

Age-Structured Absorbing Markov Modelling of Youth Fencing Dropout: Critical Windows and Mechanistic Drivers

Jeremiah Liu

Lexington High School
251 Waltham St, Lexington, MA 02421
Email: jeremiahliu8@gmail.com

ABSTRACT

Adolescent dropout from organized sport is often a progressive process rather than an instantaneous event, with temporary disengagement frequently preceding permanent exit. This study presents an age-embedded absorbing Markov framework for analyzing youth fencing participation through three states: Active, Lapsed, and Dropped. The framework is designed for mechanistic analysis and intervention-timing comparisons rather than direct calibration to USA Fencing records. Age is incorporated directly into the state index, producing a sparse, time-homogeneous transition matrix while preserving age-specific transition behavior. Age-dependent transition probabilities are specified through logistic functions driven by three interpretable components: an age-localized vulnerability term, a bounded performance factor, and an encouragement term. Closed-form retention quantities are then obtained from the fundamental matrix of the absorbing chain, including expected seasons to dropout, expected time spent in Active and Lapsed states, and the full dropout-age distribution. In the illustrative baseline scenario, sensitivity analyses indicate that retention is most responsive to the timing and duration of age-localized vulnerability and to transitions involving temporary lapse. Approximately half of modeled dropout occurs through the Lapsed state, highlighting temporary disengagement as an important pathway rather than a minor deviation from continuous participation. Budget-constrained intervention experiments further show that, under equal total intervention magnitude, reducing lapse entry during early-to-mid adolescence produces larger retention gains than delayed or uniformly distributed strategies. The framework provides a tractable tool for examining dropout timing, pathway composition, and age-targeted intervention design in participation systems with reversible disengagement.

Keywords: youth sports retention, dropout modelling, absorbing Markov chains, age-structured dynamics, participation states.

Mathematics Subject Classification: 60J10, 60J20

Computing Classification System: I.6.1

1. INTRODUCTION

Sustained participation in youth sport is associated with a wide range of physical, psychological, and social benefits (Bengtsson et al., 2025). Yet continued engagement is often difficult to maintain across adolescence, a period marked by changing academic demands, shifting peer and family priorities, identity development, and increasing opportunity costs (Zhang et al., 2024). As a result, dropout remains a persistent concern across youth sport settings (Back et al., 2022, Lara-Bercial et al., 2025). Importantly,

withdrawal from sport is rarely a purely one-step event. For many participants, disengagement unfolds through intermediate stages such as reduced involvement, temporary inactivity, and possible re-entry before permanent exit (Battaglia et al., 2024, Eliasson and Johansson, 2021). Treating dropout as a single terminal outcome can therefore obscure pathway structure and limit insight into when intervention may be most effective.

From an applied mathematical perspective, youth-sport retention can be viewed as a structured population-dynamics problem in which participants transition among discrete participation states over successive seasons. This perspective closely parallels reliability and degradation modeling, where an individual may remain operational, enter a degraded condition, recover, or fail irreversibly. In participation systems, the corresponding states may be interpreted as active involvement, temporary disengagement, recovery, and permanent withdrawal. Such a formulation is useful because intermediate states can substantially alter long-run absorption behavior and can create strong sensitivity to intervention timing. Two practical questions then arise: at what ages does dropout tend to occur, and through which pathways does it emerge? These questions are naturally addressed within an absorbing Markov framework with age-dependent transition dynamics.

Fencing provides a useful setting for studying these issues. As a youth sport, it combines progressive technical development, individualized competitive feedback, variable entry ages, and a ranking structure that makes performance progression especially visible (Roi and Bianchedi, 2008). In USA Fencing, competitive status is further organized through a letter-rating system that contributes to seeding and reflects advancement. Participation may be sustained by enjoyment, challenge, community, and identity (Ganakas and Peden, 2023), but it may also be undermined by cost, time burden, performance frustration, and competing obligations. Because fencing is fundamentally an individual sport, performance setbacks and perceived stagnation may be experienced more directly than in team-based settings, making temporary disengagement a plausible intermediate stage before full withdrawal. These features suggest that dropout in fencing is better understood as a multi-stage process than as a single observed exit age. Instead, dropout timing and pathway composition may depend on how pressures accumulate across adolescence and on how temporary lapses emerge and resolve over time (Turner et al., 2014).

Despite the practical importance of retention for clubs, coaches, and governing bodies, fencing-specific research on persistence remains limited and is dominated by descriptive studies or cross-sectional intention models, rather than age-resolved, stage-based trajectory analyses (Kim and Kim, 2021). More broadly, the fencing literature has been dominated by performance- and biomechanics-oriented research (Chen et al., 2017), leaving a gap in quantitative models of participation over time. In the wider youth-sport literature, dropout is often interpreted through motivational and behavioral theories—commonly self-determination theory and the theory of planned behavior—linking continued participation to enjoyment, social influence, intention, and perceived behavioral control (Zhang et al., 2022). These approaches are valuable for identifying correlates and risk factors, but they do not directly yield age-resolved predictions of when dropout occurs, how temporary inactivity contributes to later exit, or which

ages provide the greatest leverage for prevention (Back et al., 2022). For policy design and resource allocation, these timing- and pathway-resolved quantities are central.

To address this gap, the present study develops a cohort-level, age-structured absorbing Markov chain framework (Norris, 1997) for youth fencing participation with three states: Active, Lapsed, and Dropped. A central modeling feature is that age is incorporated directly into the state index. This construction yields a time-homogeneous transition matrix at the matrix level while preserving age-specific transition interpretation. The resulting system is sparse, structured, and analytically tractable because age advances deterministically from one season to the next except at the terminal age bin. This formulation allows retention dynamics to be studied through standard absorbing-chain analysis while retaining explicit age structure.

Age-specific transition probabilities are specified using logistic mappings (Jr. et al., 2013) driven by three interpretable components: an age-localized vulnerability term representing concentrated developmental or contextual pressures, a bounded performance factor representing gradual and saturating competitive development, and an encouragement term summarizing support-related influences. Within this structure, the absorbing-chain fundamental matrix yields closed-form expressions for key retention quantities, including expected seasons until dropout, expected time spent in Active and Lapsed states, and the full dropout-age distribution. The same framework also supports controlled sensitivity analyses and intervention experiments, such as shifting the timing and breadth of vulnerability or allocating fixed intervention budgets across age windows.

The purpose of this study is not to claim a uniquely calibrated dropout curve for USA Fencing, but rather to introduce a transparent and mechanistically interpretable framework for participation systems in which temporary disengagement may precede permanent exit. In this sense, fencing serves as a concrete testbed for a broader class of retention problems. The analysis focuses on three questions: how age-localized vulnerability shapes dropout timing, how strongly retention depends on lapse-mediated pathways, and how fixed intervention resources should be timed across adolescence to maximize retention benefit. By addressing these questions within a closed-form absorbing Markov structure, the study aims to provide a useful bridge between interpretable stochastic modeling and practical retention design in youth participation systems.

2. MATHEMATICAL MODEL

Fencing participation is modeled as an age-embedded absorbing Markov chain (Norris, 1997) with three states (Active, Lapsed, Dropped). The formulation supports both closed-form retention metrics and controlled sensitivity/intervention studies through age-targeted modifications to transition probabilities. The model is intended for structural insight and intervention-timing analysis rather than direct forecasting for a specific program. Baseline values (**Appendix A**) are chosen to avoid near-certain transitions and to enable meaningful comparison of mechanisms across scenarios. Results are therefore reported in terms of relative sensitivities, pathway decompositions (direct versus lapse-

mediated dropout), and changes in retention metrics under targeted shifts, emphasizing qualitative patterns that persist across scenarios.

