

Intricacies of Duality: Binary Intuitionistic Open Sets Explored

A.G.Rose Venish¹, L.Vidyarani² and M.Vigneshwaran³

^{1,2,3}Department of Mathematics

Kongunadu Arts and Science College(Autonomous)

Coimbatore-641 029, Tamilnadu, India.

¹venishya123@gmail.com, ²vidyarani16@gmail.com,

³vigneshmaths@kongunaducollege.ac.in

ABSTRACT

The basics of topology includes the most important sets called open sets. Intuitionistic open sets exists as the basic sets for Intuitionistic Topological Space(ITS). In this article different kinds of intuitionistic open set in a special space namely binary intuitionistic topological space is introduced. Binary intuitionistic open-sets (B_1OS) namely binary intuitionistic semi- open-sets, binary intuitionistic pre- open-sets, binary intuitionistic α - open-sets are newly defined and their features existing are studied. Binary intuitionistic interior and binary intuitionistic closure of the these open sets are defined and few of its characteristics are discussed.

Keywords: BIP, BVIP, binary intuitionistic neighborhood structures and BITS.

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1 Introduction

Zadeh. L. A introduced fuzzy sets (L.A.Zadeh, 1965), which has numerous applications both theoretically and practically. Furthermore the eminence of fuzzy theory, intuitionistic fuzzy set came into existence, which is a notion by Atanassov and D.Coker (K. Atanassov, 1986)(D.Coker, 1996). Different types of open sets namely intuitionistic semi, pre and α - open sets in intuitionistic topology (IT) were introduced by authors (S.Girija and Gnanambal Ilango, 2014). The properties of these sets were studied in detail earlier. Binary topology applications in soft sets are discussed in (S.Hussain, 2020). Further binary intuitionistic open sets were introduced and their properties of neighborhoods were discussed in (L.Vidyarani and A.G.Rose Venish, 2023). As an extension of this work binary intuitionistic semi open, binary intuitionistic pre- open and binary intuitionistic α - open sets respectively in binary intuitionistic topology were introduced in this article. Description of such sets which exists naturally were studied and the behaviour of these sets in fundamental topological spaces were discussed.

2 Preliminaries

A remark illustrates theorems and fundamental definitions are conferred in this section in order to illustrate the idea introduced in this article.

Remark 2.1. For definitions of intuitionistic set and IT refer (D.Coker, 1996), intuitionistic (α -open, semi-open, pre-open and β -open regular-open) sets (S.Girija and Gnanambal Ilango, 2014), binary-intuitionistic set, topology, interior, closure, binary intuitionistic point, their properties (L.Vidharani and A.G.Rose Venish, 2023).

3 Binary intuitionistic open sets

This section brings out the definition of different forms of binary intuitionistic open- sets namely binary intuitionistic semi- open- sets (B_IOS), binary intuitionistic pre- open sets (B_IPOS) & binary intuitionistic α - open- sets ($B_I\alpha OS$) and definitions respective to their interior and closure are also defined.

Definition 3.1. Let (T, S, B_I) be a BITS and let (D, C) be a binary intuitionistic set in (T, S) then (D, C) is a binary intuitionistic semi-open set (BISOS) if $(D, C) \subseteq B_Icl(B_Iint(D, C))$. Complement of BISOS is binary intuitionistic semi closed (BISCS).

Definition 3.2. Let (T, S, B_I) be a BITS and let (C, D) be a binary intuitionistic set in (T, S) then (C, D) is a binary intuitionistic pre-open set (BIPOS) if $(C, D) \subseteq B_Iint(B_Icl(C, D))$. Complement of BIPOS is binary intuitionistic pre closed (BIPCS).

Definition 3.3. Let (T, S, B_I) be a BITS and let (C, D) be a binary intuitionistic set in (T, S) then (C, D) is said to be a binary intuitionistic α open set ($B_I\alpha OS$) if $(C, D) \subseteq B_Iint(B_Icl(B_Iint(C, D)))$. The complement of every $(B_I\alpha OS)$ is said to be binary intuitionistic α closed set ($(B_I\alpha CS)$).

