

Optic disk detection in South Africans: a contribution based in wavelets and morphological operations

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ABSTRACT

Retinography is a non-invasive medical imaging technique for analyzing the human visual system. As one of the top three global causes of blindness, diabetic retinopathy's early identification is critical, with exudates often being the initial visible indicator. The presence of the optic disc can interfere with the detection of bright lesions, particularly exudates. For this reason, accurate detection of the optic disc is considered essential. The primary goal of this research is the detection and segmentation of the optic disc. This work presents a new model for the automatic localization of the optic disc in retinal images from a South African patient cohort. The method utilized images sourced from the Eye Centre fundus photo database. An efficient method for segmenting the optic disc for retinal image analysis is described. This approach integrates wavelet and wavelet packet denoising techniques with morphological operations. The algorithm was tested on a set of 100 images, which included 60 normal images and 40 images presenting exudates. The roundness metric was measured, achieving values of 0.8 or higher in 90% of the normal images and in 87.5% of the images with exudates. The algorithm demonstrated a high degree of accuracy in detecting the optic disc in the majority of cases. In contrast to other methods found in the literature, this algorithm's effectiveness is not compromised by the presence of exudates in retinal images.

Keywords: Optic disk, wavelet packet, exudates, image segmentation

Mathematics Subject Classification: 42C40, 68U10, 94A12

1. INTRODUCTION

Diabetic retinopathy (DR) is a complication of the eye that arises from diabetes. It results from damage to the blood vessels within the retina. Preventing the advancement of DR currently represents a significant challenge in the healthcare field.

Non-proliferative retinopathy is a form of DR where blood vessels can enlarge, leading to the formation of small sacs or exudates, which are among the earliest visible indicators of the condition. Exudate refers to the fluid that leaks from blood vessels into adjacent tissues. This fluid is composed of cells, proteins, and other solid materials. It can seep from wounds or areas affected by infection or inflammation. Typically, exudates appear as whitish or yellowish patches that vary in their location, size, and shape.

The use of digital imaging in ophthalmology has enabled the processing of retinal images, which aids in providing accurate diagnoses and suitable clinical therapies (Alshayegi et al., 2016). Indeed, advancements in software and the growing computational capabilities of modern computers allow for the digital analysis and identification of features like exudates (Annunciata et al., 2016). Fundus images, also known as retinographies, are now an essential instrument for contemporary medical diagnostics. Automating the analysis of these visual records could lead to significant and vital time savings, facilitating the prompt administration of early treatment.

In retinographies, the optical disk (OD) appears as a reflection corresponding to the head of the optic nerve. For a set of images with a specific resolution, the radius of the optic nerve head can be treated as a constant. This assumption simplifies the task of locating it, which is a critical step in various procedures. Detecting and subsequently segmenting the OD is crucial to prevent its high intensity from interfering with later processes, especially when identifying bright lesions such as exudates (Cook, 2014).

The OD stands out as one of the most significant landmarks within a retinography. Its identification can serve as an initial step for the early detection of glaucoma. The diameter of the OD is commonly employed as a reference metric to measure other sizes and distances within these retinal images (Rudas et al, 2011). Furthermore, pinpointing the OD's position helps in approximating the location of the fovea—the center of human vision—which in turn aids in detecting pathologies affecting the macular region (Sinthanayothin, 1999). Figure 1 displays a fundus image, highlighting the most relevant retinal components, such as the blood vessels, macula, exudates, and the OD.

Akita (1982) proposed one of the earliest methods for detecting and extracting the primary components of a retinography, a technique in which the OD is located by analyzing the convergence of blood vessels. A different approach by Lu and Lim (2010) involves identifying multiple potential candidates for the optic disk. Their process begins by estimating a retinal background surface using an iterative smoothing technique. Following this, the discrepancies between the original retinal image and this estimated background are analyzed. A significant limitation of this technique, however, is that many retinal images are affected by different kinds of retinal lesions. These abnormalities can include drusen, exudates, microaneurysms, and hemorrhages. Additionally, imaging artifacts like haze, eyelashes, or inconsistent lighting can create areas of brightness or high variation that are comparable to or exceed that of the OD, complicating its detection (Lu and Lim, 2011).

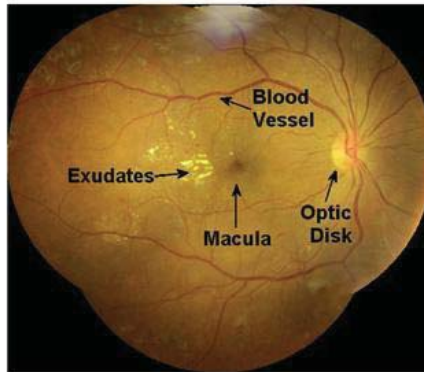


Figure 1. Macula, Blood Vessels, Exudates and Optic Disk.