2.1. State space and Markov structure

Let $a \in \{a_1, a_2, \dots, a_n\}$ denote discrete age bins spanning ages 8 to 30 ($n=23$). At each age bin a , a fencer is in one of two transient states: A_a , Active at age a (active in competing during the season), L_a , Lapsed at age a (did not compete during the season but may return). A single absorbing state D denotes permanent dropout. The full state space is

$$\mathcal{S} = \{A_1, \dots, A_n, L_1, \dots, L_n, D\}, \quad |\mathcal{S}| = 2n + 1 \quad (1)$$

At season t , the cohort distribution over states is represented by the row vector

$$x(t) = (\underbrace{x_{A_1}(t), \dots, x_{A_n}(t)}_{\text{Active}}, \underbrace{x_{L_1}(t), \dots, x_{L_n}(t)}_{\text{Lapsed}}, x_D(t)) \quad (2)$$

which is a probability distribution satisfying $\sum_{s \in \mathcal{S}} x_s(t) = 1$. Season-to-season evolution is governed by a fixed transition matrix $P \in \mathbb{R}^{(2n+1) \times (2n+1)}$:

$$x(t+1) = x(t)P \quad (3)$$

The dropout state is absorbing, so $P_{DD} = 1$ and $P_{D,s} = 0$ for all $s \neq D$. As illustrated in **Fig. 1**, age advancement is embedded in the state index: transitions occur between seasons, and probability mass moves from age bin a at season t to age bin $a+1$ at season $t+1$.

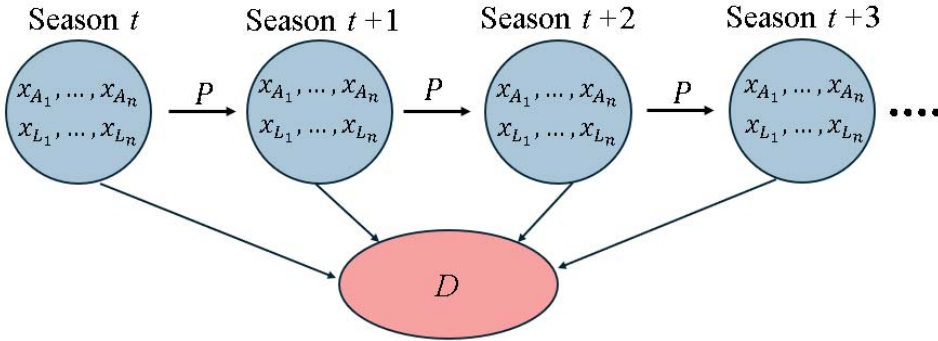


Figure 1. Schematic of the age-embedded absorbing Markov process. Each node represents the cohort-level state vector $x(t)$, consisting of the probability distribution across all age-indexed Active and Lapsed states at season t . Transitions between nodes correspond to the application of the time-homogeneous transition matrix P , advancing the cohort by one season.

2.2. Structure of the transition matrix

With states ordered as $(A_1, \dots, A_n, L_1, \dots, L_n, D)$, the transition matrix has the block form

$$P = \begin{bmatrix} Q_{AA} & Q_{AL} & R_A \\ Q_{LA} & Q_{LL} & R_L \\ 0 & 0 & 1 \end{bmatrix} \quad (4)$$

where Q_{AA} , Q_{AL} , Q_{LA} , and Q_{LL} describe transitions among transient Active and Lapsed states, and R_A and R_L represent transitions from these states to the absorbing Dropped state D . Because each season increments age by one bin, the transient blocks are first-superdiagonal, with the terminal age row diagonal to prevent further age advancement, e.g.,

$$Q_{AA} = \begin{bmatrix} 0 & p_{AA}(1) & 0 & \cdots & 0 \\ 0 & 0 & p_{AA}(2) & \cdots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & p_{AA}(n-1) \\ 0 & \cdots & \cdots & 0 & p_{AA}(n) \end{bmatrix} \quad Q_{AL} = \begin{bmatrix} 0 & p_{AL}(1) & 0 & \cdots & 0 \\ 0 & 0 & p_{AL}(2) & \cdots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & p_{AL}(n-1) \\ 0 & \cdots & \cdots & 0 & p_{AL}(n) \end{bmatrix} \quad (5)$$

$$Q_{LA} = \begin{bmatrix} 0 & p_{LA}(1) & 0 & \cdots & 0 \\ 0 & 0 & p_{LA}(2) & \cdots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & p_{LA}(n-1) \\ 0 & \cdots & \cdots & 0 & p_{LA}(n) \end{bmatrix} \quad Q_{LL} = \begin{bmatrix} 0 & p_{LL}(1) & 0 & \cdots & 0 \\ 0 & 0 & p_{LL}(2) & \cdots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & p_{LL}(n-1) \\ 0 & \cdots & \cdots & 0 & p_{LL}(n) \end{bmatrix} \quad (6)$$

The dropout vectors are

$$R_A = \begin{bmatrix} p_{AD}(1) \\ p_{AD}(2) \\ \vdots \\ p_{AD}(n) \end{bmatrix} \quad R_L = \begin{bmatrix} p_{LD}(1) \\ p_{LD}(2) \\ \vdots \\ p_{LD}(n) \end{bmatrix} \quad (7)$$

For each age bin a , season-to-season transition probabilities satisfy

$$p_{AA}(a) = 1 - p_{AL}(a) - p_{AD}(a), \quad p_{LL}(a) = 1 - p_{LA}(a) - p_{LD}(a) \quad (8)$$

To guarantee valid transition probabilities and avoid near-certain transitions in any single step, a cap is enforced:

$$p_{AL}(a) + p_{AD}(a) \leq \text{cap}, \quad p_{LA}(a) + p_{LD}(a) \leq \text{cap} \quad (9)$$

implemented via proportional scaling when the sum exceeds $\text{cap}=0.95$. The remaining probability mass defines $p_{AA}(a)$ and $p_{LL}(a)$. This construction produces a sparse, row-stochastic P with nonzeros only along shifted blocks and the absorbing column.

2.3. Transition probability specification

All age-specific transition probabilities are mapped to $[0,1]$ using the logistic function (Jr. et al., 2013)

$$\sigma(z) = 1/(1 + \exp(-z)) \quad (10)$$

Transition dynamics are driven by three age-dependent inputs: an encouragement probability $p_E(a)$ that mediates age-dependent support/engagement effects, a bounded performance factor $P(a)$, and an age-localized vulnerability term $B(a)$.

Encouragement is modeled as

$$p_E(a) = \sigma(\theta_0 + \theta_P P(a) + \theta_B B(a)) \quad (11)$$

where θ_0 sets baseline encouragement and θ_P, θ_B control its dependence on performance and vulnerability. This term is used as a convenient summary of support-related effects, rather than a claim of a unique causal mechanism.

The performance signal $P(a)$ represents a bounded, age-dependent measure of competitive performance relative to peers. In the present sensitivity studies, it is modeled as

$$P(a) = 0.05 \tanh\left(\frac{a-13}{4}\right) \quad (12)$$

which captures gradual improvement that levels off at higher ages. In data-driven applications, $P(a)$ can be replaced by an empirical rating or normalized performance metric.

Vulnerability is modeled as a Gaussian-shaped risk term

$$B(a) = s \exp\left(-\frac{(a-c)^2}{w^2}\right) \quad (13)$$

where c is the vulnerability center, w controls the temporal spread across adjacent ages, and s sets the overall amplitude. Here c represents the age at which vulnerability is most concentrated, while larger w corresponds to a broader, more sustained vulnerability window. In fencing, $B(a)$ summarizes age-dependent pressures (e.g., school transitions, time burden, and rising competitive demands) that increase lapse and dropout risk.

2.4. State transition functions

From Active state at age a , the probabilities of lapsing and dropping are

$$p_{AL}(a) = \sigma(\alpha_{AL} + \beta_P P(a) + \beta_E p_E(a) + \beta_B B(a)) \quad (14)$$

$$p_{AD}(a) = \sigma(\alpha_{AD} + \gamma_P P(a) + \gamma_E p_E(a) + \gamma_B B(a)) \quad (15)$$

Intercepts α_{AL} and α_{AD} set baselines; other coefficients weight $P(a)$, $p_E(a)$, and $B(a)$ (Table A1). From Lapsed state at age a , the probabilities of returning to Active participation or dropping permanently are

$$p_{LA}(a) = \sigma(\alpha_{LA} + \rho_P P(a) + \rho_E p_E(a) + \rho_B B(a)) \quad (16)$$

$$p_{LD}(a) = \sigma(\alpha_{LD} + \delta_B B(a)) \quad (17)$$

In Eq. (17), dropout from the Lapsed state depends only on a baseline term and vulnerability, not on current performance or encouragement, because those factors mainly affect lapse entry while Active and return from lapse. Together with the cap and normalization rules above, these functions specify a complete age-dependent absorbing Markov model.

