

## Unsteady Dispersion of Air Pollutant from a Point Source Forming a Secondary Pollutant Due to Temperature Variation in an Urban Area

Vivek Joseph<sup>1</sup> and Manju Agarwal<sup>2</sup>

<sup>1</sup>Department of Mathematics  
Lucknow Christian Degree College, Lucknow  
E-Mail: [joseph27in@gmail.com](mailto:joseph27in@gmail.com)

<sup>2</sup>Department of Mathematics and Astronomy  
University of Lucknow, Lucknow  
E-Mail: [manjuak@yahoo.com](mailto:manjuak@yahoo.com)

### ABSTRACT

*The present study is an attempt to understand the role of temperature of an urban area in shaping the dispersion of air pollutants. We know that there are certain gases in the form of air pollutants which are highly reactive and temperature dependent and converted into the secondary pollutants. In the present model, we considered that the inter-conversion process takes place during the urban heat island effect while taking the rate of reaction as temperature dependent in the form of Arrhenius equation. Urban heat island intensity is taken into an account while considering the temperature. The overall structure is divided into two intervals where the first interval reflects the inter-conversion process while the second interval is the non-existence of inter-conversion process. The two-dimensional advective diffusion equations along with the appropriate boundary conditions are used to define the phenomenon. It was observed that not only the temperature but also the urban length plays a vital role in shaping the distribution of pollutants in both intervals.*

**Keywords:** urban heat island intensity, primary and secondary pollutant, chemical reaction, temperature.

**Mathematics Subject Classification:** 97M10

### 1. INTRODUCTION

The problem of air pollution in an urban area is one of the global concerns over a few decades. Various studies have been made by scientists and researchers to understand the effect of the air pollutants on the environment. Focus was also made on the various measures to control this problem. In past, mathematicians focussed their attention through mathematical modelling on air pollution. In this regard Gaussian plume model is still widely used (Hanna et.al. 1982, Seinfeld 1986). The impact of air pollution

usually observed in an urban area. Various factors are responsible for the dispersion of pollutants like wind speed, diffusivity, temperature, humidity. In recent time, various study has been made on understanding how temperature influence the dispersion of pollutants (Zhao et. al. 1993, Elbayouml et. al. 2014, Cichowicz et. al. 2017, Joseph et. al. 2023). In fact, the temperature of urban area is usually higher than the rural area. Urban area which are densely populated having huge buildings, industries retain the heat energy for longer period, whereas the adjacent areas loose heat energy more rapidly (Rao 2014). The difference in temperature between urban and rural area creates the urban heat island effect in the urban area. In recent times, a significant recognition has been made in regarding with the distribution of air pollution under urban heat island effect (Chander 1968, Griffith 1970, Dilly and Yen 1971, Agarwal and Tandon 2010, Joseph and Yadav 2021). Several meteorological parameters affecting the UHI include solar fluxes, topography, wind speed, energy fluxes, (Landsberg 1981). Summer (1964) expressed UHII (urban heat island intensity), as the temperature difference with respect to the “surrounding” area in the form  $\Delta T = T_u - T_r = \left( \frac{2H_0\gamma X}{\rho C_p U_m} \right)^{1/2}$  where  $T_u, T_r$  represent the urban and rural temperature,  $H_0$  is the surface heat flux,  $\gamma$  represent the potential temperature gradient in the approach flow in the vertical direction,  $\rho$  is the air density,  $C_p$  is the specific heat and  $U_m$  is the layer-averaged wind speed. Lee et.al. (2012) describe the scaling of the urban heat island effect using the fundamental energy balance equation (Lee and Ho, 2010). The UHI intensity affect the boundary layer structure and makes the condition favourable for the transformation of the reactive pollutants (Swamy et. al. 2020).

During last few decades, several studies have been made on understanding and predicting the distribution of chemically reactive and non-reactive air pollutants in the atmosphere. Heines & Peters (1973) carried out an investigation regarding the chemical reaction on the air pollutant. The inter-conversion of primary to secondary pollutant due to the chemical reaction from the time-dependent point source has been studied by Shukla and Chauhan (1988). Naresh and Nath (1991) in his paper assumed the rate of chemical reaction to vary in accordance with height. and studied its impact on the dispersion of pollutant emitted from an area source. Shekhu and Sulochana (2010), proposed an unsteady state model for chemically reactive pollutants emitted from a line source. Joseph and Agarwal (2020) studied the interconversion of primary to secondary pollutant under mesoscale wind where the rate of reaction is assumed to vary with height.

It was observed that the gases emitted from the industrial area, power plants, vehicles are highly reactive during high temperature e.g. sulphur dioxide, oxide of nitrogen etc. Sulochana and Shekhu (2010) showed that the rise of temperature enhances the formation of secondary pollutant. Under such situation the formation of secondary pollutant due to temperature dependent rate of reaction produce a direct impact on the air quality.

We therefore, proposed a mathematical model by considering the emission of reactive pollutant from a point source in presence of urban heat island effect. We assumed that due to UHII when the temperature of urban area is high and persist to a desirable temperature favourable for inter-conversion then the process take place which exist for a certain interval of time i.e.  $t = t_1$ . Finally, for  $t > t_1$  i.e.

when the temperature is less than the required temperature for the chemical reaction, then the interconversion process stops. The whole problem is presented in two intervals by considering advective-diffusion equations with appropriate initial and boundary conditions. Analytical approach is adopted to obtain the required solution using the concept of Laplace Transform and Green's Function.

## 2. MATHEMATICAL FORMULATION

The urban heat island intensity is the temperature difference between urban temperature ( $T_u$ ) and the rural temperature ( $T_r$ ) i.e.  $(T_u - T_r) = \Delta T$ . Considering, the paper (Lee et.al. 2012), the temporal evolution of the UHI intensity is given by

$$\Delta T = T_u - T_r = (T_u - T_r)_0 \exp\left(-k \frac{u}{L} t\right) \quad (2.1)$$

where  $(T_u - T_r)_0$  is the initial temperature difference,  $L$  is the urban length scale and  $u$  is the wind speed.  $k$  is considered as a constant which is taken to be 0.45 corresponding to the average wind speed as 3.3 m/sec (Lee et.al. 2012).

We know that there are certain pollutants which are highly reactive during high temperature. For example, sulphur dioxide, oxides of nitrogen, ozone etc. We know that the rate of reaction is temperature dependent and given by Arrhenius equation as

$$\alpha = A e^{-\frac{E_a}{RT_u}} \quad (2.2)$$

where  $\alpha$  is the rate of reaction,  $A$  is Arrhenius constant,  $E_a$  is activation energy for the reaction,  $R$  is universal gas constant,  $T$  is the temperature in absolute scale. Thus, we considered a case of interconversion of the primary pollutant emitted from a point source into secondary pollutant when the temperature of urban area exists to be high and UHI is high.

When the temperature of urban area drops and becomes less then we assumed that the reaction stops.

We therefore considered the two phases where in the 1st phase, we assumed the inter-conversion of primary pollutant to secondary pollutant takes place due to significant urban temperature which persist for the time interval  $0 \leq t \leq t_1$ . In the second phase, when  $t \geq t_1$ , we assumed the temperature drops and hence there is no inter-conversion of pollutants takes place.

**1st Interval:**  $0 \leq t \leq t_1$

We assumed that the mean concentration of primary pollutant is constant along crosswind distance, and hence the governing advective-diffusion equation is considered as two-dimensional advective diffusion equation.

$$\frac{\partial C_1}{\partial t} + u \frac{\partial C_1}{\partial x} = \frac{\partial}{\partial z} \left( K_z \frac{\partial C_1}{\partial z} \right) - \alpha(t) C_1 \quad (2.3)$$

where  $C_1$  is the concentration of primary pollutant at any point  $(x, z)$ ,  $u$  represent the wind speed along downwind distance and is considered as constant,  $K_z(z)$  represent the diffusivity of the pollutant along

the vertical direction,  $\alpha(t)$  represent the rate of conversion/reaction from primary pollutant to secondary pollutant which is temperature sensitive and time dependent and is represented by (2.2).