An alternative methodology approximates the optic disk's shape as either a circle or an ellipse, based on the observation that its contour is typically round or has a slight vertical ovalness (Lalonde et al, 2001). In this method, the OD's shape was determined using Hausdorff-based template-matching, which compares detected edges against circular patterns of varying sizes.

A method based on mathematical morphology was introduced by Walter and Klein (2001) for identifying the disk boundary. Their work also addresses the identification of the vascular tree in low-contrast, noisy color fundus images. The OD detection is carried out in two stages: first, an initial approximation of the OD's position is calculated, followed by the precise outlining of its contours via the watershed transformation.

Li and Chutatape (2004) employed Principal Component Analysis (PCA) to locate the OD. Their work also proposed an approach for detecting exudates that integrates both region growing and edge detection techniques.

Capitalizing on the circular morphological structure of the OD in retinographies, the work by Rudas (2011) utilizes the Hough transform to identify patterns that can be described by mathematical models. This procedure is noted for its robustness against noise and gaps along the object's boundary. For edge detection, the Canny algorithm is employed, after which the Hough transform for circles is applied to each pixel identified as part of an edge.

An alternative strategy that combines multiple algorithms was presented by Qureshi et al. (2012). They put forward a procedure that utilizes the maximal outputs from a series of OD/fovea detectors. The evaluation of their methods was based on two criteria: the detection's location falling within manually selected optic disc regions and its proximity to the manually identified OD center.

To detect the OD, Vimala and Mohideen (2013) employed a combination of Fuzzy C-Means Clustering and line operator techniques.

2. MOTIVATION

Many algorithms existing in the literature are efficient for the detection of the OD in fundus images with absence of exudates. However, in general the accuracy achieved is not adequate when both elements are presented. The aim of this paper is to find a good solution for this problem. In all studied images with the algorithm here described, the OD was detected independently of the presence of exudates.

The algorithm here proposed consists of the technique based on image segmentation. Image segmentation is to partition an image into significant parts, which generally consist of an object and a background. With this technique, an image is separated into disjoint regions which union is the whole image. To eliminate the noise that can appear when analyzing a signal or image the wavelet transform is employed.

A first algorithm was developed by Boix and Cantó (2013) for segmenting blood cells with some diseases. In that work the Haar's mother wavelet showed the segmentation with the largest area in the cell. Based on slight modifications of this algorithm, it has been possible to detect the OD with an important accuracy. This new algorithm is described in the next section.

The validation of the presented algorithm was performed on a set of images contributed by Dr. Steve Cook, an ophthalmologist from The Eye Centre in East London. From this center, the African Eye Foundation was founded, an organization committed to research, professional training, and the provision of no-cost surgery for patients with the most critical needs.

The dataset for the study was composed of 100 images in total, with 40 of them showing the presence of exudates. The images were provided in JPEG format and varied in size; however, this size difference did not influence the final results. Because each image has unique characteristics, the matrices generated at each stage of the algorithm also differ, leading to a variable processing time for each image depending on the specific calculations involved. An average computation time of 4.6 seconds per image was recorded, which is a sufficient speed for the algorithm to be incorporated into a computer-aided diagnostic tool for in-depth analysis. Additionally, it is anticipated that the execution time would decrease if the algorithm were implemented in a more optimized programming language, a consideration that may be pivotal when selecting a technique for detecting hard exudates.

The paper will be sequenced as follows. Firstly, we will start remembering the main definitions and operations used in general algorithms for analyzing retinal images.

Secondly, in the section IV we will describe all the required stages in the algorithm proposed by the authors. Tables and figures with the different results are shown for illustrating the selections made for the type of wavelet used as well as the appropriate decomposition level.

Finally, we will complete the paper with several conclusions about the improvements obtained with the method and parameters used.

3. PRELIMINARIES AND DEFINITION

3.1. Fundamental operations

In this subsection the main morphological operations will be introduced.

To begin, a digital binary image can be described as a two-dimensional representation in which every pixel is assigned one of only two possible values: 0 or 1. By convention, the pixels corresponding to the object or objects are typically assigned the value 1, whereas pixels belonging to the background are assigned the value 0.

Throughout this paper, let A be an image seen as a subset of $\mathbb{Z}^2$. We will call complement of A , denoted by A^c , the set defined by

$$A^c := \{a \in \mathbb{Z}^2; a \notin A\}$$

The reflection of A , denoted by A^R , is given by

$$A^R := \{-a \in \mathbb{Z}^2; a \in A\}$$

The translation of A by the vector z (z belonging to $\mathbb{Z}^2$), denoted by A_z , is defined by

$$A_z := \{b \in \mathbb{Z}^2; b = a + z, a \in A\}$$

Within the field of mathematical morphology, erosion and dilation are considered the two most fundamental operations.