2.5. Absorbing Markov chain analysis

The transient states consist of Active and Lapsed at each age, $\mathcal{T} = \{A_1, \dots, A_n, L_1, \dots, L_n\}$, giving $2n$ transient states in total. Let $Q \in \mathbb{R}^{(2n) \times (2n)}$ be the submatrix of the full transition matrix P restricted to transitions among these transient states. The fundamental matrix is

$$N = (I - Q)^{-1} \quad (18)$$

where N_{ij} is the expected number of visits to transient state j before absorption, starting from transient state i (Kemeny and Snell(John G. Kemeny, 1976)). Starting from initial state s (Active at age 8 in all

studies), the expected number of seasons until dropout equals the expected total number of visits to transient states (the row sum of N corresponding to the initial state s):

$$\mathbb{E}[T|X_0 = s] = \sum_{j \in \mathcal{T}} N_{sj} \quad (19)$$

Similarly, the expected number of seasons spent Active and Lapsed are

$$\mathbb{E}[T_A|X_0 = s] = \sum_{a=1}^n N_{s,A_a}, \quad \mathbb{E}[T_L|X_0 = s] = \sum_{a=1}^n N_{s,L_a} \quad (20)$$

where N_{s,A_a} and N_{s,L_a} are the expected numbers of seasons spent in A_a and L_a starting from s . The dropout-age distribution is obtained by combining (i) the expected time spent in each age-indexed transient state and (ii) the probability of dropping out from that state at that age. Because dropout can occur from A_a with probability $p_{AD}(a)$ and from L_a with probability $p_{LD}(a)$ in that season, the expected dropout mass assigned to age bin a is

$$m(a) = N_{s,A_a} p_{AD}(a) + N_{s,L_a} p_{LD}(a) \quad (21)$$

Normalizing $m(a)$ over age bins yields the dropout-age probability distribution,

$$\Pr(\text{dropout at age } a) = \frac{m(a)}{\sum_{k=1}^n m(a_k)} \quad (22)$$

The terminal age bin (30+) aggregates dropout at or beyond the maximum modeled age; it can be omitted from plots to emphasize within-range age dynamics.

3. RESULTS AND DISCUSSION

3.1. Study A: Performance-driven perturbations and leverage windows for performance support

The first study examines how competitive performance affects fencing retention in the age-embedded absorbing Markov model by perturbing the performance driver $P(a)$ (Eq. 12) while keeping all other model parameters unchanged. Recall that performance affects the transition probabilities directly (Eqs. 14-16) and also indirectly through encouragement (Eq. 11). Thus, worse performance (lower $P(a)$) can raise the lapse probability $p_{AL}(a)$ and the dropout probability $p_{AD}(a)$, thereby affecting the expected time to dropout $\mathbb{E}[T]$, the dropout-age distribution, and the share of dropout mass originating from lapsed states.

In Study A, a global additive shift is applied to the performance driver uniformly across ages: $P(a) \mapsto P(a) + \Delta_p$, where Δ_p is a scalar (“perfShift”). **Fig. 2A** plots the resulting change in expected seasons until dropout, $\Delta\mathbb{E}[T]$, relative to baseline. Retention changes monotonically: decreasing performance (more negative Δ_p) shortens expected time to dropout, while improving performance increases retention. **Fig. 2B** shows the corresponding shifts in the dropout-age distribution (ages ≤ 29): worse performance increases dropout probability at younger ages and modestly amplifies the primary dropout peak, indicating a higher likelihood of earlier exit from Active fencing participation. This reflects a greater tendency to transition out of the Active state under degraded performance.

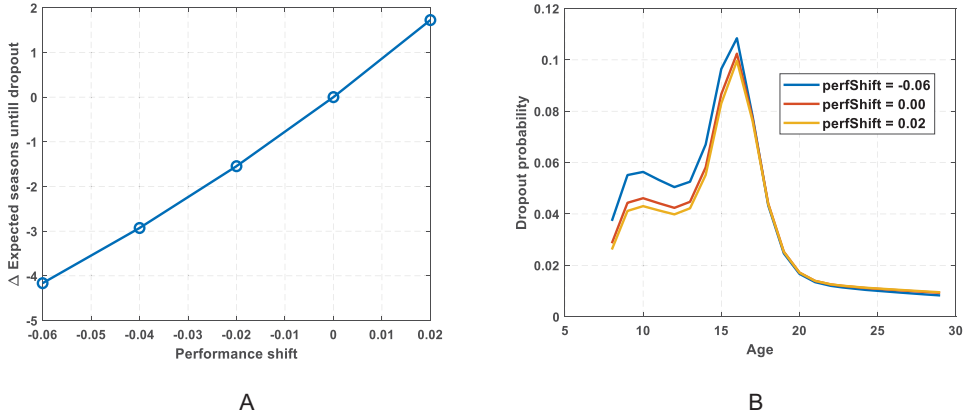


Figure 2. Effects of a global performance shift on retention and dropout timing. (A) Change in expected seasons until dropout, $\Delta E[T]$, relative to baseline, as the performance signal is shifted uniformly across all ages by an additive performance shift (perfShift). (B) Corresponding dropout-age distributions over ages ≤ 29 under representative shifts ($\text{perfShift} = -0.06, 0, 0.02$).

Next, age-specific sensitivity is evaluated by imposing a transient performance “slump,” modeled as a Gaussian-shaped dip subtracted from the baseline performance signal $P(a)$. The slump is centered at age c_P with width w_P and depth s_P :

$$P_{\text{slump}}(a) = P(a) - s_P \exp\left(-\left(\frac{a - c_P}{w_P}\right)^2\right) \quad (23)$$

With $s_P = 0.06$, **Fig. 3A** reports $\Delta E[T]$ over a grid of slump centers and widths, and **Fig. 3B** reports the corresponding lapse-mediated dropout share (fraction of dropout mass originating from the Lapsed state) over the same grid. **Fig. 3A** indicates that age-localized performance slump is most consequential during mid-adolescence: slumps centered in the mid-teen range yield the largest reductions in $E[T]$, whereas later-centered slumps produce smaller retention losses. In contrast, **Fig. 3B** shows that pathway composition is comparatively stable across the parameter range, with the lapse-mediated dropout share remaining close to ~ 0.52 . Together, these results suggest that performance perturbations mainly change how much retention is lost, while the dominant pathway structure remains similar—dropout continues to be largely mediated through the Lapsed state even under localized performance degradation.

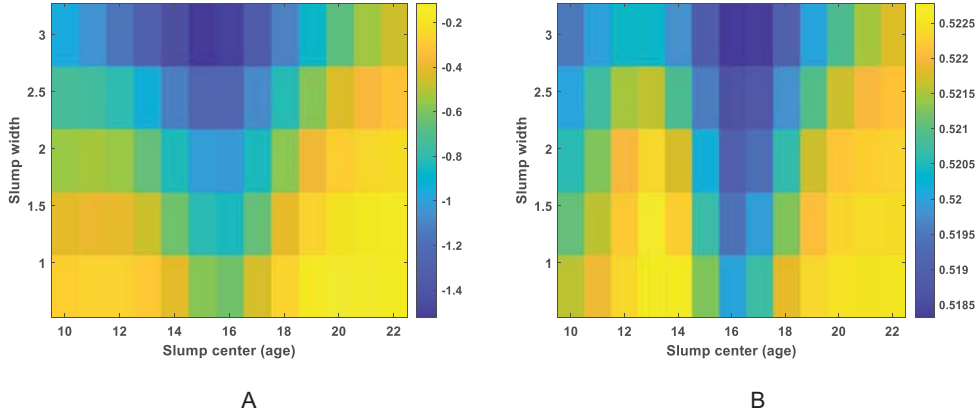


Figure 3. Effects of an age-localized performance slump on retention and pathway composition ($s_p = 0.06$ in Eq. 23). (A) Heatmap of $\Delta\mathbb{E}[T]$ (change in expected seasons until dropout relative to baseline) as a function of slump center (x-axis) and slump width (y-axis). (B) Heatmap of the fraction of dropout events occurring from the Lapsed state under the same slump parameter, showing how the pathway composition changes across slump timing and duration.

