

Pinpointing the Unique Exponent Configuration for a Specific Exponential Diophantine Equation

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ABSTRACT

We established the complete solution set for a specific exponential Diophantine equation involving polynomial bases with three unknown exponents (k, m, n) . Our analysis demonstrated that the equation possesses a unique positive integer solution for the exponents (k, m, n) , namely $(1, 1, 2)$, which holds true for every positive integer value of the parameter 'w', subject to a set of arithmetic constraints. The proof is constructed through a rigorous synthesis of established arithmetic methods (such as modular congruences and the Quadratic Reciprocity Law) and advanced analytic tools derived from transcendental number theory, specifically leveraging a powerful bound on p -adic measurement of logarithmic sums developed by Bugeaud. The proof's numerical bounds and specific calculations were performed and confirmed using Python code.

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1 Introduction

Exponential Diophantine equations (EDE's) are a class of Diophantine problems where the unknowns appear as exponents rather than as mere coefficients or base-level terms. Suppose that $m, n, p \in \mathbb{Z}_+$ with $m > 1, n > 1, p > 1$ are fixed numbers and are relatively prime. The following equation

$$m^k + n^r = p^h \quad (1.1)$$

is examined by numerous authors. It is observed that there exists limited possible values of (k, r, h) satisfying (1.1), where $k, r, h \in \mathbb{Z}_+$ and all such solutions may be derived through

the application of Baker's techniques of Logarithmic linear combinations. Alan(Alan, 2020) considered the equation

$$(s^2 + s + 1)^x + s^y = (s + 1)^z$$

with $s \geq 1$ and proved that there exist no solution in $\mathbb{Z}_+$ other than $(1, 1, 2)$ when $s \notin \{1, 2, 3\}$. On the other hand, Jinyan He(Jinyan He, 2022) et.al analyzed the equation

$$(b(b - c)u^2 + 1)^u + (bcu^2 - 1)^v = (bu)^w \quad (1.2)$$

where $b, c, u \in \mathbb{Z}_+$ with $b \equiv 1 \pmod{2}$ and $b^2u^2 \equiv -2 \pmod{q}$, q is a prime factor of c . The authors proved a theorem that it has a solution of $(u, v, w) = (1, 1, 2)$ and if $b = c$, then (1.2) has the solutions $(u, v, w) = (n, 1, 2)$, $n \in \mathbb{N}$. The conclusion is rigorously demonstrated through a synthesis of elementary mathematical principles, the application of the Bilu–Hanrot–Voutier theorem (which addresses the presence of primitive divisors within certain recurrent number sets), and auxiliary theorems developed for expanded Ramanujan–Nagell-type Diophantine problems. The same kind of equation can also be solved by Shuanglin Fei(S. Fei, 2022) with some other constraints. Earlier, Terai(Terai, 2012) established a result that the EDE

$$(4s^2 + 1)^w + (5s^2 - 1)^h = (3s)^k \quad (1.3)$$

has one possible solution $(1, 1, 2)$ in $\mathbb{Z}_+$ under the constraint $s \not\equiv 3 \pmod{6}$. He verified the result for $1 \leq s \leq 20$. The same equation is later analysed by J. Su (J. Su, 2014) and it is proven that for $s > 90$ and $3|s$, the equation (1.3) has $(1, 1, 2)$ as the only positive solution and Bertok (Bertok, 2016) found the complete solution for the problem of Terai(Terai, 2012) by considering the case $20 \leq s \leq 90$ and $s \equiv 3 \pmod{6}$. Ruiqin Fu, Hai Yang(R. Fu, 2017) solved the following EDE

$$(aw^2 + 1)^x + (bw^2 - 1)^y = (rw)^z \ni r|w, w > 36r^3 \log r \quad (1.4)$$

and found that $(1, 1, 2)$ are the only values that the variables (x, y, z) takes. Later, Alan(Alan, 2018) established the result using Baker's method for the equation

$$(18u^2 + 1)^v + (7u^2 - 1)^w = (5u)^r.$$

Further, Terai and Hibino(N. Terai, 2016) solved the equation $a^x + lb^y = c^z$ with l is prime such that $l \equiv 3 \pmod{8}$ and $\gcd(a, lb) = 1$. It is proved that this equation has $(2, 2, 1)$ as the only solution which is ascertained using Minimum estimates for linear expressions involving two logarithms found by Laurent(Laurent, 2008).