Let A and B be two images in $\mathbb{Z}^2$. The operation of binary erosion of an image A by a structuring element B , which is represented as $A \ominus B$, is defined by the following set operation (Van den Boomgard and Van Balen, 1992)

$$A \ominus B := \{z \in \mathbb{Z}^2; B_z \cap A \neq \emptyset\}$$

This can be explained as the collection of all pixel locations (z) for which the structuring element, when translated to that location B_z , has an overlap that is contained entirely within the foreground pixels of image A (The MathWorks, 2016). Essentially, applying the erosion operator to a binary image results in the progressive shrinking of the boundaries of foreground pixel regions (Qidwai and Cheng, 2009).

Conversely, the binary dilation of image A by B , symbolized as $A \oplus B$, is defined through the set operation below (Gonzalez et al., 2010):

$$A \oplus B := \{z \in \mathbb{Z}^2; \hat{B}_z \cap A \neq \emptyset\}$$

where $\hat{B}$ is the reflection of the structuring element B . In other words, it is the set of pixel locations z , where the reflected structuring element overlaps with foreground pixels in A when it is translated to z (TheMathWorks, 2016).

The basic effect of the dilation operator on a binary image is to gradually enlarge the boundaries of regions of foreground pixels (Qidwai and Cheng, 2009).

3.2. Image segmentation

As indicated above, image segmentation is the partitioning of the image I into non-overlapping regions according to some common features. Segment is to determine connected sets $S_i \subset I$ whose union is the full image, that is:

$$I = \bigcup_{i=1}^n S_i; S_i \cap S_j = \emptyset, i \neq j$$

The general process consists of using the wavelet thresholding technique to smooth the original image while filtering out noise and prepare it for appropriate segmentation.

4. METHOD

The algorithm proposed by the authors can be developed by nine phases shown in the flowchart of the Fig. 2. Throughout the following paragraphs each of the phases are described.

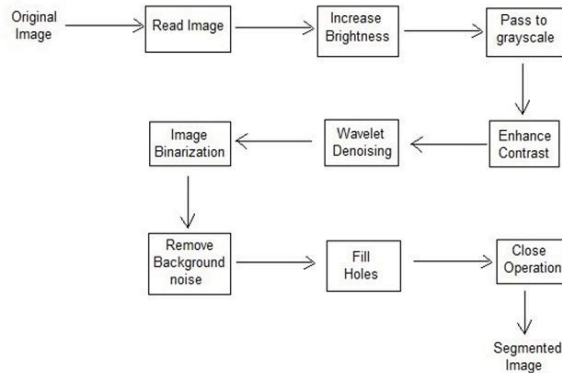


Figure 2. Process Scheme.

1). Read the image

The *Matlab* environment allows us to read an image and store it in an array. This process makes it possible to load the initial image's three distinct color channels

In the Matlab environment, what is known as an RGB or truecolor image is represented by a three-dimensional data array with dimensions m-by-n-by-3. This array structure specifies the respective red, green, and blue values for every pixel in the image.

2). Modify Brightness

Each of the red, green, and blue components within an RGB image is represented by an integer value that lies in the inclusive range from 0 to 255. We have verified that for a better detection it is desirable that the contribution of the green channel reaches its maximum value. For this purpose is necessary to multiply by a scalar p ($p = 5$) the value of each pixel. The Fig. 3 shows the change produced.

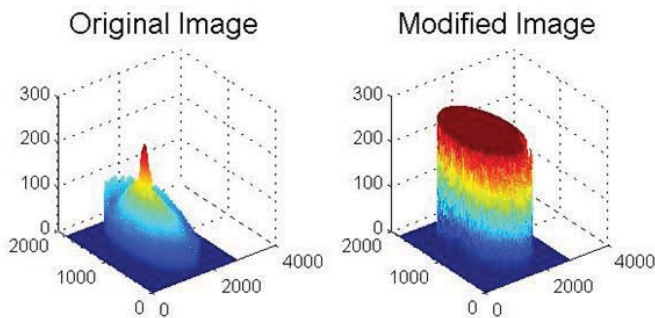


Figure 3. Representation of the green channel (3d).

3). Transform the image to grayscale

Initially, for easier processing, it is beneficial to convert the image into a new grayscale version. This new image is stored as a two-dimensional array containing only integer values from 0 to 255, a format that simplifies subsequent manipulation.

An image rendered in shades of gray is also known by the alternative terms gray-scale, grayscale, or gray-level image. The image is transformed to grayscale by removing the hue and saturation information while retaining the luminance. For conversion RGB values to grayscale ones is used the weighted sum of the R, G and B components given for *Matlab*:

$$0.2989 * R + 0.5870 * G + 0.1140 * B$$

4). Contrast enhancement

The brightness of the image corresponds directly to its gray level. Contrast serves as a measure of how spread out the gray levels are within the image. A lack of sharp differentiation between the black and white tones results in an image with zero contrast.