From equation (2.1) one can represent the urban temperature as

$$T_u = \lambda \exp\left(-k \frac{u}{L} t\right) + T_r \text{ where } (T_u - T_r)_0 = \lambda \quad (2.4)$$

Considering the value of  $k \frac{u}{L} t$  to be small, we approximate  $\exp\left(-k \frac{u}{L} t\right)$  with the linear term  $\left(1 - k \frac{u}{L} t\right)$

Hence,

$$T_u = \lambda \left(1 - k \frac{u}{L} t\right) + T_r \quad (2.5)$$

Hence, equation (2.2) becomes

$$\alpha = A \exp\left(-\frac{E_a}{R} \left(\lambda \left(1 - k \frac{u}{L} t\right) + T_r\right)^{-1}\right)$$

Further,  $\alpha = A \exp\left(-\frac{E_a}{RT_r} \left(\frac{\lambda}{T_r} \left(1 - k \frac{u}{L} t\right) + 1\right)^{-1}\right)$

Using, the Binomial expansion we get

$$\begin{aligned} \alpha &= A \exp\left(-\frac{E_a}{RT_r} \left(1 - \frac{\lambda}{T_r} \left(1 - k \frac{u}{L} t\right)\right)\right) \\ \alpha &= A \exp\left(-\frac{E_a}{RT_r} \left(1 - \frac{\lambda}{T_r} + \frac{\lambda k}{T_r L} ut\right)\right) = A e^{-\frac{E_a}{RT_r}} \cdot e^{\frac{E_a \lambda}{RT_r^2}} \cdot e^{-\left(\frac{E_a \lambda k u}{RT_r^2 L} t\right)} \end{aligned} \quad (2.6)$$

Assuming,  $\alpha = \omega e^{-(\tau t)}$  where  $\omega = A e^{-\frac{E_a}{RT_r}} \cdot e^{\frac{E_a \lambda}{RT_r^2}}$ ,  $\tau = \frac{E_a \lambda k u}{RT_r^2 L}$  (2.7)

Hence, equation (2.3) becomes

$$\frac{\partial C_1}{\partial t} + u \frac{\partial C_1}{\partial x} = \frac{\partial}{\partial z} \left( K_z \frac{\partial C_1}{\partial z} \right) - \omega e^{-(\tau t)} C_1 \quad (2.8)$$

It was assumed that initially there was no contamination and have zero concentration of primary pollutant. Thus,

$$C_1(x, z, t) = 0, \quad t = 0, x \geq 0, z \geq 0 \quad (2.9)$$

It was assumed that the primary pollutant is emitted at a constant rate from a point source located at  $x = 0$ , and height  $h_s$  from the ground level. Thus,

$$C_{1t}(0, z, t) = \frac{Q}{u} \delta(z - h_s) \text{ at } x = 0, t > 0 \quad (2.10)$$

The primary pollutant emitted from the source are assumed to be perfectly reflected when contacted with the ground, which can be mathematically expressed as

$$K_z \frac{\partial C_1}{\partial z} = 0 \quad \text{at } z = 0 \quad (2.11)$$

It is assumed that the primary pollutant could not penetrate through the top of the inversion/mixed layer and therefore the mathematical expression becomes

$$K_z \frac{\partial C_1}{\partial z} = 0 \quad \text{at } z = H \quad (2.12)$$

Corresponding to the secondary pollutant, the advection-diffusion equation regarding the concentration of secondary pollutant is represented as

$$\frac{\partial C_2}{\partial t} + u \frac{\partial C_2}{\partial x} = \frac{\partial}{\partial z} \left( K_{z_1} \frac{\partial C_2}{\partial z} \right) + \omega e^{-(\tau t)} C_1 \quad (2.13)$$

where  $C_2$  represent the concentration of secondary pollutant,  $K_{z_1}$  represent the diffusivity coefficient of secondary pollutant.

It was assumed that initially there was no existence of primary pollutant and therefore, no conversion took place. Thus,

$$C_2(x, z, t) = 0, \quad t = 0, x \geq 0, z \geq 0 \quad (2.14)$$

Further, the secondary pollutant are assumed to be perfectly reflected when contacted with the ground, which can be mathematically expressed as

$$K_{z_1} \frac{\partial C_2}{\partial z} = 0 \quad \text{at } z = 0 \quad (2.15)$$

It is assumed that the secondary pollutant could not penetrate through the top of the inversion/mixed layer and therefore the mathematical expression becomes

$$K_{z_1} \frac{\partial C_2}{\partial z} = 0 \quad \text{at } z = H \quad (2.16)$$

**II<sup>nd</sup> Interval:**  $t \geq t_1$

In second phase, we assumed that due to the drop in temperature after certain time the conversion stops. In this situation, the advection-diffusion equations corresponding to primary and secondary pollutant becomes

$$\frac{\partial C'_1}{\partial t} + u \frac{\partial C'_1}{\partial x} = \frac{\partial}{\partial z} \left( K_z \frac{\partial C'_1}{\partial z} \right) \quad (2.17)$$

$$\frac{\partial C'_2}{\partial t} + u \frac{\partial C'_2}{\partial x} = \frac{\partial}{\partial z} \left( K_{z_1} \frac{\partial C'_2}{\partial z} \right) \quad (2.18)$$

At  $t = t_1$ , it was assumed that the concentration of primary and secondary pollutant in both the intervals are same

$$C_1(x, z, t) = C'_1(x, z, t) \quad \text{at } t = t_1 \quad (2.19)$$

$$C_2(x, z, t) = C'_2(x, z, t) \quad \text{at } t = t_1 \quad (2.20)$$

In second interval, the primary and secondary pollutant are assumed to be perfectly reflected when contacted with the ground, which can be mathematically expressed as

$$K_z \frac{\partial C'_1}{\partial z} = 0 \quad \text{at } z = 0 \quad (2.21)$$

$$K_{z_1} \frac{\partial C'_2}{\partial z} = 0 \quad \text{at } z = 0 \quad (2.22)$$

Also, in second interval, the primary and secondary pollutant could not penetrate through the top of the inversion/mixed layer and therefore the mathematical expression becomes

$$K_z \frac{\partial C_1'}{\partial z} = 0 \quad \text{at } z = H \quad (2.23)$$

$$K_{z_1} \frac{\partial C_2'}{\partial z} = 0 \quad \text{at } z = H \quad (2.24)$$

### 3. METHOD OF SOLUTION

**1st Interval:**  $0 \leq t \leq t_1$

In this paper we consider that the coefficient of diffusivity is time dependent and temperature dependent and therefore, for simplicity we consider it as  $K_z = K_0 e^{-(\tau t)}$ ,  $K_{z_1} = K_1 e^{-(\tau t)}$  where  $K_0$  and  $K_1$  are the diffusivity coefficient of primary and secondary pollutants at a reference point.