(Le. M., 2020) displays that Jesmanowicz conjecture on basic Pythagorean triples is covered by Baker's technique. Some special cases of (1.4) can be found in (N. Terai, 2021),(Terai, 2020),(N. Terai, 2020). Some Diophantine equations can be solved using Catalan's conjecture which can be found from (P.B. Borah, 2022),(N. Viriyapong, 2022),(W. Tangjai, 2021),(N. Viriyapong, 2021),(W. Orosram, 2020).

Also, Terai(N. Terai, 2015) found that only $(1, 1, 2) \in \mathbb{Z}_+$ satisfies the equation

$$(12w^2 + 1)^c + (13w^2 - 1)^d = (5w)^e$$

under the condition $w \not\equiv 17, 33 \pmod{40}$ using Baker's method. Hasanalizade(Hasanalizade, 2023) found that through the use of Baker's method, certain basic techniques, and p-adic Logarithmic linear expressions, Diophantine equation

$$(44w^2 + 1)^m + (5w^2 - 1)^n = (7w)^r$$

is solved. Recently, Shanmuga Priya and Janaki (S. Shanmuga Priya, 2025) have conducted a comparative investigation of two distinct exponential Diophantine equations

$$4(C_{14,k})^r + (16Pr_k)^m = n^2 \quad \text{and} \quad (C_{14,k})^r + (4Pr_k)^m = n^2,$$

both constructed using the specialized bases of centered tetradecagonal numbers $(C_{14,k})$ and pronic numbers (Pr_k) . The primary finding reveals a sharp contrast in their behavior: while the first equation possesses a singular integer solution when its internal parameter exceeds one, the second equation is shown to yield a predictable sequence of solutions for all positive integers. The supporting analysis of specific cases reinforces these general solution patterns. In this article, the following equation is considered and solved using Baker's method.

$$(60w^2 + 1)^k + (61w^2 - 1)^m = (11w)^n \quad (1.5)$$

In section 2, preminilaries required for proving the main theorem are mentioned whereas in section 3, main results and lemmas were proved to enhance the establishment of the main theorem. The entire theorem is proved in the section 4.

2 Prolegomena

The task of bounding the solution set for Pillai's equation is accomplished by implementing established analytical findings regarding the smallest non-zero magnitude attainable by a linear combination involving two specific algebraic logarithms. This analysis operates under the condition that the two real algebraic numbers, denoted γ_1 and γ_2 , each satisfy a size requirement: their absolute values must be at least one.

$$\Lambda = d_2 \log \gamma_2 - d_1 \log \gamma_1,$$

where $d_1, d_2 \in \mathbb{Z}_+$. The logarithmic height of degree t acts as a structural invariant for any algebraic number γ , which is defined as

$$h(\gamma) = \frac{1}{t} \left(\log |c_0| + \sum_{j=1}^t \log \max\{1, |\gamma^{(j)}|\} \right)$$

The constant c_0 serves as the numerical factor tied to the maximal exponent of the variable (i.e., the coefficient of the highest degree term, $x^{\deg(\gamma)}$) in the unique irreducible polynomial for γ that has integer coefficients in $\mathbb{Z}[x]$. and $(\gamma^{(j)})_{1 \leq j \leq t}$ are the conjugates of γ . Let $B_1, B_2 \in \mathbb{R}$ such that $B_1 > 1$ and $B_2 > 1$ with

$$\log B_k \geq \max \left\{ h(\gamma_k), \frac{|\log \gamma_k|}{D}, \frac{1}{D} \right\}, \text{ for } k = 1, 2,$$

where D represents the dimension of the compositum $\mathbb{Q}(\gamma_1, \gamma_2)$ over $\mathbb{Q}$.

Now define

$$d' = \frac{d_1}{D \log B_2} + \frac{d_2}{D \log B_1}$$

Considering the corollary 2 produced by Laurent [(Laurent, 2008)] in which with $m = 10$ and $C = 25.2$, the following statement is obtained.

Statement 1:(Laurent(Laurent, 2008)) Let Λ be defined as before with $\gamma_1, \gamma_2 > 1$. If γ_1 and γ_2 are multiplicatively independent. Then

$$\log |\Lambda| \geq -25.2 D^4 \left(\max \left\{ \log d' + 0.38, \frac{10}{D} \right\} \right)^2 \log B_1 \log B_2$$

The proof invokes Bugeaud's theorem (Bugeaud, 1999) regarding p -adic linear forms, specifically under the restriction that the auxiliary parameters are set to unity ($y_1 = y_2 = 1$).