Finally, performance support is treated as a limited resource, and the goal is to determine how a fixed total support budget should be timed across ages to maximize fencing retention. Support is represented as an age-specific additive boost $\Delta P(a)$ applied to the baseline performance signal:

$$P(a) \mapsto P(a) + \Delta P(a), \quad \sum \Delta P(a) = D_p \quad (24)$$

where $D_p > 0$ denotes the fixed total performance-support budget. To isolate timing effects, the same total budget is allocated uniformly over a consecutive intervention age window $\mathcal{W}$ of length L :

$$\Delta P(a) = \begin{cases} D_p/L, & a \in \mathcal{W} \\ 0, & a \notin \mathcal{W} \end{cases} \quad (25)$$

Fig. 4 reports $\Delta\mathbb{E}[T]$ as $\mathcal{W}$ is slid across ages (x-axis denotes the $\mathcal{W}$ window midpoint age). There exists a strong timing effect: for fixed L and D_p , retention gains peak when support is concentrated in the mid-teen years and then decline as the same window is shifted to later ages. This indicates that, within the present parameterization, performance-support interventions are most effective when timed to early-to-mid adolescence. In practice, the performance-support budget may be interpreted broadly as age-targeted support that improves competitive experience and perceived progress; specific policy examples are discussed in Section 3.5.

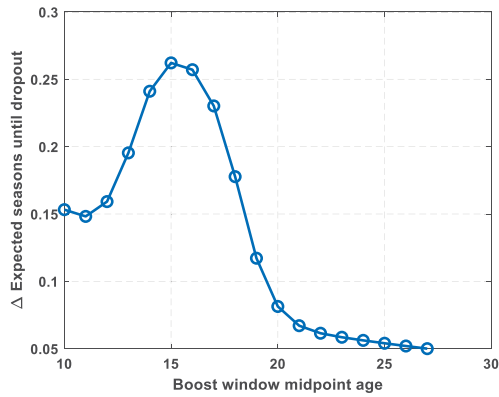


Figure 4. Timing of performance-support under a fixed performance-support budget. Change in expected seasons until dropout, $\Delta \mathbb{E}[T]$, as a fixed-length performance-support window is slid across ages. The x-axis is the window midpoint age; the y-axis is the retention gain relative to baseline.

3.2. Study B: Effects of age-dependent vulnerability on fencing retention and dropout

Study B examines age-localized vulnerability using a Gaussian-shaped risk term $B(a)$ (Eq. 13) parameterized by a center c and width w , with amplitude s held fixed. For each (c, w) pair, closed-form outcomes are computed, including expected seasons until dropout $\mathbb{E}[T]$, expected time spent Active and Lapsed, the dropout-age distribution, and the fraction of dropouts originating from the Lapsed state.

In the model, the risk center c represents when population-level pressures associated with disengagement are most concentrated (e.g., school transitions or rising competitive demands). Shifting c moves the same vulnerability profile earlier or later along the age axis. **Fig. 5** shows a monotonic timing effect: moving the risk center earlier (smaller c) shortens the expected time to dropout, whereas delaying the peak into later adolescence increases expected retention. This means that the timing of vulnerability matters: the same overall risk can have different effects depending on whether it occurs earlier or later. **Fig. 6** isolates the effect of vulnerability width w while holding the center fixed ($c = 16$). As w increases, $\Delta \mathbb{E}[T]$ decreases monotonically, indicating that vulnerability spread across multiple seasons shortens expected participation more than a similarly scaled but shorter-lived increase in vulnerability. This suggests that sustained moderate risk may have a greater effect on dropout than a brief high-risk period.

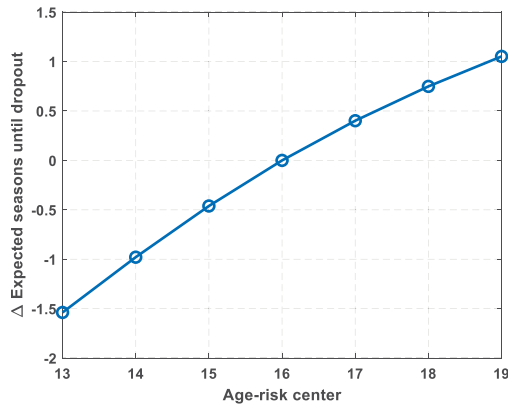


Figure 5. Sensitivity of retention to the timing of age-localized vulnerability. Each point reports the change in expected seasons until dropout, $\Delta \mathbb{E}[T]$, relative to baseline as the vulnerability center c is varied.

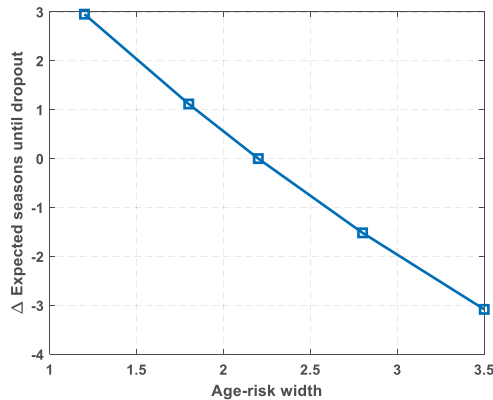


Figure 6. Sensitivity of retention to age vulnerability width. Each point shows the change in expected seasons until dropout, $\Delta \mathbb{E}[T]$, relative to the baseline case ($w = 2.2$) as the vulnerability width w is varied while holding the center fixed ($c = 16$).

Beyond its effect on $\Delta \mathbb{E}[T]$, vulnerability width w also reshapes the timing of dropout. **Fig. 7** compares dropout-age probability curves under a fixed vulnerability center and amplitude ($c = 16$, $s = 1.0$) while varying w (narrow: $w = 1.2$; baseline: $w = 2.2$; wide: $w = 3.5$). As w increases, vulnerability extends across more seasons and dropout probability is distributed over a broader age range, flattening the modal peak and increasing probability at adjacent ages. By contrast, a narrow vulnerability window produces a sharper and more concentrated dropout peak. These patterns suggest that narrow risk windows favor precisely timed support, whereas broader risk windows favor support sustained across multiple neighboring ages.

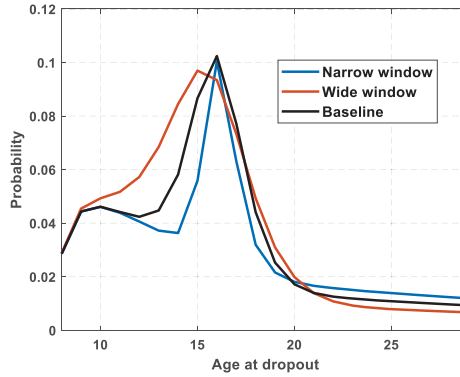


Figure 7. Probability distributions of dropout age under narrow and wide age-risk windows compared with the baseline case.

Extending the one-factor analyses, **Fig. 8** maps $\Delta E[T]$ jointly over the risk center c and width w . The heatmap displays a clear directional trend: increasing w decreases $\Delta E[T]$, while increasing c increases $\Delta E[T]$. The diagonal contours of constant $\Delta E[T]$ indicate a timing–duration tradeoff, where delaying peak vulnerability can partially offset broader vulnerability windows. In particular, when vulnerability is narrow ($w \approx 1 - 1.5$), shifting c later yields a large retention benefit (contours are widely spaced along c); when vulnerability is broad ($w \gtrsim 3$), $\Delta E[T]$ remains negative across the full c -range, indicating that persistent vulnerability depresses retention even if the peak occurs later.