Definition 3.4. Let (T, S, B_I) be a BITS and let C be a binary intuitionistic set, then the binary intuitionistic semi interior ($B_I sint$), binary intuitionistic semi closure ($B_I scl$), the binary intuitionistic pre interior ($B_I pint$), binary intuitionistic pre closure ($B_I pcl$), the binary intuitionistic α interior ($B_I\alpha int$), the binary intuitionistic α closure ($B_I\alpha cl$) of C are defined as:

1. The binary intuitionistic semi interior ($B_I sint(C)$) of C is largest binary intuitionistic semi open set $\subseteq C$.
2. The binary intuitionistic semi closure C is the smallest binary intuitionistic semi closed set containing C and it is denoted by $B_I scl(C)$.
3. The binary intuitionistic pre interior of C is largest BIPOS $\subseteq C$ and it is denoted by $B_I pint(C)$.
4. The binary intuitionistic pre closure C is the smallest binary intuitionistic pre closed set containing C and it is denoted by $B_I pcl(C)$.
5. The binary intuitionistic α interior of C is largest $B_I\alpha OS \subseteq C$ and it is denoted by $B_I\alpha int(C)$.

6. The binary intuitionistic α closure ($B_I\alpha cl(C)$) C is the smallest binary intuitionistic α closed set containing C .

Theorem 3.1. Let (T, S, B_I) be a BITS and $\{E_\alpha/\alpha \in J\}$ be a family of BIOS. Then $\cup\{E_\alpha/\alpha \in J\}$ is BIOS.

Proof For each α , since E_α is a BIOS, there exists a BIOS F_α such that $F_\alpha \subset E_\alpha \subset B_I cl(F_\alpha)$. Therefore $\cup F_\alpha \subset \cup E_\alpha \subset \cup B_I cl(F_\alpha)$, since $F_\alpha \subset \cup F_\alpha$, $B_I(F_\alpha) \subset B_I cl(F_\alpha)$ and hence $\cup B_I cl(F_\alpha)$. Therefore $\cup F_\alpha \subset B_I cl(\cup F_\alpha \subset \cup E_\alpha \subset \cup B_I cl(F_\alpha) \subset B_I cl(\cup F_\alpha)$ which implies that $\cup E_\alpha$ is a BIOS.

4 Properties of binary intuitionistic semi open sets

Few properties of binary intuitionistic semi open sets are studied in this section.

Theorem 4.1. A subset E in a BITS (T, S, B_I) is a BISOS iff $E \subset B_I cl(B_I int(E))$.

Proof. Necessity :Let E be BISOS. Then $\exists$ a BIOS F , s.t $F \subset E \subset B_I cl(F)$, but $F \subset B_I int(E)$ and thus $B_I cl(F) \subset B_I cl(B_I int(E))$. Hence $E \subset B_I cl(F) \subset B_I cl(B_I int(E))$.

Sufficiency : Let $E \subset B_I cl(B_I int(E))$. We have $B_I int(E) \subset E$. Then for $B_I int(E) \subset E \subset B_I cl(B_I int(E))$. Take $U = B_I int(E)$, we get $U \subseteq E \subseteq B_I cl(U) \Rightarrow E$ is BISOS. $\square$

Remark 4.1. Intersection of 2 BISOS need not be BISOS.

Example 4.2. Suppose $T = \{a\}$ and $S = \{1, 2\}$ and let (T, S, B_I) be any binary space and $B_I = \{(\underline{T}, \underline{S}), (\phi, \phi),$

$(\langle T, \phi, \{a\} \rangle, \langle S, \phi, \{2\} \rangle), (\langle T, \phi, \phi \rangle, \langle S, \phi, \phi \rangle)\}$ Here the BISOS are $\{(\underline{T}, \underline{S}), (\phi, \phi), (\langle T, \{a\}, T \rangle, \langle S, \{a\}, \{a\} \rangle), (\langle T, \phi, T \rangle, \langle S, \phi, \{1\} \rangle), (\langle T, \phi, T \rangle, \langle S, \phi, \{2\} \rangle),$

$(\langle T, \phi, \phi \rangle, \langle S, \phi, \phi \rangle), (\langle T, \phi, \phi \rangle, \langle S, \phi, \{1\} \rangle), (\langle T, \phi, \phi \rangle, \langle S, \phi, \{2\} \rangle)\}$.