Let q be an odd prime. Let $b_1, b_2 \in \mathbb{Z}_+ - \mathbb{N}$ which are prime to q . Let $g \in \mathbb{Z}_+$ be the minimum number such that the value of $\text{ord}_q(b_1^g - 1)$ and $\text{ord}_q(b_2^g - 1)$ are atleast 1 where $\text{ord}_p(\cdot)$ denotes the p -adic valuation. Suppose that there occurs $T \in \mathbb{R}$ such that

$$\frac{1}{q-1} < T \leq \text{ord}_q(b_1^g - 1)$$

Select the integer

$$\Lambda = b_1^{d_1} - b_2^{d_2},$$

where $d_1, d_2 \in \mathbb{Z}_+$. Let $B_1, B_2 \in \mathbb{R}$ and $B_1 > 1, B_2 > 1$ with

$$\log B_k \geq \max\{\log |b_k|, T \log q\} \quad (k = 1, 2),$$

and set $d' = \frac{d_1}{\log B_2} + \frac{d_2}{\log B_1}$.

Statement 2((Bugeaud, 1999)) Assuming the same notation as above, if b_1 and b_2 are multiplicatively independent, then the upper estimate is given by

$$\text{ord}_q(\Lambda) \leq \frac{36.1g}{T^3(\log q)^4} (\max\{\log d' + \log(T \log q) + 0.4, 6T \log q, 5\})^2 \log B_1 \log B_2$$

3 Main Results

Lemma 3.1. $(k, m, n) = (1, 1, 2) \in \mathbb{Z}_+$ is a unique set of values which satisfies the equation

$$61^k + 60^m = 11^n \quad (3.1)$$

Proof. Taking the equation (3.1) modulo 11,

$$(-5)^k + 5^m \equiv 0 \pmod{11}$$

The above equation implies that both k and m must be odd. Hence by the lemma of Scott, which shows that if $(m, n) = 1$ with $m, n > 1$ and p is prime and an odd number, then the $m^x + n^y = p^z$ has maximum one solution $(x, y, z) \in \mathbb{Z}_+$ while assuming predefined parity for x and y , aside from $(m, n, p) = (3, 10, 13)$ (taking $m < n$), it is obtained that $(k, m, n) = (1, 1, 2)$ is a only set of values which satisfies (3.1). $\square$

Lemma 3.2. *In the equation (1.5), m is odd.*

Proof. Due to the constraint $n \geq 2$, the next step involves an arithmetic analysis of equation (1.5) using congruence modulo w^2 .

$$\begin{aligned} 1 + (-1)^m &\equiv 0 \pmod{w^2} \\ \Rightarrow m &\text{ is odd.} \end{aligned} \quad \square$$

Lemma 3.3. *The equation (1.5) is satisfied by only one solution $(k, m, n) = (1, 1, 2) \in \mathbb{N}$ for even values of w .*

Proof. For $n \leq 2$, it is observed from (1.5) that $(k, m, n) = (1, 1, 2)$ is the solution. Hence, assume that $n \geq 3$, then on taking (1.5) modulo w^3 gives

$$\begin{aligned} 60w^2k + 1 + 61w^2m - 1 &\equiv 0 \pmod{w^3} \\ \Rightarrow 60k + 61m &\equiv 0 \pmod{w} \end{aligned}$$

This is impossible because of the fact that w is even and m is odd. Hence, one can conclude that only $(k, m, n) = (1, 1, 2)$ satisfies (1.5) $\square$

Lemma 3.4. *If w is odd with $w \not\equiv_{11} 0$ satisfying the following conditions*

- $w \equiv_{11} \pm 1$
- $w \equiv_{11} \pm 2$ and $w \not\equiv_8 1$.

then $m = 1$ and k is odd.

Proof. For any odd integer w that satisfies the existing limitations, the task is to formally establish the parity of the exponents, showing that k must be odd (i.e., $k \equiv 1 \pmod{2}$) and n must be even (i.e., $n \equiv 0 \pmod{2}$). The entire procedure is naturally segmented into two separate analytical cases:

Case (i) If $w \equiv \pm 1 \pmod{11}$, then $\left(\frac{61w^2 - 1}{60w^2 + 1}\right) = 1$ and $\left(\frac{11w}{60w^2 + 1}\right) = -1$

where $\left(\frac{*}{*}\right)$ indicates the Jacobi symbol. Since $w \equiv \pm 1 \pmod{11}$, taking (1.5) modulo $(60w^2 + 1)$ gives

$$1 \equiv (-1)^n \pmod{61}$$

This implies n must be even.