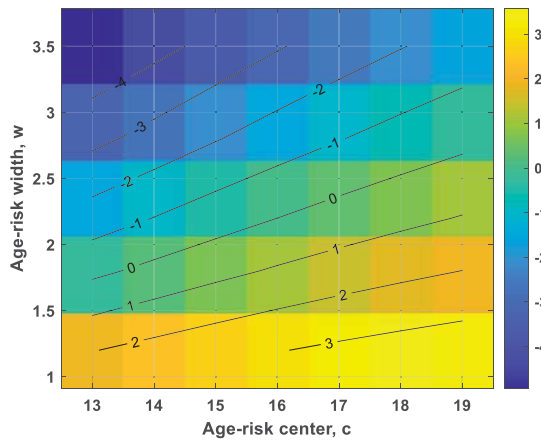


Figure 8. Joint sensitivity of retention to the timing and duration of age-related vulnerability.

To connect the retention effects in **Figs. 5–8** to dropout pathway structure, **Fig. 9** reports the fraction of dropout events originating from the Lapsed state as a function of (c, w) . The fraction is tightly bounded (≈ 0.52 – 0.53), indicating that roughly half of dropouts occur via the Lapsed-state route across a wide range of risk timings and durations. Systematic variation is modest but clear: the lapsed-origin share increases with broader vulnerability windows (larger w), while dependence on the risk center c is weak, showing only slight shifts across the tested range. Practically, this reinforces Lapsed as a consequential

intermediate state: interventions that reduce entry into lapse or improve return from lapse can influence fencing dropout pathways even as overall retention changes with c and w .

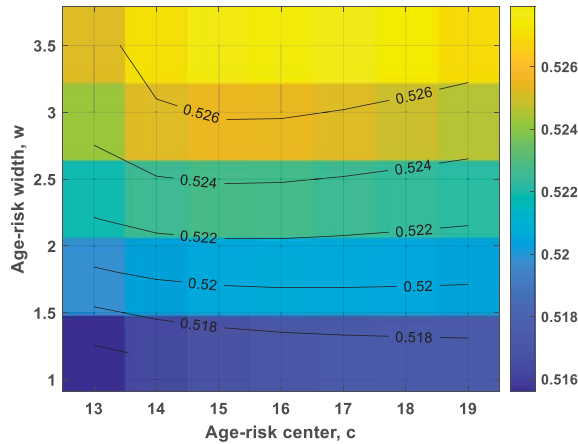


Figure 9. Joint sensitivity of the fraction of dropouts originating from the Lapsed state to the timing and duration of age-related vulnerability.

Overall, Study B shows that retention in the illustrative baseline scenario is strongly shaped by the placement and breadth of age-localized vulnerability, with changes in the center c and width w affecting both expected retention and the dropout-age distribution. Temporary lapse remains an important pathway across the tested parameter range, with roughly half of modeled dropout occurring through the Lapsed state. These findings suggest that age-targeted support and reduced barriers to re-entry may merit closer empirical evaluation than age-uniform approaches.

3.3. Study C: Lapse as a dominant leverage point in fencing retention dynamics

Building on Study B's vulnerability analyses, Study C shifts attention to a more actionable mechanism: temporary lapse as the key intermediate state before dropout. Interventions are modeled as additive shifts to the transition probabilities, i.e., Active→Lapsed (Δp_{AL}) and Lapsed→Active (Δp_{LA}), applied either globally or within selected age windows. The analysis begins with a global shift in lapse probability. **Fig. 10A** quantifies the sensitivity of expected seasons until dropout, $\Delta E[T]$, to a uniform additive shift Δp_{AL} applied across non-terminal ages. Expected retention responds monotonically: reducing lapse probability ($\Delta p_{AL} < 0$) increases expected seasons until dropout, whereas increasing lapse ($\Delta p_{AL} > 0$) shortens participation. Over the tested range, the response is near-linear with mild curvature, with slightly smaller marginal losses at larger positive Δp_{AL} . **Fig. 10B** decomposes $\Delta E[T]$ into changes in expected seasons spent Active and Lapsed. Most of the gain from reducing lapse appears as increased Active time ($\Delta E[T_A]$), while expected Lapsed time ($\Delta E[T_L]$) changes only modestly. This indicates that lowering p_{AL} primarily extends participation by keeping fencers Active longer, rather than by shifting time into prolonged lapse.

While **Fig. 10** proves that lapse is influential overall, **Fig. 11** shows that its impact depends strongly on age. In **Fig. 11A**, the same lapse reduction $\Delta p_{AL} = -0.05$ is applied to one age bin at a time, and the

resulting $\Delta E[T]$ is plotted against the targeted age. Sensitivity varies markedly across the age axis, with the largest gains occurring from late childhood through mid-adolescence (peaking in the early-to-mid teen bins), followed by a step-down and a gradual decline at older ages. Preventing lapse in this age range has a larger effect because it keeps more fencers Active in subsequent seasons, reducing later opportunities to progress toward dropout. **Fig. 11B** shows that these gains arise mainly from additional Active seasons, while expected Lapsed time varies only weakly and is often slightly reduced—consistent with interventions that reduce entry into lapse rather than prolonging disengagement. Thus, the modeled retention benefit of lapse reduction is strongly age-dependent and is largest before late adolescence.

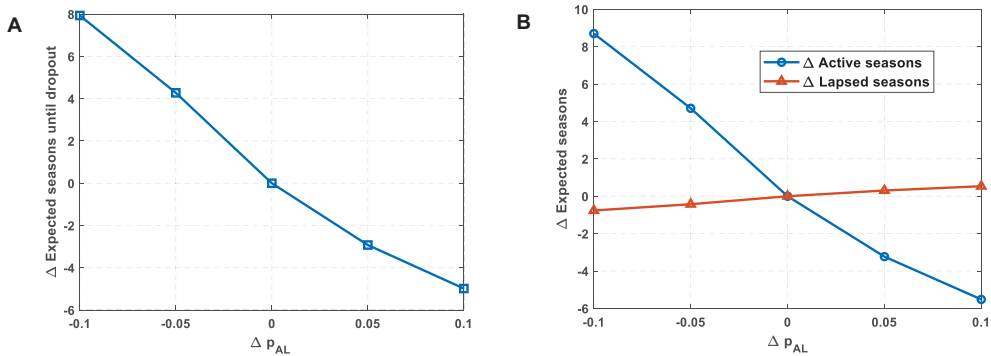


Figure 10. Effect of a global lapse change on retention and its Active/Lapsed decomposition.

(A) Change in expected seasons until dropout ($\Delta E[T]$) under a uniform additive shift Δp_{AL} to the Active→Lapsed transition probability applied across all non-terminal ages (≤ 29). (B) Decomposition of the same effect into changes in expected seasons spent Active and Lapsed versus Δp_{AL} . By construction, $\Delta E[T] = \Delta E[T_A] + \Delta E[T_L]$.

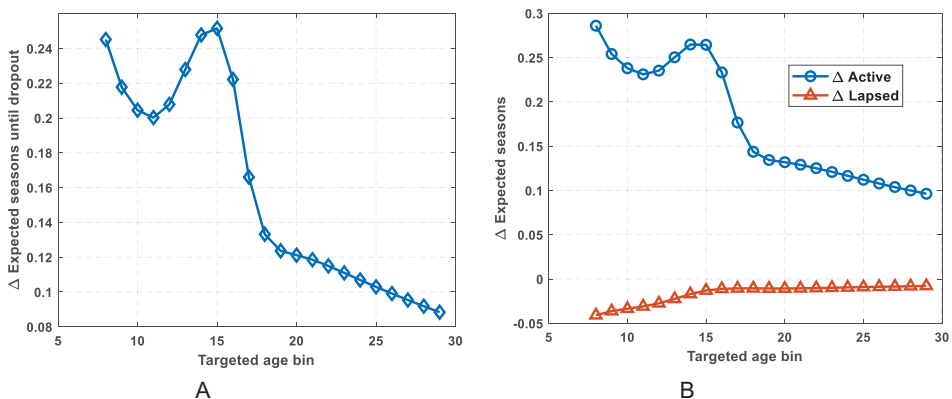


Figure 11. Age-specific impact of lapse reduction and decomposition of retention gains. (A) $\Delta E[T]$ obtained by applying a fixed additive lapse reduction ($\Delta p_{AL} = -0.05$) to the Active → Lapsed transition at one targeted age bin at a time. (B) Decomposition of the same age-targeted effect into changes in $\Delta E[T_A]$ and $\Delta E[T_L]$.