Intersection of $(\langle T, \phi, \phi \rangle, \langle S, \phi, \{1\} \rangle)$ and

$(\langle T, \phi, \phi \rangle, \langle S, \phi, \{2\} \rangle)$ is $(\langle T, \phi, \phi \rangle, \langle S, \phi, \{1, 2\} \rangle)$ which is not a BISOS.

Theorem 4.3. Let (T, S, B_I) be any binary intuitionistic space. If E is a BIOS & B , a BISOS, then $E \cap B$ is a BISOS.

Proof. $\therefore B$ is a binary intuitionistic semi open, $\exists$ binary intuitionistic open set F such that $F \subset B \subset B_I cl(F)$ and so $E \cap F \subset E \cap B \subset E \cap B_I cl(F)$. Since $E \cap F$ is a binary intuitionistic open set, $E \cap F = B_I int(E \cap F)$ and by theorem $E \cap B_I cl(F)$. Therefore $E \cap B \subset E \cap B_I cl(F) \subset B_I cl(E \cap F) \subset B_I cl(B_I int(E \cap B))$. Hence $E \cap B$ is a binary intuitionistic semi open set. $\square$

Remark 4.2. If U is binary intuitionistic open in (T, S) , then U is binary intuitionistic semi open in (T, S) . Thus (T, S, B_I) be a binary intuitionistic topological space. Let B_I be set of all BIOSs in (T, S) , then (i) $B_I \subset BISOS$

(ii) For $A \in BIOS$ and $A \subset B \subset B_I cl(A)$ then $B \in BIOS$. Converse of (i) need not be true.

Example 4.4. Suppose $T = \{a\}$ and $S = \{1, 2\}$ and let (T, S, B_I) be any binary space and $B_I = \{(\underline{T}, \underline{S}), (\phi, \phi),$

$(\langle T, T, \phi \rangle, \langle S, \{1\}, \{2\} \rangle))$, the binary intuitionistic semi open sets are $\{(T, S), (\phi, \phi), (\langle T, T, \phi \rangle, \langle S, \{1\}, \{2\} \rangle), (\langle T, T, \phi \rangle, \langle S, \{1\}, \phi \rangle)\}$.

Let $A = \langle T, T, \phi \rangle, \langle S, \{1\}, \phi \rangle$ here A is BISOS but not BIOS.

Theorem 4.5. A is binary intuitionistic semi closed set iff $\exists$ a binary intuitionistic closed set F s.t $B_{Int}F \subseteq C \subseteq F$.

Proof. Let C be a binary intuitionistic semi open then $B_{Int}(B_{Icl}(C)) \subseteq C$. Let $B_{Icl}(C) = B$. Then B is binary intuitionistic closed sets. Here $B_{Int}(B) \subseteq C \subseteq B$. Thus if C is a BISOS there exists BICS F such that $B_{Int}F \subseteq C \subseteq F$.

Conversely assume F to be BICS s.t $B_{Int}(F \subseteq C \subseteq F)$. WKT $C \subseteq F \Rightarrow B_{Icl}(C) \subseteq B_{Icl}(F)$ which implies $B_{Int}(B_{Icl}(C)) \subseteq B_{Int}(B_{Icl}(F)) = B_{Int}(F)$ (Since F is binary intuitionistic closed set). $B_{Icl}(F) = F \subseteq C$ implies C is BISOS. $\square$