Suppose that $m \geq 2$, letting (1.5) modulo 8,

$$5^k \equiv (11w)^n \equiv 1 \pmod{8}$$

Hence, k is even.

However, since $m \equiv \pm 1 \pmod{11}$, (1.5) gives

$$6^k + 5^m \equiv 0 \pmod{11}$$

This produces a contradiction to the fact that k is even and m is odd. Therefore, $m = 1$.

Also (1.5) yields

$$5^k + 4 \equiv (11w)^n \equiv_8 1$$

$\Rightarrow k$ is odd.

Case (ii) $w \equiv_{11} \pm 2$ and $w \not\equiv_8 1$

From the equation (1.5),

$$(-1)^k + 1 \equiv 0 \pmod{11}$$

$\Rightarrow k$ is odd.

Assume that $m \geq 2$. Now, taking (1.5) modulo 8 yields

$$5^k \equiv (11w)^n \equiv 1 \pmod{8}$$

The above equation implies that k is even, which is a contradiction since k is odd and $w \not\equiv 1 \pmod{8}$. Thus $m = 1$. □

Lemma 3.5. $k \geq \frac{1}{60}(w^2 - 61)$

Proof. Since $k \geq 3$, (1.5) give rise to the following inequality:

$$(11w)^n = (60w^2 + 1)^k + 61w^2 - 1 \geq (60w^2 + 1)^3 + 61w^2 - 1 > (11w)^3$$

$\Rightarrow n \geq 4$. From (1.5), it is obtained that

$$\begin{aligned} 1 + 60w^2k + 61w^2 - 1 &\equiv 0 \pmod{w^4} \\ \Rightarrow w^2(60k + 61) &\equiv 0 \pmod{w^4} \end{aligned}$$

The above equation yields our claim. □

4 Main Theorem

Theorem 4.1. Let $w \in \mathbb{Z}_+$ such that the following two constraints hold:

(i) $w \equiv \pm 1 \pmod{11}$

(ii) $w \equiv \pm 2 \pmod{11}$ and $w \not\equiv 1 \pmod{8}$,

then the EDE (1.5) has a unique solution $(k, m, n) = (1, 1, 2) \in \mathbb{Z}_+$

Proof. The proof involves the following cases:

Case (i): $w = 1$

From lemma 3.1, it is proved that when $w = 1$, the equation (1.5) has $(k, m, n) = (1, 1, 2)$ as the only solution in $\mathbb{N}$. By lemma 1, Let $w \geq 2$. Lemma 3.2 shows that m is odd in the given equation.

Case (ii): w is even

By lemma 3.3, the result follows.

Case (iii): w is odd with $w \not\equiv 0 \pmod{11}$

From lemma 3.4, it is obtained that $m = 1$ and k is odd. Suppose that $k = 1$, then from equation (1.5), it is observed that $n = 2$. Hence, assume that $k \geq 3$. As a result, equation (1.5) assumes the structure of Pillai's equation.

$$t^n - r^k = s \quad (4.1)$$

with $k \geq 3$. Here $r = 60w^2 + 1$, $s = 61w^2 - 1$ and $t = 11w$.

Now, it is essential to find the lower bound for k , which is obtained from lemma 3.5 and in order to get the maximum bound for k , it is enough to establish the result $k \leq 2521 \log t$, which is shown as follows. Take the linear expression involving two logarithms:

$$\Lambda = n \log t - k \log r \quad (> 0)$$

Utilizing the result $\log(1 + s) < s$ for $s > 0$, it is observed that

$$0 < \Lambda = \log \left(\frac{t^n}{r^k} \right) = \log \left(1 + \frac{s}{r^k} \right) < \frac{s}{r^k} \quad (4.2)$$

This equation leads to

$$\log \Lambda < \log s - k \log r \quad (4.3)$$

Now, the lower bound for Λ is obtained by applying the following result of statement 1.