Fig. 12 shows how global shifts in the Active→Lapsed probability affect the dropout-age distribution. Reducing lapse ($\Delta p_{AL} < 0$) lowers dropout probability across the modeled age range, with the largest reductions at younger ages and near the modal peak, thereby shifting dropout toward later ages, including the terminal 30+ bin. In contrast, increasing lapse ($\Delta p_{AL} > 0$) raises dropout probability at younger ages and shifts more dropout risk earlier in the participation pathway. Thus, changes in lapse probability affect not only mean retention $\mathbb{E}[T]$ but also the age distribution of dropout.

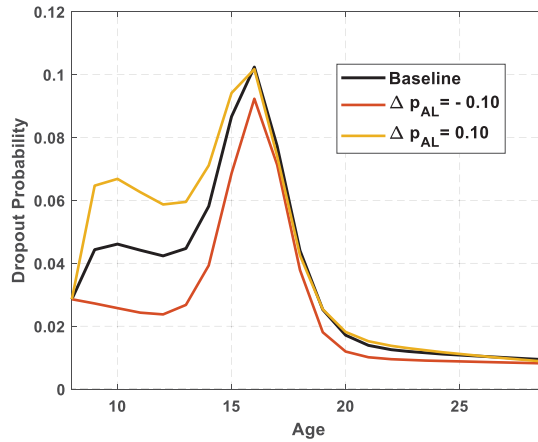


Figure 12. Dropout-age distributions under global lapse shifts. Dropout-age probability curves are shown for the baseline and for uniform additive shifts Δp_{AL} to the Active→Lapsed transition probability applied across non-terminal ages. Curves are plotted over ages $a \leq 29$ for comparability within the modeled range.

To connect these distributional shifts to pathway structure, **Fig. 13** plots the fraction of dropout events originating from the Lapsed state as Δp_{AL} varies. The lapsed-origin fraction is about half at baseline ($\Delta p_{AL} = 0$) and increases monotonically as Δp_{AL} increases. This indicates that dropout is increasingly routed through lapse rather than occurring directly from Active participation. Conversely, reducing lapse decreases the lapsed-origin share, consistent with preventing entry into the intermediate state. These results further support the structural importance of lapse incidence in shaping long-run dropout pathways.

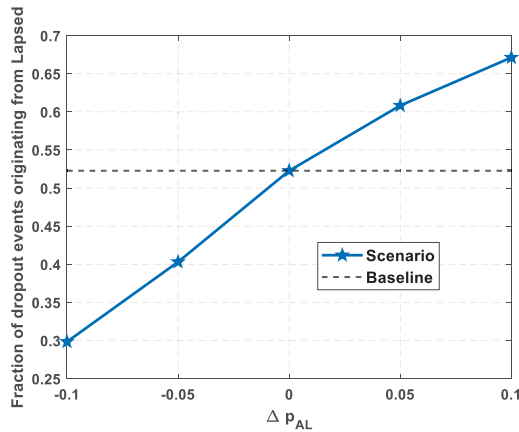


Figure 13. Fraction of dropouts originating from the Lapsed state under global lapse shifts.

The plotted value is the fraction of dropout events that occur from the Lapsed state as the uniform shift Δp_{AL} to the Active→Lapsed probability varies.

Finally, **Fig. 14** evaluates the combined effects of shifting lapse entry (Δp_{AL}) and boosting return (Δp_{LA}). The heatmap is dominated by a strong horizontal gradient: reducing lapse produces large retention gains, whereas increasing lapse substantially shortens expected participation. In contrast, changes in return probability (Δp_{LA}) have smaller effects across most of the grid, indicating that—within the tested range—retention is more sensitive to preventing lapse onset than to increasing return once lapsed. Increasing return provides incremental gains and can partially offset moderate increases in lapse, but they do not fully counteract large lapse increases. Together, **Figs. 10–14** indicate that lapse reduction is the primary intervention target, with the greatest payoff predicted by the model in the mid-teen range, while return-focused supports provide secondary benefits.

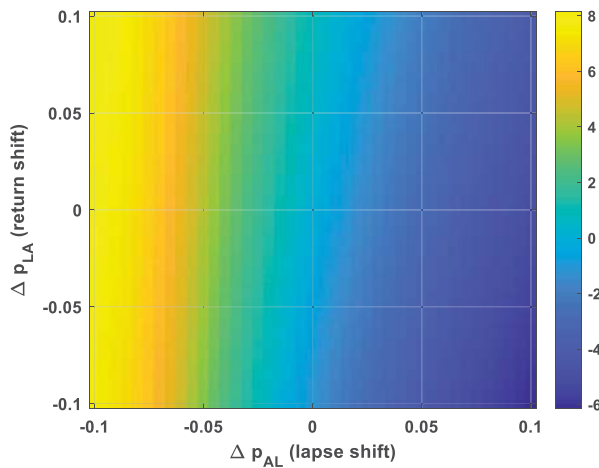


Figure 14. Joint sensitivity of retention to lapse and return interventions. The heatmap shows the change in expected seasons until dropout, $\Delta E[T]$, under simultaneous shifts in the Active→Lapsed transition probability (Δp_{AL} , x-axis;

lapse shift) and the Lapsed→Active transition probability (Δp_{LA} , y-axis; return boost).

3.4. Study D: Timing and Budgeted Allocation of Lapse-Reduction Interventions

Study D examines how a fixed lapse-reduction budget should be allocated across ages. Following the same fixed-budget, uniform-over-window formulation as **Eqs. 24–25**, the budget is implemented as an age-specific negative additive shift to the Active→Lapsed transition probability $p_{AL}(a)$. All strategies are compared at the same total budget $D_{AL} = \sum \Delta p_{AL}$ by distributing the budget uniformly across the targeted age set $\mathcal{W}$, so that each targeted age receives $D_{AL}/|\mathcal{W}|$.

Figs. 15A–B compare three ways of allocating the same total lapse-reduction budget: an Early window (ages 10–14), a Late window (15–19), and a Uniform strategy (ages 8–29). As D_{AL} becomes more negative, both outcomes improve monotonically: $\Delta \mathbb{E}[T]$ increases in **Fig. 15A** and $\Delta \mathbb{E}[T_A]$ (expected Active seasons) increases in **Fig. 15B**. The key result is that timing changes the return on the same budget. Across all budget levels, the Early window yields the largest gains, the Late window is intermediate, and the Uniform strategy is smallest. Reducing lapse earlier keeps more fencers Active in subsequent seasons, lowering later opportunities to enter lapse and progress toward dropout. By contrast, spreading the same budget across many ages reduces the per-age effect and allocates effort to ages where lapse reduction has smaller payoff. For retention planning, this implies that a fixed amount of support yields larger gains when concentrated earlier rather than delayed.

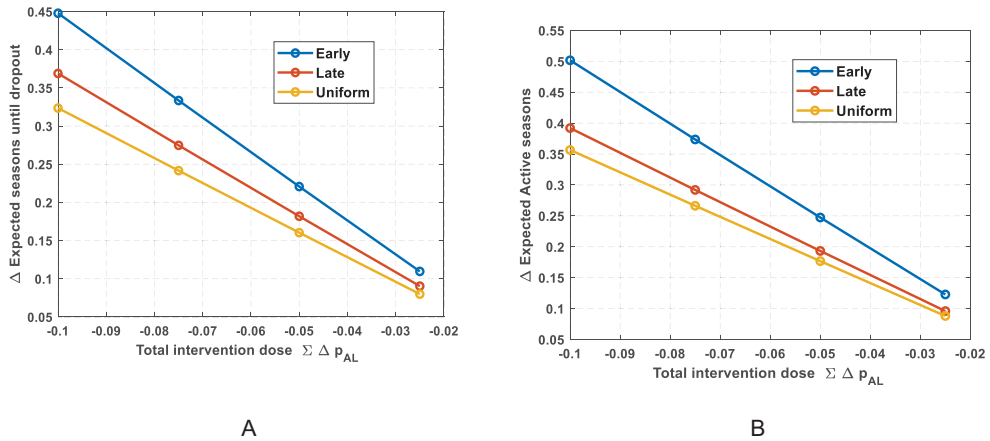


Figure 15. Timing effects under a fixed total lapse-reduction budget (retention gains). (A) $\Delta \mathbb{E}[T]$ plotted against the total intervention budget $\sum_a \Delta p_{AL}(a)$. (B) $\Delta \mathbb{E}[T_A]$ plotted against the same total budget.