In this paper the contrast is improved using histogram equalization. The process of enhancing contrast makes dark regions appear even darker while causing light regions to become brighter. An image's histogram provides a visual representation of how its various tonal values are distributed. Typically, an image is said to have good contrast when its histogram spans nearly the full spectrum of available tones. Dynamic range denotes to the range of brightness levels, from dark to light.

This procedure involves transforming the original grayscale image, I , into a new image, J , which is composed of n discrete gray levels. The mapping ensures that each of the n gray levels in the new image J is assigned a roughly equivalent number of pixels, so that the histogram of J is nearly flat. (The histogram of the image J is flatter when n is much smaller than the number of discrete levels in I), (TheMathWorks, 2016). In this work, n is taken 64.

5). Wavelet-denoising

The wavelet denoising technique allows for the elimination of noise from an image, ensuring that its fundamental features are maintained. A crucial step in this process is selecting an appropriate threshold, denoted as T , to effectively preserve these key characteristics.

If a pixel's grey value exceeds the selected threshold, it is classified as part of the region of interest and assigned a white color. Conversely, if a pixel's grey value is below the threshold, it is considered part of the background and is colored black.

Two main types of thresholding can be applied: hard and soft. With hard thresholding, the signal is x if $|x| > T$ and 0 if $|x| \leq T$, and the soft threshold signal is $\text{sign}(x)(|x| - T)$ if $|x| > T$ and 0 if $|x| \leq T$.

For this procedure, the universal threshold, T given by $T = \sigma\sqrt{2\log(mn)}$, where σ is the variance of the estimated noise and mn is the number of pixels in the image.

Regarding the wavelet families selected for this process, an analysis was conducted on Daubechies (dbn) of orders $n=1,2,10$; Coiflets (coifn) of orders $n=1,2$; and Symlets (symn) of orders $n=2,3$, where n is the order of the wavelet. These families have been chosen because they share some characteristics such as orthogonality, bi-orthogonality, compact support, and a large number of vanishing moments (except the db1 wavelet). Besides, their analysis can be made using the Discrete or Continuous Wavelet Transform.

For improving the denoising process is necessary to choose the optimum decomposition level and the best wavelet. For this selection the following steps have been made :

a) Different levels of decomposition ($N=1,2,\dots,5$) are applied on the same images (with the wavelets previously mentioned) to choose the best level of the decomposition, N . In Table I and Table II the obtained results are shown and, in consequence, the value $N=1$ will be used in the algorithm.

Table 1: Results of the area (in pixels) obtained with different wavelets and different levels of decomposition.

Normal Images	Db1	Db2 Sym2	Db10	Sym3	Coif1	Coif2
nimage01 N=1	59241	59331	59379	59303	59311	59336
nimage01 N=2	58831	58726	58477	58439	58402	58451
nimage01 N=3	58430	58131	57898	57728	58027	57624
nimage01 N=4	58630	58197	57783	57897	57888	57576
nimage01 N=5	58630	58217	58016	57981	58080	57775

Table 2: Results of the area (in pixels) obtained with different wavelets and different levels of decomposition.

Images with exudates	Db1	Db2 Sym2	Db10	Sym3	Coif1	Coif2
image01 N=1	59373	59384	59344	59426	59253	59221
image01 N=2	59056	58651	58594	58581	58628	58573
image01 N=3	58951	58492	57903	58118	58351	58013
image01 N=4	58895	58421	57926	58085	58449	57941
image01 N=5	58565	58448	58183	58041	58478	58023

b) After taking the previously set level value N , we compare the measurements obtained for some images using the wavelet families cited above. The wavelet showing the best results in area was the db1 wavelet, also called Haar's wavelet. Fig. 4 shows the graphs of the different wavelets used in this study.

Table III and Table IV show the areas results obtained with the different wavelets in images with and without exudates. These results justify the choice made by the wavelet employed in the algorithm.

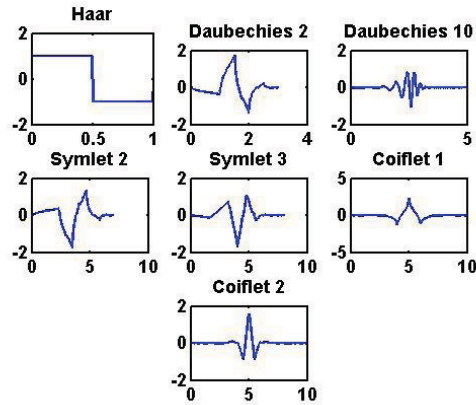


Figure 4. Different wavelets applied in the fundus images.

c) Once selected the Haar wavelet and $N=1$ as a level of decomposition, the area and the roundness metric are located. In general, the analysis can be made using other parameters. However, it has been considered important to measure the roundness metric to establish how much portion of the optic disk has been covered.