To solve the above equations (2.8), (2.13) we use the transformation

$$X = \int \frac{dx}{e^{\tau t}} = x \cdot e^{-\tau t} \quad (3.1)$$

$$T = \int_0^t \frac{dt}{e^{\tau t}} = \frac{1 - e^{-\tau t}}{\tau} \quad (3.2)$$

Using the above transformations, equation (2.8) and (2.13) becomes

$$\frac{\partial C_1}{\partial T} + u \frac{\partial C_1}{\partial X} = \frac{\partial}{\partial z} \left( K_0 \frac{\partial C_1}{\partial z} \right) - \omega C_1 \quad (3.3)$$

$$\frac{\partial C_2}{\partial T} + u \frac{\partial C_2}{\partial X} = \frac{\partial}{\partial z} \left( K_1 \frac{\partial C_2}{\partial z} \right) + \omega C_1 \quad (3.4)$$

The corresponding initial and boundary conditions (2.9) – (2.12) and (2.14) – (2.16) are transformed accordingly which are as follows:

$$C_1(X, z, T) = 0, \quad T = 0, X \geq 0, z \geq 0 \quad (3.5)$$

$$C_2(X, z, T) = 0, \quad T = 0, X \geq 0, z \geq 0 \quad (3.6)$$

$$C_1(0, z, T) = \frac{Q}{u} \delta(z - h_s) \text{ at } X = 0, T > 0 \quad (3.7)$$

$$\frac{\partial C_1}{\partial z} = 0 \text{ at } z = 0 \quad (3.8)$$

$$\frac{\partial C_1}{\partial z} = 0 \text{ at } z = H \quad (3.9)$$

$$\frac{\partial C_2}{\partial z} = 0 \text{ at } z = 0 \quad (3.10)$$

$$\frac{\partial C_2}{\partial z} = 0 \text{ at } z = H \quad (3.11)$$

We use the concept of matrix form as Astarita et.al. (1979), Alam and Seinfeld (1981) to get uncoupled equations as (dropping bar for convenience)

$$\frac{\partial C_1}{\partial T} + u \frac{\partial C_1}{\partial X} = \frac{\partial}{\partial z} \left( K_0 \frac{\partial C_1}{\partial z} \right) - \omega C_1 \quad (3.12)$$

$$\frac{\partial B}{\partial T} + u \frac{\partial B}{\partial X} = \frac{\partial}{\partial z} \left( K_1 \frac{\partial B}{\partial z} \right) \quad (3.13)$$

$$\text{where } B = C_2 + C_1 \quad (3.14)$$

The corresponding initial and boundary conditions for  $B(X, z, T)$  are as follows:

$$B(X, z, T) = 0 \text{ at } T = 0, X \geq 0, z \geq 0 \quad (3.15)$$

$$B(X, z, T) = \frac{Q}{u} \delta(z - h_s) \text{ at } X = 0 \quad (3.16)$$

$$K_1 \frac{\partial B}{\partial z} = 0, z = 0 \quad (3.17)$$

$$K_1 \frac{\partial B}{\partial z} = 0, z = H \quad (3.18)$$

Using Laplace Transform and method of separation of variable, the solution corresponding to the equation (3.12) along with the conditions (3.5), (3.7) – (3.9) is represented as

$$C_1(X, z, T) = \frac{Q}{u} \sum_{n=1}^{\infty} \frac{\cos \frac{p_n}{\sqrt{K_0}} h_s}{\int_0^H \cos^2 \frac{p_n}{\sqrt{K_0}} z dz} e^{-\frac{p_n^2}{u} X} \cdot e^{-\frac{w}{u} X} \cdot \cos \frac{p_n}{\sqrt{K_0}} z \cdot \delta \left( T - \frac{X}{u} \right) \quad (3.19)$$

where,  $\delta(\cdot)$  is Dirac-delta function.

Here,  $p_n$  are the eigenvalues and is represented as  $p_n = \frac{n\pi}{H} \sqrt{K_0}$ ,  $n = 1, 2, 3, \dots$

The solution corresponding to the (3.13) along with the conditions (3.15) – (3.18) is represented as

$$B(X, z, T) = \frac{Q}{u} \sum_{m=1}^{\infty} \frac{\cos \frac{\lambda_m}{\sqrt{K_1}} h_s}{\int_0^H \cos^2 \frac{\lambda_m}{\sqrt{K_1}} z dz} e^{-\frac{\lambda_m^2}{u} X} \cdot \cos \frac{\lambda_m}{\sqrt{K_1}} z \cdot \delta \left( T - \frac{X}{u} \right) \quad (3.20)$$

Here,  $\lambda_m$  are the eigenvalues and is represented as  $\lambda_m = \frac{m\pi}{H} \sqrt{K_1}$ ,  $m = 1, 2, 3, \dots$

**IIInd Interval:**  $t \geq t_1$

For obtaining the concentration in the second phase ( $t \geq t_1$ ), we again consider the same transformation given by equations (3.1) and (3.2). Also, we use the Green's function methodology and the concept of Laplace Transform to solve the equations (2.17) and (2.18) with their appropriate conditions.

Thus, the concentration of primary pollutant is represented as

$$C_1' = Q \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} \frac{\cos \left( \frac{\mu_k}{\sqrt{K_0}} z \right) \cdot \cos \left( \frac{p_n}{\sqrt{K_0}} h_s \right)}{\int_0^H \cos^2 \left( \frac{\mu_k}{\sqrt{K_0}} z \right) dz \cdot \int_0^H \cos^2 \left( \frac{p_n}{\sqrt{K_0}} z \right) dz} \cdot \exp(-p_n^2 T|_{t=t_1}) \cdot \exp \left( -w \frac{X}{u} \right) \cdot \left( \int_0^H \cos \left( \frac{\mu_k}{\sqrt{K_0}} z \right) \cdot \cos \left( \frac{p_n}{\sqrt{K_0}} z \right) dz \right) \cdot \cos \left( \frac{p_n}{\sqrt{K_0}} z \right) \cdot \exp \left( -\mu_k^2 \left( \frac{X}{u} - T|_{t=t_1} \right) \right) \cdot \delta \left( T - \left( \frac{X}{u} - T|_{t=t_1} \right) \right) \quad (3.21)$$

The concentration of secondary pollutant is represented as

$$\begin{aligned}
 C'_2 = & Q \sum_{m=1}^{\infty} \sum_{k=1}^{\infty} A_{m,k} \cdot \exp\left(-\mu_k^2 \left(\frac{X}{u} - T|_{t=t_1}\right)\right) \cdot \int_0^H \cos\left(\frac{\lambda_m}{\sqrt{K_1}} z\right) \cos\left(\frac{\mu_k}{\sqrt{K_1}} z\right) dz \\
 & \exp\left(-\lambda_m^2 T|_{t=t_1}\right) \cdot \cos\left(\frac{\lambda_m}{\sqrt{K_0}} z\right) \delta\left(T - \left(\frac{X}{u} - T|_{t=t_1}\right)\right) - \\
 & Q \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} B_{n,k} \cdot \exp\left(-\mu_k^2 \left(\frac{X}{u} - T|_{t=t_1}\right)\right) \cdot \int_0^H \cos\left(\frac{p_n}{\sqrt{K_0}} z_0\right) \cos\left(\frac{\mu_k}{\sqrt{K_1}} z_0\right) dz_0 \\
 & \exp\left(-w \frac{X}{u}\right) \cdot \cos\left(\frac{p_n}{\sqrt{K_0}} z\right) \cdot \exp\left(-p_n^2 T|_{t=t_1}\right) \cdot \delta\left(T - \left(\frac{X}{u} - T|_{t=t_1}\right)\right) \quad (3.22)
 \end{aligned}$$

$$\text{where } A_{m,k} = \frac{\cos\left(\frac{\lambda_m}{\sqrt{K_1}} h_s\right) \cdot \cos\left(\frac{\mu_k}{\sqrt{K_1}} z\right)}{\int_0^H \cos^2\left(\frac{\lambda_m}{\sqrt{K_1}} z_0\right) dz_0 \cdot \int_0^H \cos^2\left(\frac{\mu_k}{\sqrt{K_1}} z_0\right) dz_0} \text{ and}$$