Even with the same total intervention budget ($D_{AL} = -0.05$ for every candidate window), the payoff depends strongly on when lapse reduction is applied. **Fig. 16** plots the change in $\Delta \mathbb{E}[T]$ for a fixed-length window ($L = 5$ age bins) versus the window midpoint age. The curve rises as the window moves into early adolescence, peaks at the best window (ages 12–16), and then declines as the same window is shifted toward older age bins. Thus, equal intervention budget doesn't yield equal retention gains: the

same total lapse-reduction budget is most effective when placed in a narrow age band where dropout is most sensitive to lapse. The dashed Early/Late reference windows illustrate this contrast directly—Early lies near the high-gain region, whereas Late falls on the declining side despite identical total budget.

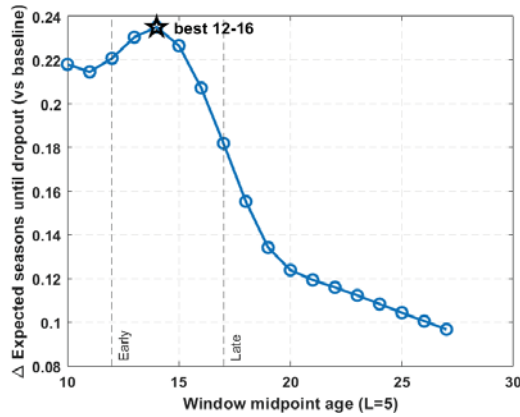


Figure 16. Timing sensitivity of lapse-reduction under a fixed total budget. $\Delta E[T]$ is obtained by sliding a fixed-length lapse-reduction window ($L = 5$ age bins) across ages $a \leq 29$ while holding the total budget fixed ($\sum_a \Delta p_{AL}(a) = -0.05$). The star marks the best-performing window (ages 12–16).

With the intervention budget held fixed ($D_{AL} = -0.05$), a natural design question is whether to spread effort across many ages or concentrate it within a shorter span. **Fig. 17** summarizes this tradeoff by constructing a “best-achievable” curve: for each coverage length L , all consecutive L -age windows are evaluated under the same total budget distributed uniformly across the targeted ages, and the maximum resulting $\Delta E[T]$ is recorded. Retention gains are largest when the intervention is concentrated in a short window, and the best-achievable $\Delta E[T]$ decreases as L increases. The star marks the best overall design, indicating that the highest-impact ages are concentrated around 14–15. This decline is expected: with fixed budget, expanding coverage reduces the per-age adjustment ($\Delta p_{AL} \propto 1/L$). Once the window extends beyond the highest-impact ages, adding extra ages contributes little and mainly weakens the intervention where it matters most.

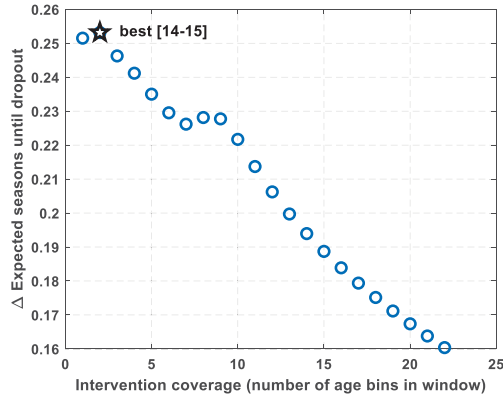


Figure 17. Best-achievable retention gain versus intervention coverage under a fixed budget. For each window length L , the plotted value is the maximum $\Delta\mathbb{E}[T]$ obtained by applying a consecutive window of length L within ages ≤ 29 , holding the total budget fixed at $\sum_a \Delta p_{AL}(a) = -0.05$. The starred point marks the single best-performing window across all L , with the corresponding age range labeled.

Given the decline in best-achievable gains with increasing L , **Fig. 18** reports where the optimal window is placed for each coverage length. For small L , the optimal windows cluster tightly in the mid-teen range, consistent with the high-gain timing band shown in **Fig. 16**. As L increases, the optimal start age remains near the earliest high-impact ages while the end age expands to include later ages. In practical terms, once a coverage length L is selected, **Fig. 18** predicts the start and end ages of the window that maximizes $\Delta\mathbb{E}[T]$ under the fixed total budget.

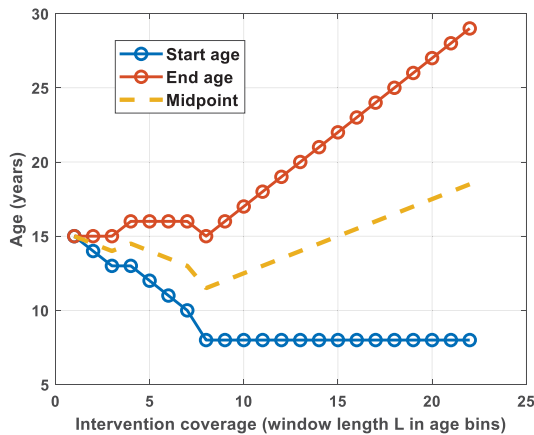


Figure 18. Optimal lapse-reduction window location versus intervention coverage under a fixed budget. For each window length L , the plot reports the start and end ages of the consecutive window that maximizes $\Delta\mathbb{E}[T]$ under $\sum_a \Delta p_{AL}(a) = -0.05$. The dashed curve shows the corresponding window midpoint age.

Across **Figs. 15–18**, holding the total lapse-reduction **budget** fixed shows that timing and concentration can matter as much as overall intervention strength. Earlier, well-timed lapse reduction yields larger retention gains than late or uniformly distributed allocations under the same budget. **Fig. 16** identifies a narrow age band where lapse reduction is most valuable, **Fig. 17** shows diminishing returns as coverage widens and the per-age effect is diluted, and **Fig. 18** predicts the optimal start and end ages for each coverage choice. In applied terms, the model indicates that earlier and more concentrated lapse reduction yields larger retention gains than delayed or diffuse targeting within the present parameterization.

3.5. Practical implications for youth fencing

Across Studies A–D, the model identifies temporary disengagement (“Lapsed”) as a structurally important intermediate stage rather than a minor detour on the pathway to dropout. In the illustrative baseline regime, the largest modeled retention gains arise from reducing entry into lapse at younger ages, whereas improving return from lapse provides smaller but still meaningful secondary benefits. In this sense, the framework points to two broad priorities for retention-oriented program design: reducing the likelihood of lapse during high-sensitivity ages and lowering barriers to re-entry for fencers who temporarily disengage.

For empirical application, lapse should be defined by observable participation markers rather than by self-reported intent. Depending on available records, a Lapsed episode may correspond to one or more seasons without competition participation, a membership non-renewal gap followed by later renewal, or a sustained absence from club activity when such data are available. Estimated lapse-mediated dropout will depend on the operational definition used, and competition-only records may under-detect temporary disengagement. Nevertheless, the central implication is robust: if a substantial share of dropout is preceded by measurable inactivity, then lapse becomes an actionable stage at which timely intervention may prevent permanent exit.

Although the present model is not calibrated to USA Fencing participation records, it suggests several broad intervention classes that may merit empirical evaluation. One class involves reducing barriers to continued participation during the high-sensitivity adolescent years, for example by lowering cost and logistical burdens, improving access to coaching or development opportunities, and reducing friction associated with competition participation. A second class involves improving competitive experiences in ways that limit discouragement and temporary withdrawal, such as formats or recognition structures that increase meaningful participation, reduce repeated one-sided outcomes, and support perceived progress. A third class involves creating low-friction re-entry pathways for athletes who lapse, including outreach, flexible reactivation opportunities, and transitional participation formats that make return easier after a period of inactivity.

