the outbreak of exposed, infected and susceptible population [(a) - (c)].



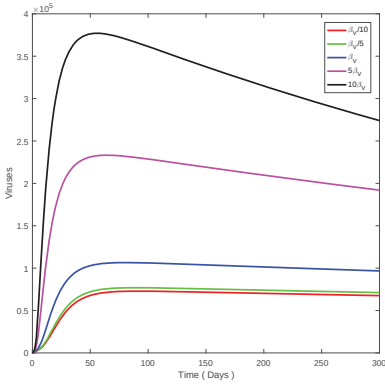
(a)



(b)



(c)



(d)

Figure 10: Effect of the variation of virus transmission rate on some specified categories of pandemic population [(a) - (d)].

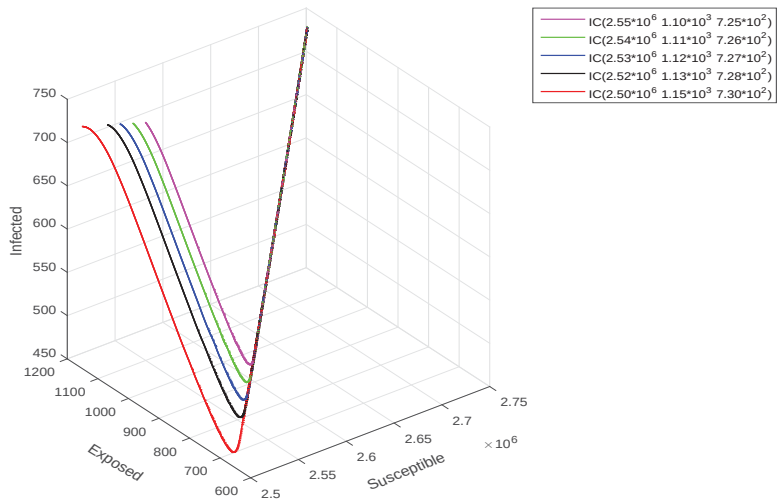


Figure 11: Stability of the equilibrium position of the reduced model in three dimensional space.

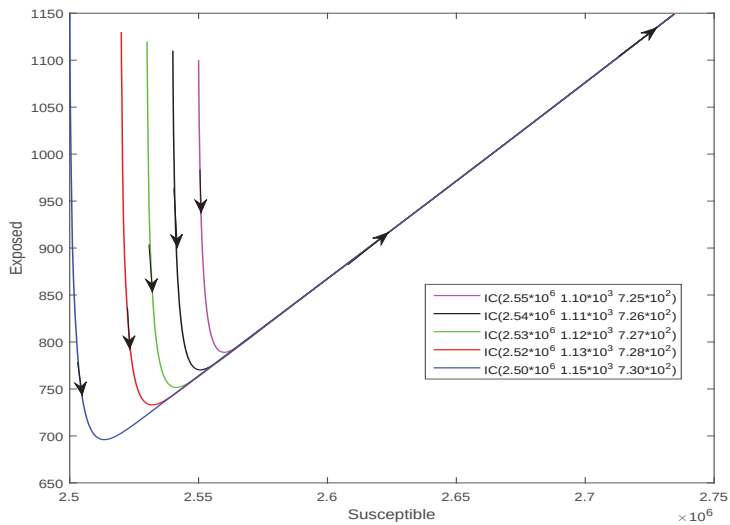


Figure 12: Two dimensional phase portrait of the reduced model in (S, E) plane.

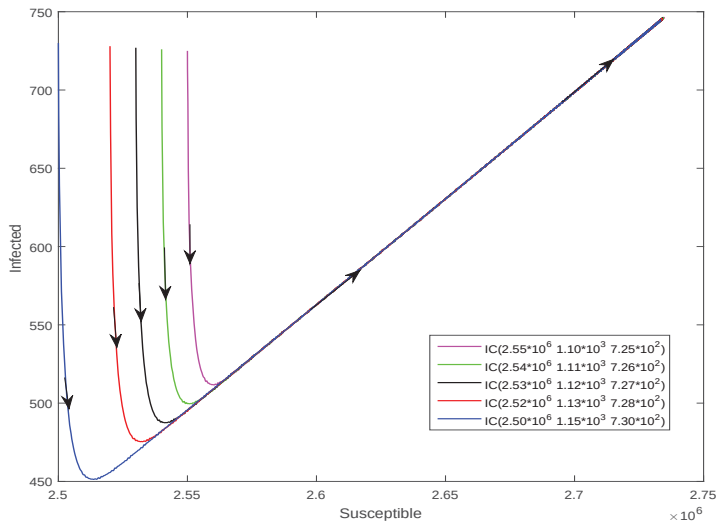


Figure 13: Two dimensional phase portrait of the reduced model in (S, I) plane.