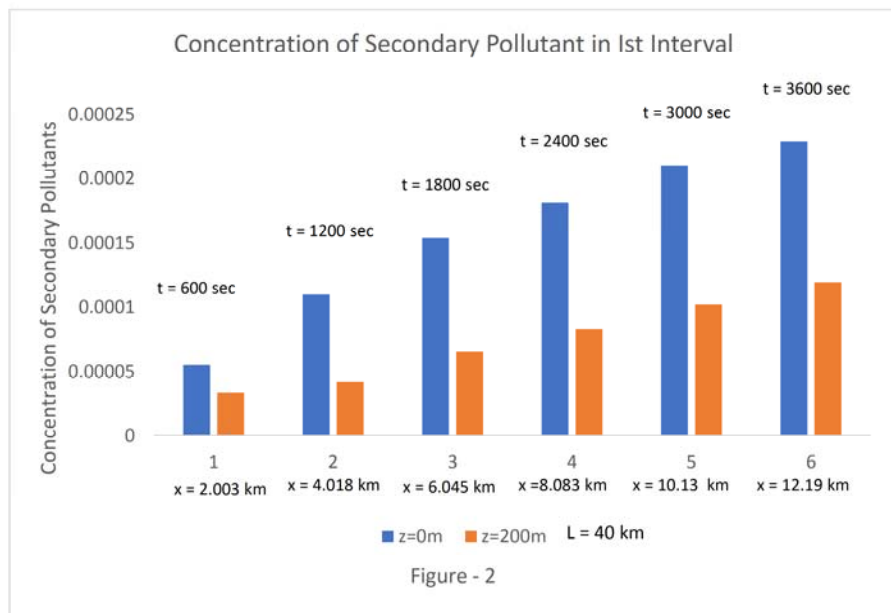
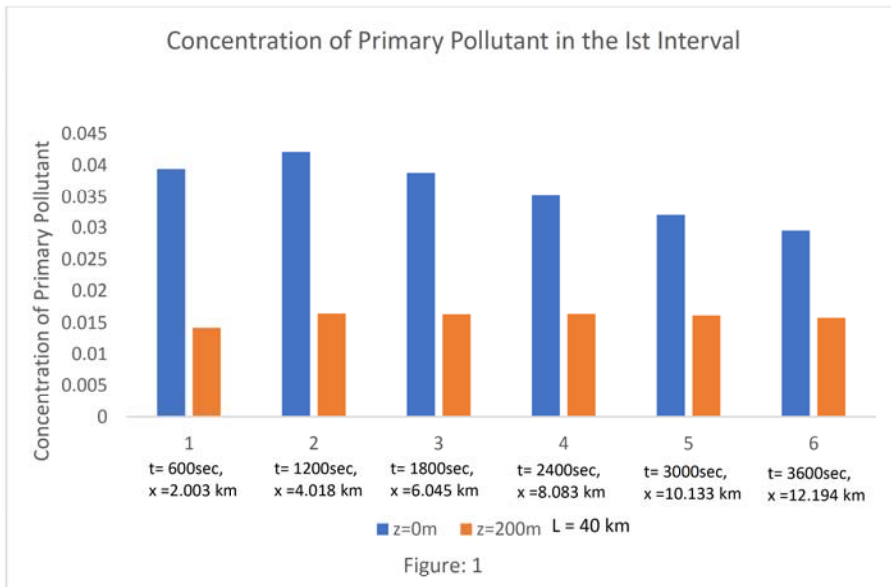
$$B_{n,k} = \frac{\cos\left(\frac{p_n}{\sqrt{K_0}} h_s\right) \cdot \cos\left(\frac{\mu_k}{\sqrt{K_1}} z\right)}{\int_0^H \cos^2\left(\frac{p_n}{\sqrt{K_0}} z_0\right) dz_0 \cdot \int_0^H \cos^2\left(\frac{\mu_k}{\sqrt{K_1}} z_0\right) dz_0}$$

#### 4. RESULTS AND DISCUSSION

The analytical solution so obtained as equations (3.19) – (3.20) in the first interval and equations (3.21) - (3.22) in the second interval are analysed graphically to understand the impact of primary and secondary pollutant in an urban area due to temperature variation. In this regard, we considered the following suitable data:  $A = 10 \text{ mol}^{-1} \text{ L s}^{-1}$ ,  $E_a = 40 \text{ KJ mol}^{-1}$ ,  $R = 8.3145 \text{ J/mol}$ ,  $\lambda = 5 \text{ K}$ ,  $T_r = 308.15 \text{ K}$ ,  $K_0 = 5 \frac{\text{m}^2}{\text{sec}}$ ,  $K_1 = 5 \frac{\text{m}^2}{\text{sec}}$ .

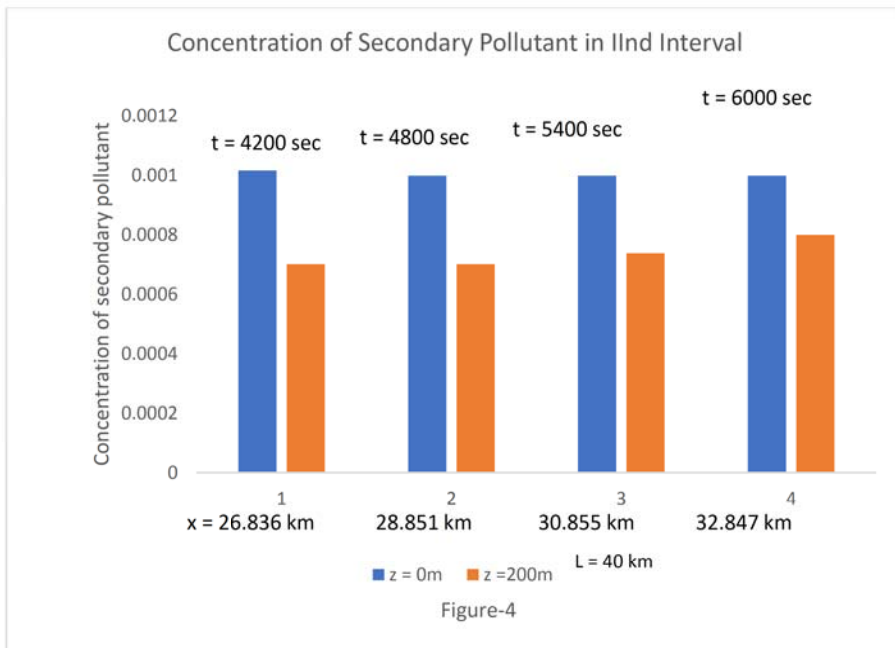
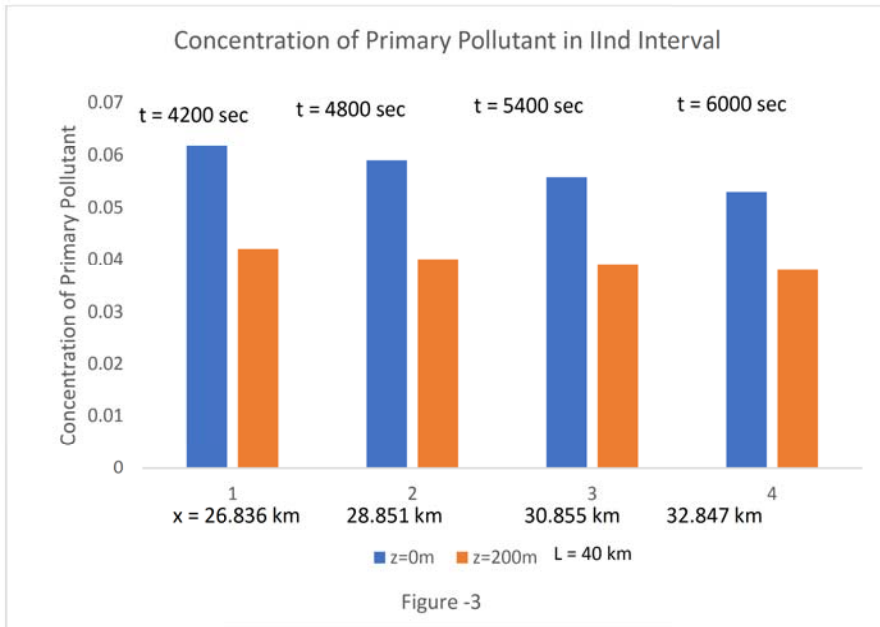
In accordance with the wind speed of 3.3 m/sec, the suitable value of  $k$  is considered as 0.45 (Lee et.al. 2012). The urban length scale is considered as  $L = 40 \text{ km}$ . The height of the chimney is taken as 100m. The inversion height is assumed to be 1 km i.e.  $H = 1000 \text{ m}$ . The emission of the pollutant from the chimney is taken as  $11.1 \frac{\text{kg}}{\text{sec}}$  (Shukla et. al. 1988). We assumed that the inter-conversion process takes places for 1 hour, i.e.  $t_1 = 3600 \text{ sec}$ . Whereas, in the second interval i.e.  $t > 3600 \text{ sec}$ , there is no inter-conversion process and interval last up to  $t = 6000 \text{ sec}$ .