These implications should be interpreted as hypothesis-generating rather than as validated policy prescriptions. Because the present study is not calibrated to longitudinal participation records, the practical value of any specific intervention will ultimately need to be tested against observed participation histories. Still, the model provides a transparent framework for identifying where such

efforts may be most consequential: around lapse onset, during key adolescent windows, and in settings where temporary disengagement is common before permanent dropout.

3.6. Limitations and calibration needs

The parameters used here are selected for controlled sensitivity analysis rather than estimated from observed fencing histories. Accordingly, the model is intended to provide structural and mechanistic insight, not quantitative forecasts for specific programs or individual athletes. Empirical calibration would require longitudinal participation records, including age-by-season competition participation, membership renewal gaps to identify lapse and re-entry, performance measures such as classification, rating, or win-loss history, and contextual variables such as club access, travel burden, event density, and fees. These data would allow estimation of age-specific transition probabilities and evaluation of whether the model-predicted high-leverage windows are consistent with observed dropout patterns. In addition, the present formulation assumes season-to-season Markovian dynamics and does not account for richer history dependence or heterogeneity across weapon, sex, club, region, or socioeconomic background.

4. CONCLUSION

This study demonstrates that youth fencing dropout is shaped not only by permanent attrition risk but also by temporary disengagement pathways that accumulate across age-specific participation trajectories. To capture this process, we developed an age-embedded absorbing Markov framework with a Lapsed state, allowing short-term participation interruptions to be distinguished from permanent dropout while enabling closed-form retention metrics, including expected seasons until dropout, expected time spent Active or Lapsed, and the full dropout-age distribution. The sensitivity and intervention analyses show that retention is most strongly influenced by the timing and duration of age-localized vulnerability, particularly during adolescence, whereas performance-related effects are comparatively secondary. Interventions that reduce entry into lapse produced the largest model-predicted gains, and under fixed budgets, earlier and more focused strategies consistently outperformed later or broadly distributed allocations. These results directly address the study's objective of identifying key mechanisms and high-leverage age windows for retention, highlighting temporary disengagement as both a structural gateway to dropout and an actionable intervention target. More broadly, the framework provides a transparent and tractable tool for comparing age-targeted retention strategies in USA Fencing and other youth participation systems. Future work should calibrate the model with longitudinal participation records to test these mechanisms empirically and support evidence-based retention policy design.

Acknowledgments

The author gratefully acknowledges support from the Harvard University Extension School High School Student Scholarship during Fall 2025 and Spring 2026. Coursework in Multivariable Calculus and Linear Algebra, along with discussions with instructors during this period, contributed to the mathematical foundation and development of the modeling framework presented in this study.

Appendix A

Table A1 summarizes all parameters used in the absorbing Markov chain model. Values were selected to represent a plausible, non-extreme regime in which no single transition dominates, and conclusions are based on relative sensitivities rather than absolute calibration.

Table A1. Model parameters used in the absorbing Markov chain analysis

(a) Global configuration parameters			
Parameter	Symbol	Value	Description
Age bins	a	8–30	Discrete age states (years)
Probability cap	—	0.95	Upper bound on competing transition probabilities
Performance scaling	—	1.0	Scaling factor for performance driver

(b) Age-localized vulnerability (risk) parameters			
Parameter	Symbol	Value	Interpretation
Risk center	c	16	Age at which population-level vulnerability is maximal
Risk width	w	2.2	Temporal spread of vulnerability across ages
Risk scale	s	1.0	Strength of age-related vulnerability

(c) Encouragement (latent mediator) parameters			
Parameter	Symbol	Value	Description
Baseline encouragement	θ_0	−0.2	Encouragement at zero performance and risk
Performance sensitivity	θ_P	2.0	Effect of performance on encouragement
Age-risk sensitivity	θ_R	−0.6	Suppression of encouragement under age risk

(d) Active → Lapsed transition (A → L)			
Parameter	Symbol	Value	Meaning
Baseline lapse bias	α_{AL}	−2.0	Baseline propensity to lapse
Performance effect	β_P	−3.0	Better performance reduces lapse
Encouragement effect	β_E	−1.2	Encouragement reduces lapse
Age-risk effect	β_R	1.0	Age risk increases lapse

(e) Active → Dropped transition (A → D)			
Parameter	Symbol	Value	Meaning
Baseline dropout bias	α_{AD}	−3.0	Baseline direct dropout probability
Performance effect	γ_P	−3.8	Poor performance increases dropout
Encouragement effect	γ_E	−1.6	Encouragement reduces dropout
Age-risk effect	γ_R	1.5	Age risk increases direct dropout

(f) Lapsed → Active transition (L → A)			
Parameter	Symbol	Value	Meaning
Baseline return bias	α_{LA}	−0.6	Baseline likelihood of return
Performance effect	ρ_P	0.6	Better performance increases return
Encouragement effect	ρ_E	0.8	Encouragement increases return
Age-risk effect	ρ_R	−0.8	Age risk suppresses return

(g) Lapsed → Dropped transition (L → D)			
Parameter	Symbol	Value	Meaning
Baseline dropout bias	α_{LD}	−1.2	Baseline dropout from lapse
Age-risk effect	δ_R	0.8	Age risk increases dropout from lapse

5. REFERENCES

- Back, J., Johnson, U., Svedberg, P., McCall, A. & Ivarsson, A. 2022. Drop-out from team sport among adolescents: A systematic review and meta-analysis of prospective studies. *Psychology of Sport and Exercise*, **61**, 102205.
- Battaglia, A., Kerr, G. & Tamminen, K. 2024. The Dropout From Youth Sport Crisis: Not as Simple as It Appears. *Kinesiology Review*, **13**, 345-356.
- Bengtsson, D., Svensson, J., Wiman, V., Stenling, A., Lundkvist, E. & Ivarsson, A. 2025. Health-related outcomes of youth sport participation: a systematic review and meta-analysis. *International Journal of Behavioral Nutrition and Physical Activity*, **22**, 89.
- Chen, T. L.-W., Wong, D. W.-C., Wang, Y., Ren, S., Yan, F. & Zhang, M. 2017. Biomechanics of fencing sport: A scoping review. *PLOS ONE*, **12**, e0171578.
- Eliasson, I. & Johansson, A. 2021. The disengagement process among young athletes when withdrawing from sport: A new research approach. *International Review for the Sociology of Sport*, **56**, 537-557.
- Ganakas, E. & Peden, A. E. 2023. Exploring why young Australians participate in the sport of fencing: Future avenues for sports-based health promotion. *Health promotion journal of Australia*, **34**, 48-59.
- John G. Kemeny, J. L. S. 1976. *Finite Markov Chains*, New York, Springer.
- Jr., D. W. H., Lemeshow, S. & Sturdivant, R. X. 2013. *Applied Logistic Regression* (3rd ed.). Wiley.
- Kim, Y.-J. & Kim, E.-S. 2021. Analysis of Korean Fencing Club Members' Participation Intention Using the TPB Model. *International Journal of Environmental Research and Public Health*, **18**, 2813.
- Lara-Bercial, S., Jowett, G. E., Gledhill, A., McKenna, J., Hill, M. & Sargent Megicks, B. 2025. Part I: Why Do Children and Young People Drop Out of Sport? Development and Initial Validation of the Youth Sport Dropout Questionnaire. *Youth*, **5**, 50.
- Norris, J. R. 1997. *Markov Chains*, Cambridge, Cambridge University Press.
- Roi, G. S. & Bianchedi, D. 2008. The Science of Fencing. *Sports Medicine*, **38**, 465-481.
- Turner, A., James, N., Dimitriou, L., Greenhalgh, A., Moody, J., Fulcher, D., Mias, E. & Kilduff, L. 2014. Determinants of olympic fencing performance and implications for strength and conditioning training. *J Strength Cond Res*, **28**, 3001-11.
- Zhang, M., Wang, X.-C. & Shao, B. 2022. Predictors of Persistent Participation in Youth Sport: A Systematic Review and Meta-Analysis. *Frontiers in Psychology*, **Volume 13 - 2022**.
- Zhang, Y., Wang, F., Szakál, Z., Bíró, Z., Kovács, M., Őrsi, B. & Kovács, K. E. 2024. Why do students drop out of regular sport in late adolescent? The experience of a systematic review. *Frontiers in Public Health*, **Volume 12 - 2024**.