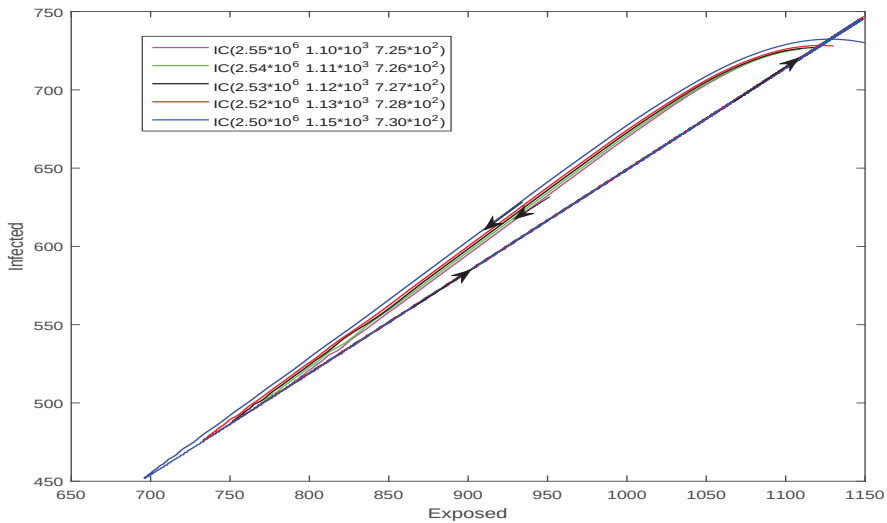


Figure 14: Two dimensional phase portrait of the reduced model in (E, I) plane.

6 Concluding Remarks

An effort is made to develop an improved mathematical model of COVID-19 for the purpose of having a complete understanding of the dynamics of the outbreak of novel coronavirus spread over worldwide. The present befitting mathematical model depicting the dynamics of epidemiology is formulated through the system of seven nonlinear ordinary differential equations governing the most pertinent categories of population. As the original model is seven dimensional signifying seven distinct classes of population, namely, susceptible, exposed, infected, recovered, dead and virus concentration itself, it becomes difficult to study the stability of the dynamical system in-depth by means of phase-portraits. For this reason, the standard method of QSSA is employed to reduce the larger original system into a smaller one in order to study more complex features of the model system without any loss of generality.

The correctness of model reduction is established with the confirmation that the steady states of both the original and the reduced one are same. The study undertaken is of adequate merit on several counts. First, the present improved model is realistic as it takes into account all the pertinent members directly participating in the dynamics of epidemiology and having relevance to the present day environment of novel coronavirus. Secondly, nonlinearity in the model formulation deals with epidemiological complexities. Thirdly, both analytical and numerical methods are adopted to solve the system of governing equations with particular emphasis on stability analysis of both the original and the reduced model systems. Lastly, the performance of the parametric variation of several key model parameters on the dynamic response of the present model system is intensively carried out for a better understanding of the model characteristics. The sensitivity of the affected population to the model parameters poses challenges to the applicability and acceptability in such a way, so that proper care needs to be exercised based on the knowledge acquired to control the pandemic. Several conclusions can be drawn right from the numerical simulations undertaken to measure the variation of parameters of relevance regarding the applicability and accuracy of the model parameters of the system. The entire investigation on epidemic model is conducted in some region around the base parameter values. The input parameters in epidemiology are often uncertain and cover large ranges which are difficult to deal with. One may conclude from the present investigation that there are certain indispensable parameters like incubation period, various transmission rates, quarantine rates and disease induced mortality rate other than natural mortality are involved in novel coronavirus which deserves to be accorded priority before one moves ahead for further investigation in this direction. Stability analysis is an important tool to conclude several physical realms of the realistic mathematical models. Since the present system is globally asymptotically stable, therefore it may be concluded that COVID-19 will accompany mankind in future, but in the long run it will be stabilized. Thus, if the administrative powers knock up with it for prolonged time, then COVID-19 may stumble from its endemic burden soon.

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