Finally, the noise present in the image is reduced in a two-stage process. The first stage involves applying a wavelet decomposition at level $N=1$ using the Haar wavelet and soft thresholding. In the second stage, a wavelet packet decomposition is applied to the image's complement, also at level $N=1$ and with the same mother wavelet, but this time using hard thresholding. This entire procedure yields a denoised version of the input image, achieved by thresholding wavelet coefficients with a global threshold.

6). Convert Image to Binary Image

To improve the detection of the Optic Disc (OD), tests have shown that using the complement of the image is more effective. Therefore, the image's complement is transformed into a binary format. This binarization is achieved by setting a luminance level, L , which is a value between 0 and 1. Any pixel with a luminance value exceeding L is set to 1 (white), while all other pixels are set to 0 (black). The specific value for L is determined automatically using Otsu's method (1979), which calculates the optimal threshold to minimize the intra-class variance between the resulting black and white pixel groups. This process yields a black-and-white image.

7). Remove Background Noise

Mathematical morphology is a widely recognized technique for extracting components from an image. The algorithm utilizes morphological functions, specifically applying a morphological opening operation to the binary image to remove small, extraneous objects. In image analysis, a group of connected pixels

in a binary image is defined as an object or a connected component. This study employs an 8-connected neighborhood, meaning pixels are considered connected if they touch at their edges or corners, as illustrated in Fig. 5.

Following the opening operation, all connected components with fewer than P pixels are eliminated, which produces a cleaner binary image. For this work, the parameter P is set to 22000.

8). Fill Image Regions and Holes

A hole is defined as a region of background pixels that cannot be reached by filling from the image's edge. Filling these holes is a necessary step to create a more uniform image region and is performed accordingly.

9). Close

Finally, a morphological closing operation is performed on the binary image. This operation consists of a dilation followed by an erosion. Both of these fundamental operations utilize the same structuring element: a disk with a radius of 50 pixels. This final step results in a binary image ready for further analysis. As a result of this step a binary matrix is obtained whose elements are 1 when the pixel belongs to the segmented object (OD), or 0 if it belongs to background.

All these nine steps or phases of the algorithm, that define the process followed to detect the OD, can be observed and summarized in Fig. 6.

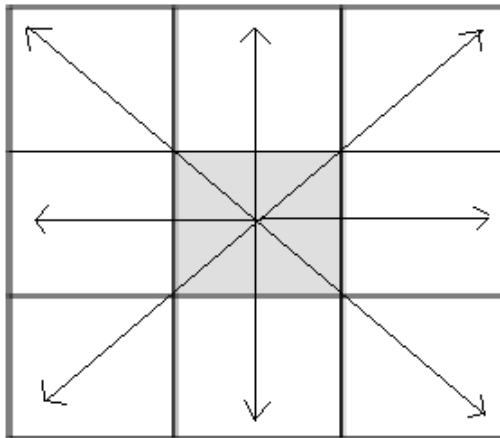


Figure 5. Pixels 8-connected.

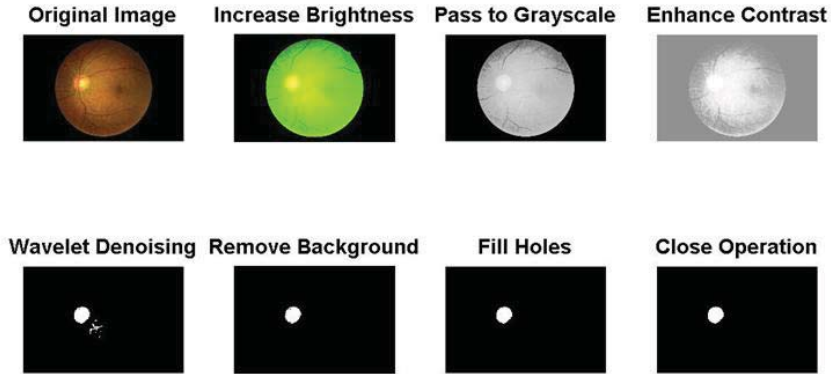


Figure 6. Segmentation process.

5. DISCUSSION

Once the method is finished and the image has been segmented, it is possible to estimate the object's area and perimeter. As we have said in the introduction, in the literature the OD contour was estimated as a circle or an ellipse. In our case, the shape of the OD will be approximated calculating the roundness metric.

The compactness of an object is frequently defined as the area divided by the square of the perimeter. If we multiply this quantity by 4π it will be equal to 1 for the circle.

Let rm be the roundness metric defined as

$$rm = \frac{4\pi \text{ area}}{\text{perimeter}^2}$$

This metric's values range from 0 to 1; a perfect circle yields a value of 1, while other shapes result in values less than 1. A value approaching 1 indicates that the object is generally circular, making this parameter a measure of roundness.