Theorem 4.6. For any BIS C in (S, T, B_I) , we have

$$(a) B_{Isc}(C^c) = B_{Isint}(C)^c$$

$$(b) B_{Isc}(C^c) = B_{Isc}(C)^c.$$

Proof. (i) $B_{Isc}(A^c) = B_{Isc}(((S, T) - A)^c) = B_{Isc}((S, T) - A)$
 $= \cup \{K / K \text{ is a BISOS in } (S, T) \text{ and } (S, T) - C \subset K\}$. Therefore $(S, T) - B_{Isc}((S, T) - C)$
 $= \{(S, T) - K / (S, T) - K \text{ is a BISOS in } (S, T) \text{ and } (S, T) - K \subset C\}$
 $= \cup \{G / G \text{ is a BISOS in } (S, T) \text{ and } G \subset C\}$
 $= B_{Isint}(C)$. Therefore $B_{Isc}(C^c) = (B_{Isint}(C))^c$.
 (ii) $B_{Isint}(C^c) = B_{Isint}((S, T) - C) = \cup \{G / G \text{ is a BISOS in } (S, T) \text{ and } G \subset (S, T) - C\}$, $(S, T) - B_{Isint}((S, T) - C) = \cap \{(S, T) - G / (S, T) - G \text{ is a BISOS in } (S, T) \text{ and } C \subset (S, T) - G\}$
 $= \cap \{B / B \text{ is a BISOS in } (S, T) \text{ and } C \subset B\}$. Hence $(S, T) - B_{Isint}((S, T) - C) = B_{Isc}(C)$.
 Hence $(S, T) - B_{Isc}(C) = B_{Isc}(C) = B_{Isint}((S, T) - C)$. Hence $B_{Isint}(C^c) = (B_{Isc}(C))^c$. $\square$

Theorem 4.7. Let C be a subset of (T, S) . Then below are equal:

(a) C is an BISOS.

(b) $C \subset B_{Icl}(B_{Int}(C))$.

(c) $B_{Icl}(C) = B_{Icl}(B_{Int}(C))$.

Proof. (a) $\Rightarrow$ (b). Let C is a BISOS. Then $\exists$ an BIOS G s.t $G \subset C \subset B_{Icl}(G)$. $\therefore G$ is a BIOS, $G = B_{Int}(G)$ and so $C \subset B_{Icl}(B_{Int}(G))$. $\therefore G \subset C$, then $C \subset B_{Icl}(B_{Int}(C))$.

(b) $\Rightarrow$ (c). $B_{Icl}(B_{Int}(C)) \subset B_{Icl}(C)$.

Hence $B_{Icl}(C) \subset B_{Icl}(B_{Icl}(B_{Int}(C)))$

$= B_{Icl}(B_{Int}(C))$. Hence $B_{Icl}(C) = B_{Icl}(B_{Int}(C))$.

(c) $\Rightarrow$ (a) $\therefore B_{Int}(C) \subset C \subset B_{Icl}(C) = B_{Icl}(B_{Int}(C))$, C is a BISOS. $\square$

5 Properties of binary intuitionistic pre- open sets

In this section some of the binary intuitionistic pre open sets (B_IPOS) properties are illustrated.

Theorem 5.1. Let (T, S, B_I) be a BITS and $\{A_\alpha : \alpha \in K\}$ be a family of BIPOS in (T, S, B_I) . Then $\cup\{A_\alpha : \alpha \in K\}$ is a BIPOS.

Proof. Suppose $\{A_\alpha : \alpha \in K\}$ be family of BIPOS in (T, S, B_I) . Then for every α , $A_\alpha \subset B_{Int}(B_{Icl}(A_\alpha))$. Since $A_\alpha \subset \cup A_\alpha$, $B_{Int}(B_{Icl}(A_\alpha)) \subset B_{Int}(B_{Icl}(\cup A_\alpha))$ and also $A_\alpha \subset B_{Int}(B_{Icl}(\cup A_\alpha))$. Hence $\cup A_\alpha \subset B_{Int}(B_{Icl}(\cup A_\alpha))$ which implies that $\cup A_\alpha$ is a BIPOS. $\square$

Remark 5.1. Intersection of two BIPOS need not be BIPOS.

Example 5.2. Suppose $T = \{a\}$ and $S = \{1, 2\}$ and let (T, S, B_I) be any binary space and $B_I = \{(\underline{T}, \underline{S}), (\phi, \phi),$

$(\langle T, \phi, \{a\} \rangle, \langle S, \phi, \{2\} \rangle), (\langle T, \phi, \phi \rangle, \langle S, \phi, \phi \rangle)\}$