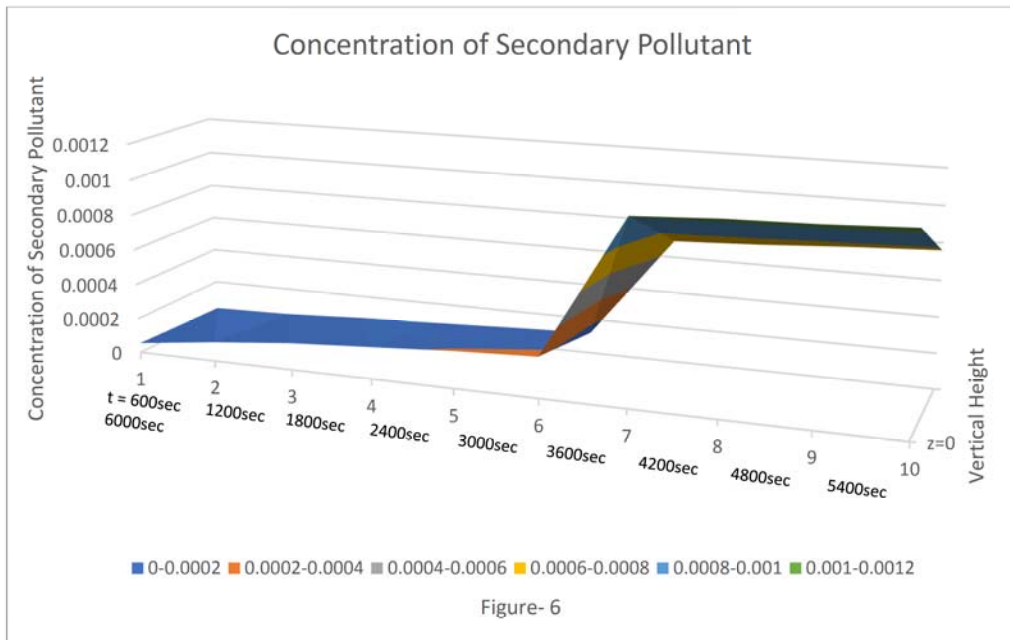
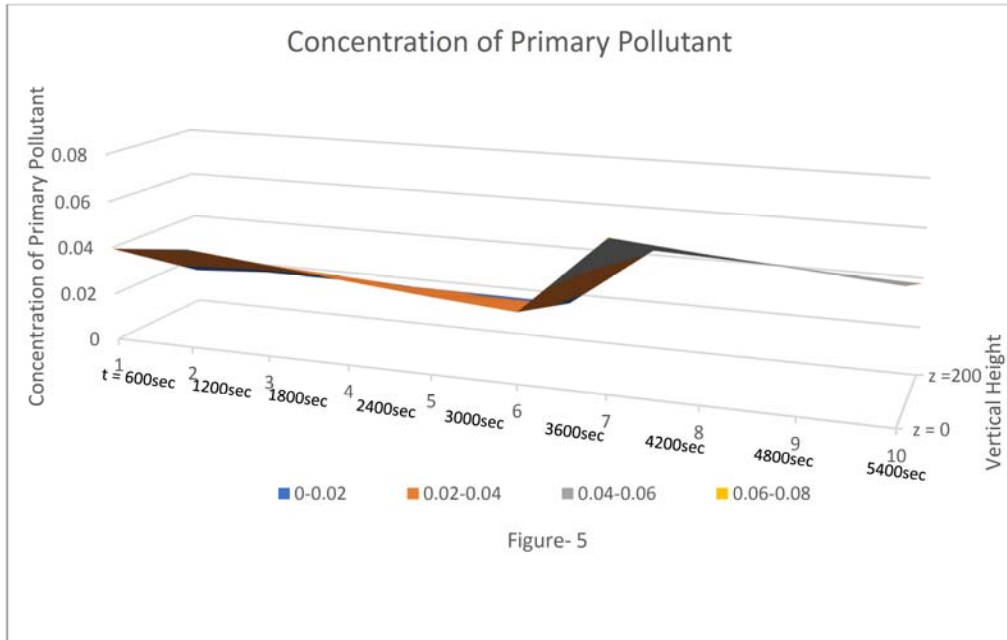
The graphical analysis from the figure 1 – 2 reveals that the ground level concentration (GLC) of primary and secondary pollutant in the 1st interval is much higher than the concentration level at  $z = 200 \text{ m}$ . Further, the concentration level of primary pollutant decreases with the increase in time and downwind distance, as shown in figure -1 but persist to considerable magnitude, whereas the concentration of secondary pollutant increases with the increase in time and downwind distance as shown in the figure -2. Further, the GLC of secondary pollutants is much higher than the concentration at  $z = 200 \text{ m}$ .



The graphs regarding the concentration level for both types of pollutants in second interval are shown in the figures 3 - 4. In figure 3, it was observed that the concentration of primary pollutant increases suddenly from  $t = 3600\text{sec}$  to  $t = 4200\text{sec}$  and then decreases with the increase in time and downwind distances for  $z = 0\text{ m}$  and  $z = 200\text{ m}$ . On other side in figure 4 the concentration of secondary pollutant for  $z = 0\text{ m}$  and  $z = 200\text{ m}$  increases sharply from  $t = 3600\text{sec}$  to  $t = 4200\text{sec}$  and then remain almost same for different downwind distance and time interval for  $z=0$ ,  $z = 200\text{ m}$ . The overall behaviour of the pollutant in both the intervals as shown in the figure 5 -6 conclude that the impact of primary and