As demonstrated in Figure 7, a roundness metric near 0.8 provides an accurate detection of the OD. Values below 0.8 lead to a less precise approximation of the OD, often due to interference from adjacent bright areas or blood vessels.

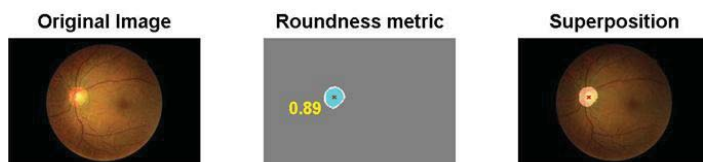


Figure 7. Superposition of the segmented OD in the original image.

6. CONCLUSION

This paper puts forward a novel and efficient method for detecting the Optic Disc (OD) in digital fundus images. The algorithm is designed to be fully automatic, fast, and precise. It achieves excellent results by combining wavelet denoising with morphological operations to segment the OD.

The method proves to be a robust estimator for OD detection, even when applied to abnormal retinal images containing exudates. The simplicity and general applicability of the algorithm suggest its potential for use in a wide range of medical applications. As evidenced by the results in the provided tables, the proposed method demonstrates a high level of accuracy in optic disc recognition.

It is important to note that the algorithm's performance relies heavily on the brightness of the image; therefore, poor illumination in a fundus image may result in an incomplete segmentation of the optic disc.

7. ACKNOWLEDMENT

The authors thank Dr Steve Cook and The Eye Center in East London the dataset provided.

Table 3: Results of the area (in pixels) obtained with different wavelets whit N=1.

Normal Images	Db1	Db2 Sym2	Db10	Sym3	Coif1	Coif2
nimage01	59241	59331	59379	59303	59311	59336
nimage02	54904	54931	54880	54919	54903	54828
nimage03	58320	58456	57933	58203	58674	58517
nimage04	35448	35512	35652	35533	35521	35469
nimage05	68544	68770	68846	68719	69727	69693
nimage06	61857	61756	62027	62066	61801	61607
nimage07	48075	48038	47893	47979	48630	48614
nimage08	45443	46097	45426	45424	45463	45314
nimage09	39682	39639	39659	39598	39652	39638
nimage10	54992	54880	54842	54895	55063	55101
nimage11	38812	39301	39426	39374	38848	38790
nimage12	39682	39639	39659	39598	39652	39638
nimage13	75977	75821	75683	75702	76535	76559
nimage14	55749	55690	55581	55654	55681	55623
nimage15	76287	76330	76173	76087	76456	76233
nimage16	46759	47107	47129	47001	45244	45283
nimage17	60797	61080	61091	60886	60810	60715
nimage18	48411	48344	48523	48311	48436	48407
nimage19	61517	61473	61528	61356	61600	61480
nimage20	70350	70003	70102	70061	69912	69969
nimage21	62540	62351	62284	62275	62460	62331
nimage22	56598	56487	56485	56557	56500	56490
nimage23	48099	47938	47350	47404	47624	47582
nimage24	50853	50882	51001	5.0795	50960	50954

nimage25	66595	66422	66528	66345	66530	66627
nimage26	54602	54721	54697	54640	54681	54595
nimage27	55554	55828	55706	55734	55623	55737
nimage28	58662	58542	58435	58486	58541	58494
nimage29	39241	39121	39287	39207	39353	39294
nimage30	35580	35642	35640	35632	35542	35604
nimage31	43955	44698	44572	44535	44567	44603
nimage32	44466	44331	44401	44418	44472	44482
nimage33	45663	45431	45740	45778	45515	45703
nimage34	51957	52210	51702	51601	51669	51301
nimage35	78263	77945	77284	77354	76968	76862
nimage36	73965	73926	73366	73435	73740	73658
nimage37	48969	48993	49159	49038	49022	48880
nimage38	43173	43178	43247	43171	43399	43416
nimage39	36257	36357	36362	36366	36209	36273
nimage40	46433	46323	46251	46672	46153	46140
nimage41	41146	41316	41601	41537	41216	41170
nimage42	54662	54823	54951	54478	54909	54842
nimage43	52906	53071	53248	53163	53228	53114
nimage44	58014	57298	57521	57332	57364	57501
nimage45	75916	74954	75207	75124	76102	76078
nimage46	50576	50558	50570	50556	50510	50487
nimage47	56773	56658	56131	56060	56220	56513
nimage48	55698	55574	55565	55834	55839	56044
nimage49	42266	42217	42265	42283	42393	42201
nimage50	61732	61567	61430	61531	61819	61641
nimage51	57460	57355	57179	57558	57460	57298
nimage52	67882	67535	67572	67690	67678	67605
nimage53	67119	67260	67260	67220	67392	67311
nimage54	49466	49531	49472	49495	49423	49473
nimage55	52605	52765	52556	52433	52595	52619
nimage56	27139	26806	26964	26770	26952	26851
nimage57	64769	64554	65138	64959	64663	65102
nimage58	53838	53921	53843	53826	53872	53744
nimage59	67548	67441	67426	67437	67419	67404
nimage60	66595	66422	66528	66345	6.6530	66627

Table 4: Results of the area (in pixels) obtained with different wavelets whit N=1.