Here the binary intuitionistic pre open sets are $\{(\underline{T}, \underline{S}), (\phi, \phi),$
 $(\langle T, \phi, \phi \rangle, \langle S, \{1\}, \{2\} \rangle), (\langle T, T, \phi \rangle, \langle S, \{1\}, \phi \rangle), (\langle T, T, \phi \rangle, \langle S, \{1\}, \{2\} \rangle),$
 $(\langle T, \{ \phi \}, T \rangle, \langle S, S, \{ \phi \} \rangle), (\langle T, \phi, T \rangle, \langle S, \phi, \phi \rangle), (\langle T, \phi, T \rangle, \langle S, \phi, \{1\} \rangle),$
 $(\langle T, \phi, T \rangle, \langle S, \phi, \{2\} \rangle), (\langle T, \phi, T \rangle, \langle S, \{1\}, \phi \rangle), (\langle T, \phi, T \rangle, \langle S, \{1\}, \{2\} \rangle),$
 $(\langle T, \phi, \phi \rangle, \langle S, S, \phi \rangle), (\langle T, \phi, \phi \rangle, \langle S, \phi, S \rangle), (\langle T, \phi, \phi \rangle, \langle S, \phi, \phi \rangle),$
 $(\langle T, \phi, \phi \rangle, \langle S, \phi, \{1\} \rangle), (\langle T, \phi, \phi \rangle, \langle S, \phi, \{2\} \rangle)\}$.

Intersection of $(\langle T, T, \phi \rangle, \langle S, \{1\}, \phi \rangle)$ and $(\langle T, \phi, \phi \rangle, \langle S, \{1\}, \phi \rangle)$ is $(\langle T, \phi, \phi \rangle, \langle S, \{1\}, \phi \rangle)$ which is not a BIPOS.

Theorem 5.3. Let (T, S, B_I) be a BITS. If A and B are any two BIOS in (T, S, B_I) and one of which is a BIOS, then $A \cap B$ is a BIPOS.

Proof. Let A and B be two BIPOS in (T, S, B_I) . Suppose A is a BIPOS and B is a BIOS. Then $A \subseteq B_{Int}(B_{Icl}(A))$. Now $A \cap B \subset B_I - int(B_{Icl}(A) \cap B) = B_{Int}(B_{Icl}(A)) \cap B_{Int}(B) = B_{Int}(B_{Icl}(A) \cap B) \subset B_{Int}(B_{Icl}[A \cap B])$. Hence $A \cap B$ is a BIPOS. $\square$

Theorem 5.4. Suppose (T, S, B_I) be a BITS. A be a BIPOS & B be a BIS s.t $B \subseteq A \subseteq B_{Icl}(B)$. Then B is also a BIPOS.

Proof. $A \subseteq B_{Icl}(B)$ implies that $B_{Icl}(A) \subseteq B_{Icl}(B)$ and so $A \subseteq B_{Int}(B_{Icl}(A)) \subseteq B_{Int}(B_{Icl}(B))$ and so $B \subseteq B_{Int}(B_{Icl}(B))$. Therefore B is also a BIPOS. $\square$

Theorem 5.5. Let (T, S, B_I) be a BITS and A is a BIS. Then

(a) $B_{Ipcl}(A^c) = B_{I pint}(A)$.

(b) $B_{I pint}(A^c) = (B_{Ipcl}(A))^c$.

Proof. (a) $B_{Ipcl}(A^c) = ((T, S) - A) \cup B_{Icl}(B_{Int}((T, S) - A)) = ((T, S) - A) \cup (A - B_{Int}(B_{Icl}(A))) = (B_{I pint}(A))^c$

(b) $B_{I pint}(A^c) = ((T, S) - A) \cup B_{Int}(B_{Icl}((T, S) - A)) = ((T, S) - A) \cup (A - B_{Int}(B_{Icl}(A))) = (B_{Ipcl}(A))^c$. $\square$

Theorem 5.6. Let (T, S, B_I) be a BITS and let A be a BIS. Below are equivalent:

(a) A is a BIPCS.

- (b) $B_{Int}(B_{Icl}(A)) \subseteq A$.
 (c) $B_{Icl}(B_{Int}(A^c)) \supseteq A^c$.
 (d) A^c is a BIPOS.