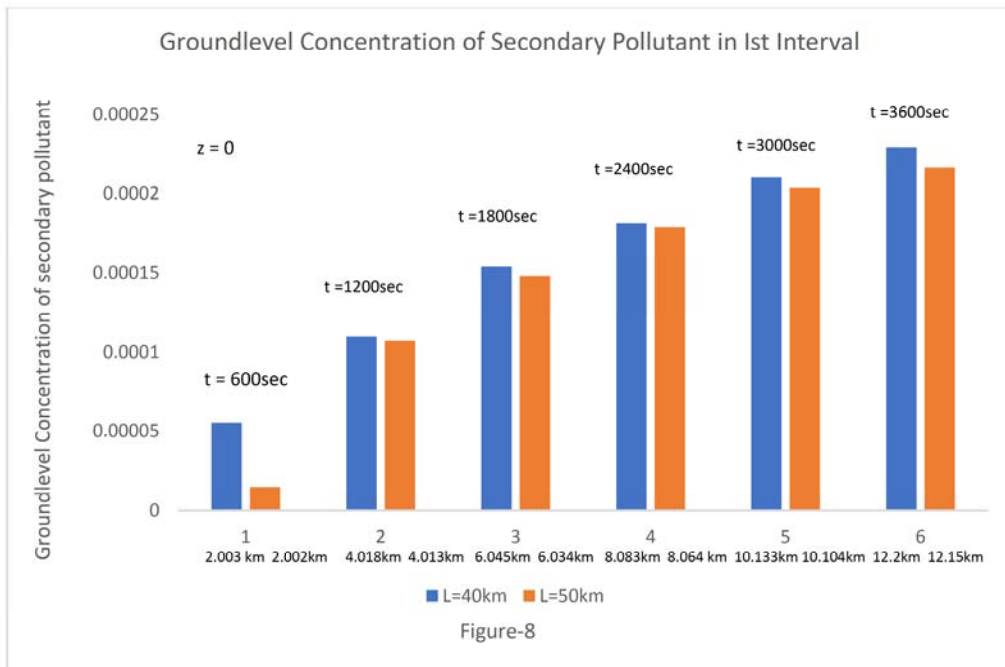
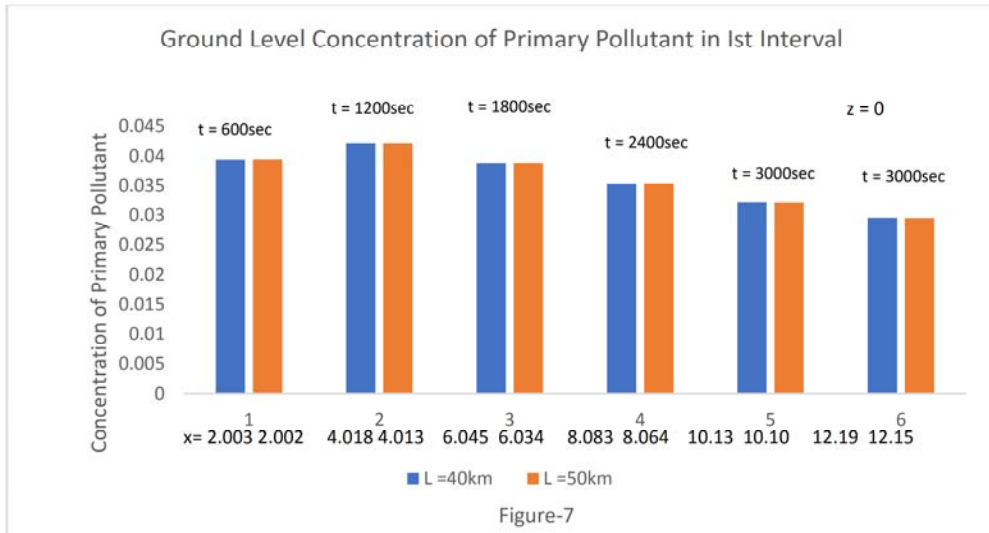
secondary pollutant is more significant in second interval due to the persistence of the concentration of pollutants even after the non-existence of chemical reaction. Hence, the primary and secondary pollutant makes a huge footprint on the environment in second interval as compared in first interval.

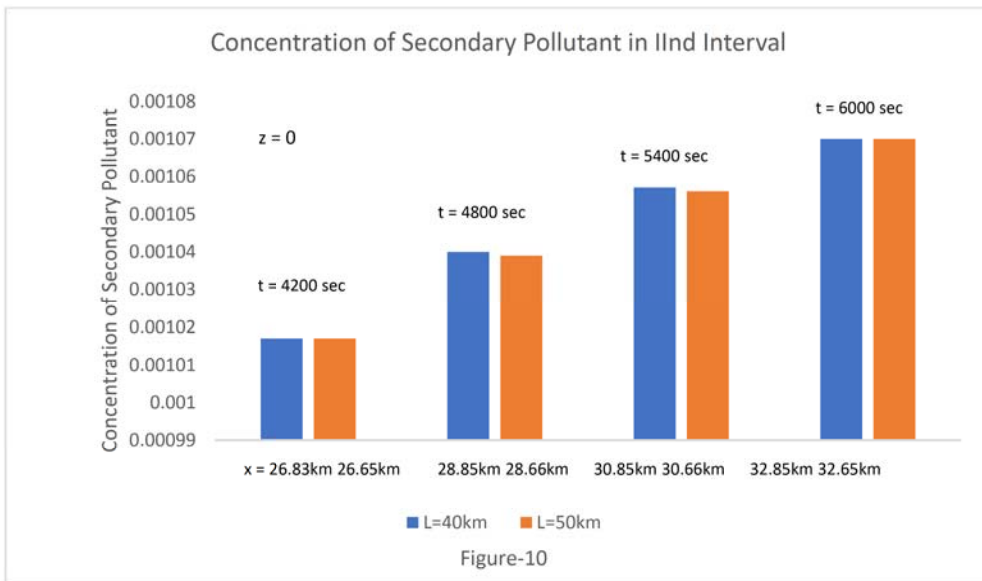
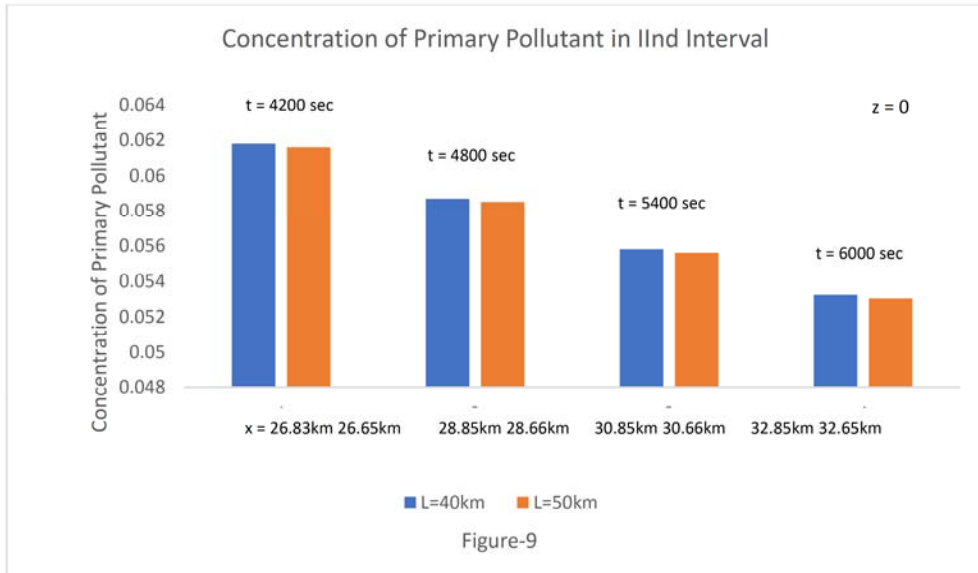




While considering the expansion of an urban area, the graphical representation regarding distribution of the primary and secondary pollutant in both the intervals are plotted while considering  $L = 40 \text{ km}$  and  $L = 50 \text{ km}$  as shown in the figure 7 - 10. In first phase, the concentration of primary pollutant decreases very marginally with the expansion of urban area as shown in the figure 7. In same way, the concentration of secondary pollutant decreases with the increase in urban length as shown in the figure 8. The same trend was observed in second interval where the concentration of primary pollutant and