Images with exudates	Db1	Db2 Sym2	Db10	Sym3	Coif1	Coif2
image01	59373	59384	59344	59426	59253	59221
image02	47649	47673	47607	47641	47645	47644
image03	72338	72623	72555	72626	72632	72651
image04	58919	59000	59457	59130	59008	59023
image05	61869	61488	61715	61798	61315	63207
image06	62555	62505	62703	62616	62751	62674
image07	59601	59583	59519	59521	59524	59512
image08	76976	76760	76776	76712	76859	76886
image09	74511	74409	74210	74304	74288	74262
image10	35568	35656	35538	35535	35432	35480
image11	45903	45899	45961	45734	45585	45699
image12	52879	52737	52679	52707	52823	52775
image13	62025	61868	61614	61585	61719	61702
image14	58702	58630	58625	58616	58538	58491

image15	35697	35945	35893	35872	35753	35791
image16	40779	40743	40714	40650	40690	40718
image17	48580	48685	48991	49094	49180	49038
image18	58352	58255	58359	58394	58447	58378
image19	47649	47673	47607	47641	47645	47644
image20	38680	38643	38644	38681	38663	38565
image21	48056	47879	47792	47827	47776	47722
image22	85213	84611	84007	84086	84812	84671
image23	70487	70418	70257	69923	70969	70171
image24	59714	59580	60069	60728	59390	59524
image25	40591	40607	40610	40555	40565	40629
image26	68856	68689	68676	68660	68758	68747
image27	77403	77270	77292	77236	77322	77276
image28	76976	76760	76776	76712	76859	76886
image29	35566	35448	35628	35247	35903	35799
image30	76605	76309	76478	76338	76082	76378
image31	50336	50535	50581	50464	50444	50430
image32	55073	55188	55181	55487	57867	58018
image33	28897	28940	28848	28799	28944	28877
image34	77499	77133	76879	76927	77288	77575
image35	69682	69452	69180	69194	69354	69367
image36	36944	36910	36907	36852	37009	37024
image37	37400	37254	37226	37250	37277	37235
image38	59601	59583	59519	59521	59524	59512
image39	58919	59000	59457	59130	59008	59023
image40	43895	43904	43956	43709	43958	43957

Table 5: Results of the area (in pixels) and rm obtained with different wavelets whit N=1 and Haar's wavelet.

Normal Images	rm	area	Images with Exudates	rm	area
nimage01	0.86	59241	image01	0.87	59373
nimage02	0.84	54904	image02	0.86	47649
nimage03	0.80	58320	image03	0.81	72338
nimage04	0.82	35448	image04	0.85	58919
nimage05	0.80	68544	image05	0.76	61869
nimage06	0.84	61857	image06	0.75	62555
nimage07	0.77	48075	image07	0.86	59601
nimage08	0.85	45443	image08	0.87	76976
nimage09	0.84	39682	image09	0.88	74511
nimage10	0.84	54992	image10	0.83	35568
nimage11	0.81	38812	image11	0.85	45903
nimage12	0.84	39682	image12	0.86	52879
nimage13	0.84	75977	image13	0.84	62025
nimage14	0.86	55749	image14	0.85	58702
nimage15	0.85	76287	image15	0.86	35697
nimage16	0.80	46759	image16	0.87	40779
nimage17	0.82	60797	image17	0.80	48580
nimage18	0.87	48411	image18	0.86	58352
nimage19	0.80	61517	image19	0.86	47649
nimage20	0.82	70350	image20	0.84	38680
nimage21	0.88	62540	image21	0.82	48056
nimage22	0.86	56598	image22	0.83	85213
nimage23	0.76	48099	image23	0.81	70487

nimage24	0.86	50853	image24	0.77	59714
nimage25	0.86	66595	image25	0.86	40591
nimage26	0.84	54602	image26	0.83	68856
nimage27	0.85	55554	image27	0.86	77403
nimage28	0.89	58662	image28	0.87	76976
nimage29	0.86	39241	image29	0.76	35566
nimage30	0.84	35580	image30	0.83	76605
nimage31	0.81	43955	image31	0.80	50336
nimage32	0.81	44466	image32	0.85	55073
nimage33	0.83	45663	image33	0.81	28897
nimage34	0.78	51957	image34	0.77	77499
nimage35	0.84	78263	image35	0.85	69682
nimage36	0.82	73965	image36	0.84	36944
nimage37	0.77	48969	image37	0.86	37400
nimage38	0.83	43173	image38	0.86	59601
nimage39	0.86	36257	image39	0.85	58919
nimage40	0.81	46433	image40	0.86	43895
nimage41	0.85	41146			
nimage42	0.82	54662			
nimage43	0.82	52906			
nimage44	0.75	58014			
nimage45	0.74	75916			
nimage46	0.87	50576			
nimage47	0.80	56773			
nimage48	0.81	55698			
nimage49	0.87	42266			
nimage50	0.84	61732			
nimage51	0.82	57460			
nimage52	0.82	67882			
nimage53	0.82	67119			
nimage54	0.83	49466			
nimage55	0.85	52605			
nimage56	0.85	27139			
nimage57	0.86	64769			
nimage58	0.87	53838			
nimage59	0.80	67548			
nimage60	0.86	66595			