$$\log \Lambda \geq -25.2 (\max\{\log d' + 0.38, 10\})^2 (\log r)(\log t) \quad (4.4)$$

where $d' = \frac{k}{\log t} + \frac{n}{\log r}$. Note that $r^{k+1} > t^n$. Indeed,

$$r^{k+1} - t^n = r(t^n - s) - t^n = (r - 1)t^n - rs \geq 60w^2 \cdot 121w^2 - (60w^2 + 1)(61w^2 - 1) > 0$$

$$\text{Hence } d' < \frac{2k + 1}{\log t}$$

Put $P = \frac{k}{\log t}$. Combining (4.3) and (4.4) gives

$$k \log r < \log s + 25.2 \left(\max \left\{ \log \left(2P + \frac{1}{2} \right) + 0.38, 10 \right\} \right)^2 (\log r)(\log t),$$

so,

$$P < 1 + 25.2 \left(\max \left\{ \log \left(2P + \frac{1}{2} \right) + 0.38, 10 \right\} \right)^2$$

Since $\log t = \log(11w) \geq \log 33 > 2$, It is obtained that $P < 2521$. This proves the fact

$$k < 2521 \log t$$

If follows from the above inequality and Lemma 3.5, One may observe that

$$\frac{1}{60}(w^2 - 61) < 2521 \log(11w) \quad (4.5)$$

This results in the fact that $w \leq 1197$.

Also, (4.2) yields the inequality

$$\left| \frac{\log r}{\log t} - \frac{n}{k} \right| < \frac{s}{kr^k \log t}$$

This implies

$$\left| \frac{\log r}{\log t} - \frac{n}{k} \right| < \frac{1}{2k^2} \quad (4.6)$$

Thus, with the help of Python Programming, we checked that the inequality (4.6) does not hold for any w in the range $3 \leq w \leq 1197$.

Case (iv): $w \equiv_{11} 0$

Assuming $w \in \mathbb{N}$ divisible by 11, and utilizing the solution (k, m, n) of equation (1.5), we establish the odd parity of m simply by examining the congruence modulo w . Applying statement 2, by setting $q = 11, b_1 = 60w^2 + 1, b_2 = 1 - 61w^2, d_1 = k, d_2 = m$ and

$$\Lambda := (60w^2 + 1)^k - (1 - 61w^2)^m$$

Consider $g = 1, T = 2, B_1 = 60w^2 + 1, B_2 = 61w^2 - 1$. Thus, it follows that

$$2n \leq \frac{36.1}{8(\log 11)^4} (\max\{\log d' + \log(2 \log 11) + 0.4, 12 \log 11\})^2 \log(60w^2 + 1) \log(61w^2 - 1)$$

where $d = \frac{k}{\log(61w^2 - 1)} + \frac{m}{\log(60w^2 + 1)}$.

Suppose that $n \geq 4$. Taking (1.5) modulo w^4 , it gives

$$60k + 61m \equiv 0 \pmod{w^2}$$

It is also found that $P = \max\{k, m\} \geq \frac{w^2}{121}$. Since $n \geq P$ and $d' \leq \frac{P}{\log w}$, one may get

$$2P \leq \frac{36.1}{8(\log 11)^4} \left(\max \left\{ \log \left(\frac{P}{\log w} \right) + \log(2 \log 11) + 0.4, 12 \log 11 \right\} \right)^2 \times \log(60w^2 + 1) \log(61w^2 - 1) \quad (4.7)$$

For $w \geq 30236669$,

$$2P \leq \frac{36.1}{8(\log 11)^4} \left(\left\{ \log \left(\frac{P}{\log w} \right) + \log(2 \log 11) + 0.4 \right\} \right)^2 \log(60w^2 + 1) \log(61w^2 - 1)$$

This inequality further provides the following equation due to the result that $w^2 \leq 121 P$.

$$2P \leq 0.136(\log P - \log(\log 30236669) + 1.968)^2 \log(7260P + 1) \log(7381P - 1)$$

$\Rightarrow P \leq 387$, which is a contradiction to the fact that $P \geq \frac{w^2}{121} \geq 7555835968724$.

If $w \leq 30236669$, then the inequality (4.7) becomes

$$\frac{2w^2}{121} \leq 113 \log(60w^2 + 1) \log(61w^2 - 1)$$

This results in $w \leq 1554$. Hence all the values of k, m, n are bounded. There is no values of (w, k, m, n) satisfying (1.5) which is verified using Python Programming language. Thus, it is concluded that $n \leq 3$. In this case, It is simple to demonstrate that $(k, m, n) = (1, 1, 2)$, bringing the proof to its logical completion. $\square$

Conclusion

Thus, it is established that the exponential Diophantine equation (1.5) has $(k, m, n) = (1, 1, 2)$ as the only solution in $\mathbb{Z}_+$ which is verified for different values of w utilizing p-adic logarithms. The entire result is verified and found with the help of Python Programming Software.

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