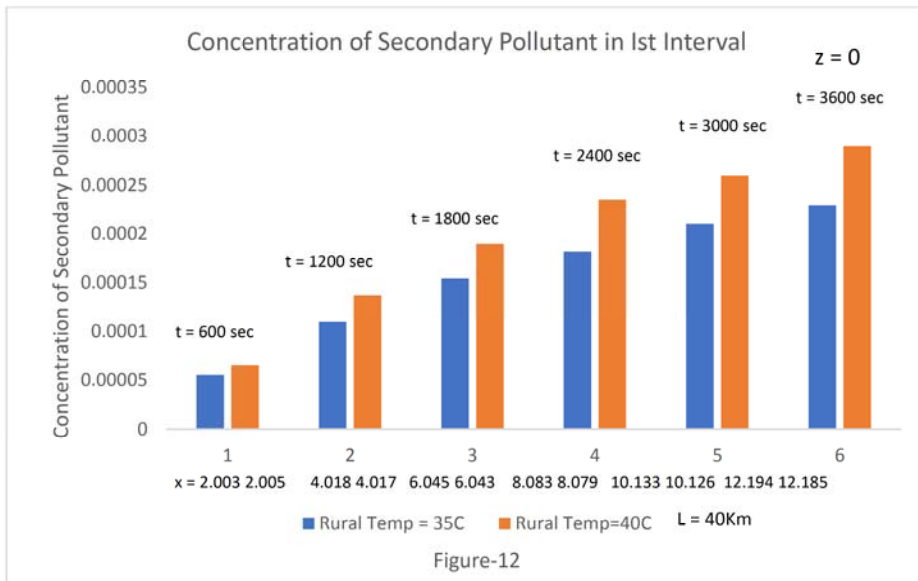
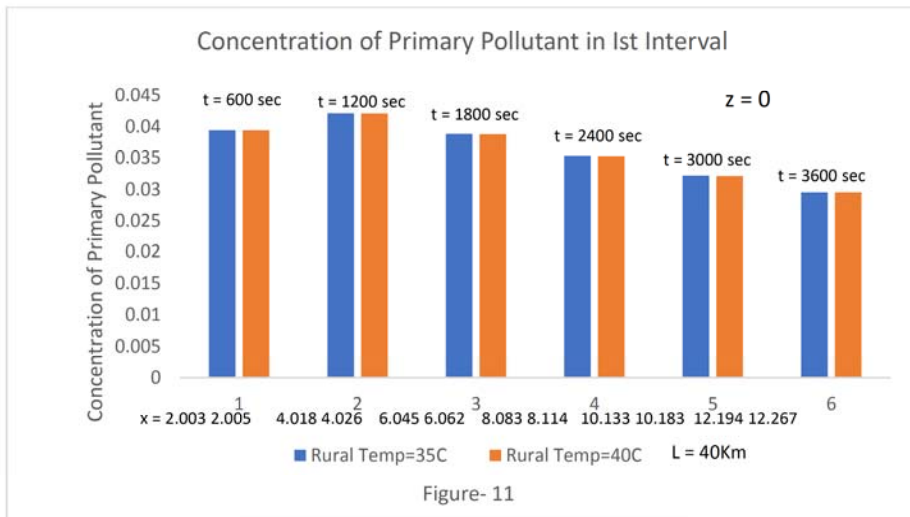
secondary pollutant decreases very slightly with the increase in the urban length. Also, it was noticed that the concentration level of both types of pollutants persist to a significant level at a far downwind distance and higher time interval as shown in the figure 9-10. The overall conclusion drawn from figure 7 -10 is that the concentration level of the pollutants decreases with the expansion of the urban length.

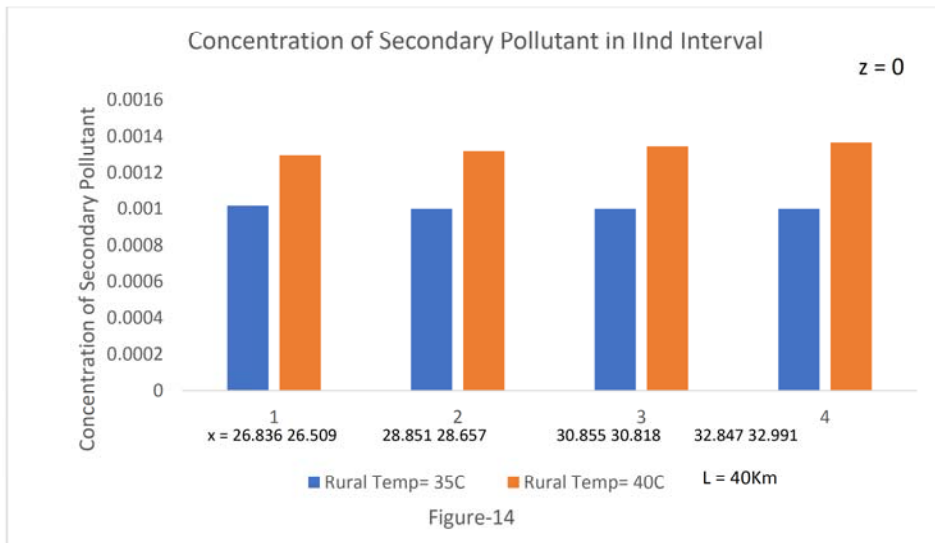
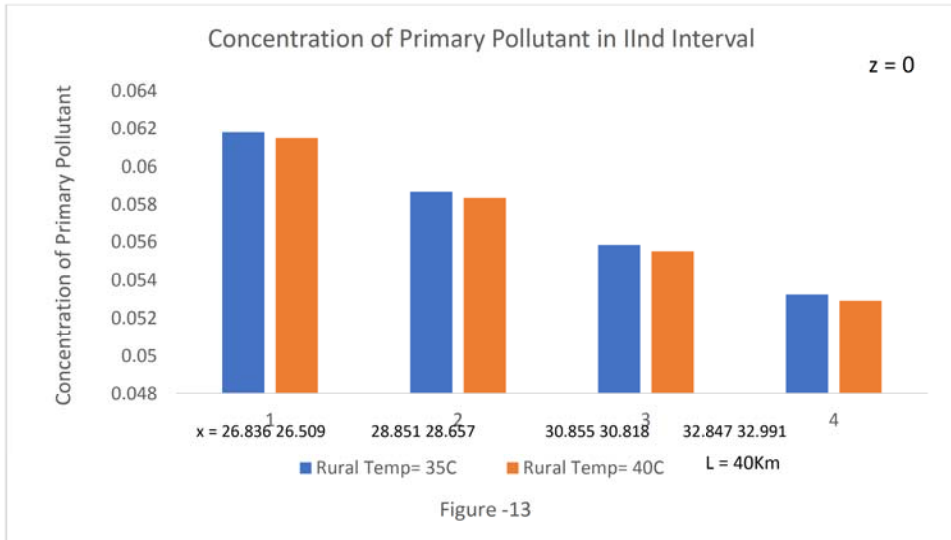




The temperature variation plays an important role in the distribution of pollutants. To understand such phenomenon, we consider two different rural temperature as  $T_r = 308.15\text{ K}$  &  $313.15\text{ K}$ . The overall behaviour of the pollutants in both intervals are plotted in figure 11 -14. It was observed that in the first interval while the rural temperature increases ( $\lambda$  remains same) the ground-level concentration (GLC) of primary pollutant decreases slightly. It was found that the difference of ground-level concentration (GLC) for  $T_r = 308.15\text{ K}$  &  $313.15\text{ K}$  respectively increased with the increase in time and downwind distance. Further, the ground-level concentration (GLC) of secondary pollutant in the first interval increases with the increase in rural temperature. Also, the difference of ground-level concentration (GLC) of secondary pollutant for  $T_r = 313.15\text{ K}$  &  $308.15\text{ K}$  respectively increases with the increase in

time and downwind distance. This shows that with the increase in temperature the impact of secondary pollutant during chemical reaction becomes more significant in the surrounding. While in absence of inter-conversion i.e. in second interval, the ground-level concentration of primary pollutant still decreases marginally with the rise in temperature of rural area as shown in the figure – 13 and show the same trend of increasing in the difference of concentration at rural temperature  $T_r = 308.15\text{ K}$  &  $313.15\text{ K}$  respectively. On other hand the concentration of secondary pollutant in absence of inter-conversion still increases (marginally) with the increase in rural temperature as shown in figure -14. However, the concentration level of secondary pollutant persists to a significant level at greater downwind distance and time.