5. REFERENCES

Akram, M.U., & Khan, S.A., 2013, Multilayered thresholding-based blood vessel segmentation for screening of diabetic retinopathy. *Eng. Comput.* **29**(2), 165–173.

Alshayegi, M., et al., 2016, Optic disc detection in retinal fundus images using gravitational law-based edge detection. *Med. Biol. Eng. Comput.*, 1–14.

Annunciata, R., et al., 2016, Leveraging multiscale hessian-based enhancement with a novel exudate inpainting technique for retinal vessel segmentation. *IEEE J. Biomed. Health Inform.* **20**(4), 1129–1138.

Cook, S., et al., 2014, Quality assurance in diabetic retinal screening in South Africa. *S. Afr. Med. J.* **104**(10), 700–704.

- Rudas, J., et al., 2011, Una técnica para la localización del disco óptico en retinografías. *Av. Sist. Informát.* **8**(3), 85–90.
- Sinthanayothin, C., et al., 1999, Automated localisation of the optic disc, fovea and retinal blood vessels from digital colour fundus images. *Br. J. Ophthalmol.* **83**(8), 902–910.
- Akita, K., & Kuga, H., 1982, Pattern recognition of blood vessel networks in ocular fundus images. *Med. Imaging Image Interp. Proc.*, 436–441.
- Lu, S., & Lim, J.H., 2010, Automatic optic disc detection through background estimation. *IEEE Int. Conf. Image Process.* **1**, 833–838.
- Lalonde, M., et al., 2001, Fast and robust optic disk detection using pyramidal decomposition and Hausdorff-based template matching. *IEEE Trans. Med. Imag.* **20**(11), 1193–1200.
- Walter, T., & Klein, J.C., 2001, Segmentation of color fundus images of the human retina: detection of the optic disc and the vascular tree using morphological techniques. In: *ISMDA Lect. Notes Comput. Sci.* Springer, London, 282–287.
- Li, H., & Chutatape, O., 2004, Automated feature extraction in color retinal images by a model-based approach. *IEEE Trans. Biomed. Eng.* **51**(2), 246–254.
- Vimala, G.S.A.C., & Mohideen, S.K., 2013, Automatic detection of optic disc and exudate from retinal images using clustering algorithm. *Intell. Syst. Control (ISCO)*, 280–284.
- Boix, M., & Cantó, B., 2013, Using wavelet denoising and mathematical morphology in the segmentation technique applied to blood cells images. *Math. Biosci. Eng.* **10**(2), 279–294.
- Van den Boomgard, R., & Van Balen, R., 1992, Methods for fast morphological image transforms using bitmapped images. *CVGIP: Graph. Models Image Process.* **45**(3), 252–258.
- Gonzalez, R.C., et al., 2010, *Digital Image Processing Using MATLAB*, 2nd ed., McGraw-Hill Education.
- Otsu, N., 1979, Threshold selection method from gray-level histograms. *IEEE Trans. Syst. Man Cybern.* **9**(1), 62–66.
- Roddy, G.W., et al., 2013, Pathology of the retina. In: Tasman, W., Jaeger, E.A. (eds), *Duane's Found. Clin. Ophthalmol.* vol. 3, chap. 13. Lippincott Williams & Wilkins, Philadelphia.
- Soille, P., 1999, *Morphological Image Analysis: Principles and Applications*. Springer-Verlag, 173–174.
- Lu, S., & Lim, J.H., 2011, Automatic optic disc detection from retinal images by a line operator. *IEEE Trans. Biomed. Eng.* **58**(1), 88–94.
- The MathWorks, Inc., 2016, *Image Processing Toolbox User's Guide*.
- Qidwai, U., & Chen, C.H., 2009, *Digital Image Processing: An Algorithm Approach with MATLAB*. Taylor & Francis Group.
- Qureshi, R.J., Kovacs, L., Harangi, B., Nagy, B., Peto, T., & Hajdu, A., 2012, Combining algorithms for automatic detection of optic disc and macula in fundus images. *Comput. Vis. Image Underst.* **116**(1), 138–145.