Proof. (a) $\Rightarrow$ (b) If A is a BIPOS, then $(T, S) - A$ is a BIPOS and so $(T, S) - A \subset B_{Int}(B_{Icl}((T, S) - A))$. Hence $(T, S) - B_{Int}(B_{Icl}((T, S) - A)) \subset A \Rightarrow$ that $B_{Icl}(B_{Int}((T, S) - A)) \subset A$.
 (b) $\Rightarrow$ (c) WRT (b) $B_{Int}[B_{Icl}(A)] \subseteq A$ and so $(T, S) - A \subset (T, S) - B_{Icl}(B_{Int}(A)) \Rightarrow B_{Icl}(B_{Int}([T, S] - A)) \supseteq A$.
 (c) $\Rightarrow$ (d) By definition A, A^c is a BIPOS.
 (d) $\Rightarrow$ (a) using definition, A is a BIPCS. □

Theorem 5.7. Let (X, Y, B_I) be a BITS and let A and B be BIS. Then the following are true.

- (a) $B_{Ipcl}(\phi) = \phi$ and $B_{Ipcl}(X, Y) = (X, Y)$.
 (b) A is a BIPOS iff $A = B_{Ipcl}(A)$.
 (c) $B_{Ipcl}(B_{Ipcl}(A)) = B_{Ipcl}(A)$.
 (d) $A \subseteq B \Rightarrow B_{Ipcl}(A) \subseteq B_{Ipcl}(B)$.
 (e) $B_{Ipcl}[A \cap B] \subseteq B_{Ipcl}(A) \cap B_{Ipcl}(B)$.
 (f) $B_{Ipcl}[A \cup B] = B_{Ipcl}(A) \cup B_{Ipcl}(B)$.

Proof. (a) Let A be a BIOS. Then $A = B_{Int}(A)$. Now $B_{Icl}(A) = B_{Icl}(B_{Int}(A)) \subset B_{Icl}(B_{Int}(B_{Icl}(A)))$. hence $B_{Icl}(A) \subset B_{Icl}(B_{Int}(B_{Icl}(A)))$.

Again $B_{Icl}(B_{Int}(B_{Icl}(A))) \subset B_{Icl}(A)$.

Thus $B_{Icl}(B_{Int}(B_{Icl}(A))) = B_{Icl}(A)$.

(b) If K is BICS then $(X, Y) - K$ is BIOS and so $B_{Icl}((X, Y) - K)$ is BIRCS by (a). Hence $(X, Y) - (B_{Icl}((X, Y) - K))$ then $B_{Int}(K)$ is a BIOS.

(c) Using (b) and since $B_{Icl}(A)$ is a BICS, then $B_{Int}(B_{Icl}(A))$ is BIOS.

(d) Using (a) and since $B_{Int}(A)$ is a BIOS, then $B_{Icl}(B_{Int}(A))$ is BIOS.

Using the definitions of B_{Int} and B_{Icl} (e) and (f) may be proved. □

6 Properties of binary intuitionistic α -open sets

In this section some of the binary intuitionistic α open sets (B_IPOS) properties are illustrated.

Theorem 6.1. Let (T, S, B_I) be a BITS and $\{A_\alpha : \alpha \in K\}$ be a family of $B_{I\alpha}OS$ in (T, S, B_I) . Then $\cup\{A_\alpha : \alpha \in K\}$ is a $B_{I\alpha}OS$.

Proof. Suppose $\{A_\alpha : \alpha \in K\}$ be a family of binary intuitionistic α open set in (T, S, B_I) . Then for each A_α , $A_\alpha \subset B_{Int}(B_{Icl}(B_{Int}(A_\alpha)))$. Since $A_\alpha \subset \cup A_\alpha$, for each α , $A_\alpha \subset B_{Int}(B_{Icl}(B_{Int}(A_\alpha))) \subset B_{Int}(B_{Icl}(B_{Int}(\cup A_\alpha)))$ and so $\cup A_\alpha \subset B_{Int}(B_{Icl}(B_{Int}(\cup A_\alpha)))$. □

Theorem 6.2. Suppose A be a subset of (T, S, B_I) . Then A is $B_{I\alpha}OS$ iff $\exists$ a BIOS such that $B \subseteq A \subseteq B_{Int}(B_{Icl}(B))$.