## 5. CONCLUSION

The study explores how the temperature plays an influential role in the inter-conversion process from primary to secondary pollutant under urban heat island intensity. In the present study we considered the rate of chemical reaction to be temperature dependent and follows the Arrhenius equation. With this consideration, the present study assumed two intervals in which the first interval is taken to be phase of chemical reaction i.e. the process of inter-conversion while the second interval is non-existence of chemical reaction i.e. when the inter-conversion process stops. The temperature so considered in the Arrhenius equation follows urban island heat intensity formula. The overall process is studied by defining the two-dimensional advective diffusion partial differential equations along with the appropriate

boundary conditions. Analytical solutions are obtained by using the concept of Laplace Equation, separation of variables and Green's Function. The suitable parameters were considered to understand the overall situation. The following findings are stated below:

- (i) In first interval, the concentration of primary pollutant decreases with the increase in time and downwind distance whereas the concentration of secondary pollutant increases with the increase in time and downwind distance.
- (ii) In second interval, the concentration of primary pollutant increases suddenly from  $t = 3600\text{sec}$  to  $t = 4200\text{sec}$  and then decreases slowly. On other hand, the concentration of secondary pollutant increases from  $t = 3600\text{sec}$  to  $t = 4200\text{sec}$  and then remain almost constant.
- (iii) On the expansion of urban area, the ground-level concentration (GLC) of primary and secondary pollutants decreases in both the intervals but persist to a significant level at far downwind distance.
- (iv) Due to the increase in temperature, the GLC of primary pollutant decreased (marginally) in the first interval whereas the GLC of secondary pollutant increased in the first interval. Thus, increase in the temperature helps in lifting the concentration level of secondary pollutant in the first interval.
- (v) With the increase in the rural temperature the impact of secondary pollutant during the chemical reaction becomes more significant in the surrounding.
- (vi) The GLC of primary pollutant in second interval decreased with the increase in temperature whereas GLC of secondary pollutant in second interval increased (marginally) with the increase in temperature and persist to a greater downwind distance and time.
- (vii) It was observed that the GLC of secondary pollutant in second interval (in absence of chemical reaction) persist to a significant level for a greater downwind distance and longer time duration.
- (viii) The impact of primary and secondary pollutants are more significant in second interval due to the persistence of the concentration of pollutants even after the non-existence of chemical reaction.

## 6. REFERENCES

- Agarwal M., Tandon A., 2010, Modeling of the urban heat island in the form of mesoscale wind and of its effect on air pollution dispersal. *Applied Mathematical Modelling*. **34(9)**, 2520-2530.
- Alam M. K. and Seinfeld J. H., 1981, Solution of steady state three-dimensional atmospheric diffusion equation for sulphur dioxide and sulphate dispersion from point sources. *Atmospheric Environment*. **15**, 1221- 1225.
- Astarita G., Wei J., Iorio G., 1979, Theory of dispersion, transformation and deposition of atmospheric pollutant using modified Green's functions. *Atmospheric Environment*. **13**, 239-246.
- Chandler T. J., 1968, Discussion of the paper by marsh and foster. The bearing of the urban temperature field upon urban pollution patterns. *Atmospheric Environment*. **2**, 619 - 620.

Chung C. Y., Yang J., Yang X., He J., 2022, Mathematical modelling in the health risk assessment of air pollution-related disease burden in China: A review. *National Library of Medicine, Front Public Health*, doi: 10.3389/fpubh.1060153

Cichowicz R., Wielgosiński G, Fetter W., 2017, Dispersion of atmospheric air pollution in summer and winter season. *Environmental Monitoring and Assessment*, 189: 605.

Dilley J. F., Yen K. T., 1971, Effect of a mesoscale type wind on the pollutant distribution from a line source. *Atmospheric Environment*. **6**, 843-851.

Elbayoumi M., Ramli N. A., Fitri N. F., I Madhoun W. Al., 2014, The effect of seasonal variation on indoor and outdoor carbon monoxide concentration in Eastern Mediterranean climate. *Atmospheric Pollution Research* (5), 315 -324.

Griffiths J. F., 1970, Problems in urban air pollution. *American Institute of Aeronautics and Astronautics*.

Hanna, S. R., 1984, Applications in air pollution modelling. *Atmospheric turbulence and air pollution modelling*. Springer, Dordrecht, 275-310.

Heines T. S. and Peters L. K., 1973, An analytical investigation of effect of a first order chemical reaction on the dispersion of pollutants in the atmosphere. *Atmospheric Environment*. **7**, 153-162.

Joseph V., Agarwal M., 2023, A Study on variation in the concentration of Carbon Monoxide in an urban city to the seasonal variation with temperature dependent meteorological parameters. *Ganita*. **73(2)**, 195 -210.

Joseph V., Yadav R., 2021, Impact of Pollutants in the Patchy Atmosphere Due to Mesoscale Wind under Multiple Point Sources. *International Journal of Ecological Economics and Statistics*. **42**, 84 - 98.

Joseph V., Agarwal M., 2020, Mathematical Modelling on the Effect of Chemical Reaction on the Air Pollutant under Mesoscale Wind. *Ganita*. **70(2)** 73 -84.

Lee T.-W., Lee J. Y., Wang Z.-H., 2012, Scaling of the urban heat island intensity using time-dependent energy balance. *Urban Climate*. **2**, 16 -24.

Lee, T.-W., Ho, A., 2010, Scaling of the urban heat island effect based on the energy balance. nighttime minimum temperature increases vs urban length scale of Phoenix and Tucson, AZ, USA. *Climate Research*. **42**, 209 –216.

Naresh R. and Nath R., 1991, The effects of chemical reactions on the dispersion of air pollutants over area sources. *Earth, Moon and Planets*. **48**, 99 -111.

Rao V. L., 2014, Effects of urban heat island on air pollution concentrations. *International Journal of Current Microbiology and Applied Sciences*. **3(10)**, 388 - 400.

Seinfeld, J. H., 1986, Atmospheric Chemistry and Physics of Air Pollution, *John Wiley & Sons, New York*.

Shekhu M., Sulochana C., 2011, Time dependent model for chemically reactive pollutants emitted from various line sources into a stable atmospheric boundary layer. *Journal of Petroleum and Gas Exploration Research*. **1(1)**, 2011, 22-33.

Shukla J. B. and Chauhan R. S., 1988, Unsteady state dispersion of air pollutant from a time dependent point source forming a secondary pollutant., *Atmospheric Environment*. **22**, 2573-2578.

Sulochana C., Shekhu M., 2010, Unsteady state mathematical model of chemically reactive pollutants from an instantaneous line source into a stable atmospheric boundary layer. **217**, 2757 – 2767.

Summers P. W., 1964, An Urban Ventilation Model Applied to Montrea. *Department of Meterology, McGill University, Montreal*.

Swamy G.S.N.V.K.S.N., Nagendra S. M., Schlink U., 2020, Impact of urban heat island on meteorology and air quality at microenvironments. *Journal of the Air & Waste Management Association*. **70**, 876 - 891.

Ulfah S., Awalludin S. A., Wahidin, 2018, Advection-diffusion model for the simulation of air pollution distribution from a point source emission. *IOP Conf. Series: Journal of Physics: Conf. Series* **948**, 012067, doi :10.1088/1742-6596/948/1/012067.

Zhao W., Khalil M. A. K., 1993, The Relationship between the Precipitation and the Temperature over the contiguous United States, *Journal of Climate*. **6**, 1232 – 1236.