Proof. Suppose that $\exists$ a BIOS B such that $B \subseteq A \subseteq B_{Int}(B_{Cl}(B))$. Since $A \subseteq B_{Int}(B_{Cl}(B)) = B_{Int}(B_{Cl}(B_{Int}(B))) \subseteq B_{Int}(B_{Cl}(B_{Int}(A)))$, by the definition and by hypothesis, A is $Bl\alpha OS$. On the other hand, let A be a $Bl\alpha OS$. Then $A \subseteq B_{Int}(B_{Cl}(B_{Int}(A)))$. Let $B_{Int}(A) = B$. Since $B_{Int}(A) \subseteq A$, we have $B \subseteq A$ and also $A \subseteq B_{Int}(B_{Cl}(B))$. Hence $\exists$ a BIOS such that $B \subseteq A \subseteq B_{Int}(B_{Cl}(B))$. $\square$

Theorem 6.3. *Let A be a subset of a BITS (T, S, B_I) . Then A is a $Bl\alpha OS$ iff $B_{Cl}(B_{Int}(B_{Cl}(A))) \subseteq A$.*

Proof. Let A be a subset of a BITS (T, S, B_I) . So A^c is a $Bl\alpha OS$. Therefore by definition, $A^c \subseteq B_{Int}(B_{Cl}(B_{Int}(A^c)))$
 $= B_{Int}(B_{Cl}(B_{Int}(A^c))) = B_{Int}[B_{Int}(B_{Cl}(A))]^c$
 $= [B_{Cl}(B_{Int}(B_{Cl}(A)))]^c$ which gives $B_{Cl}(B_{Int}(B_{Cl}(A))) \subseteq A$. By the definitions and by hypothesis converse part is true. $\square$

Theorem 6.4. *Let A be a subset of a BITS (T, S, B_I) . Then A is $Bl\alpha OS$ iff $\exists$ BICS B s.t $B_{Int}(B_{Cl}(B)) \subseteq A \subseteq B$.*

Proof. Let A be a $Bl\alpha OS$. Then $B_{Cl}(B_{Int}(B_{Cl}(A))) \subseteq A$. Let $B_{Cl}(A) = B$. Then B is BICS. Since $A \subseteq B_{Cl}(A)$, $A \subseteq B$ and $B_{Cl}(B_{Int}(B_{Cl}(A))) \subseteq A \Rightarrow B_{Cl}(B_{Int}(B)) \subseteq A$. Thus there exists BICS B s.t $B_{Cl}(B_{Int}(B)) \subseteq A \subseteq B$. Conversely assume that there exists BICS B s.t $B_{Cl}(B_{Int}(B)) \subseteq A \subseteq B$.

Since B is BICS, then $B_{Cl}(B) = B$. By hypothesis, $B_{Cl}(B_{Int}(B)) \subseteq A \Rightarrow B_{Cl}(B_{Int}(B_{Cl}(B))) \subseteq A$.

Since $A \subseteq B \Rightarrow B_{Cl}(A) \subseteq B_{Cl}(B)$

$\Rightarrow B_{Cl}(B_{Int}(B_{Cl}(A))) \subseteq B_{Cl}(B_{Int}(B_{Cl}(B))) \subseteq A \Rightarrow A$ $Bl\alpha OS$. $\square$

Theorem 6.5. *Let (T, S, B_I) be a BITS and let D and C be BITS in (T, S) . Then*

(a) D is a $Bl\alpha OS$ in (T, S, B_I) iff $D = B_{I\alpha cl}(D)$.

(b) D is a $Bl\alpha CS$ in (T, S, B_I) iff $D = B_{I\alpha int}(D)$.

(c) $B_{I\alpha cl}(\phi) = \phi$ and $B_{I\alpha cl}((T, S)) = (T, S)$.

(d) $B_{I\alpha int}(\phi) = \phi$ and $B_{I\alpha int}((T, S)) = (T, S)$.

(e) $B_{I\alpha int}(B_{I\alpha int}(D)) = B_{I\alpha int}(D^c)$.

(f) $(B_{I\alpha cl}(D))^c = B_{I\alpha int}(D^c)$.

(g) $B_{I\alpha int}(D)^c = B_{I\alpha cl}(D^c)$.

(h) $D \subseteq C \Rightarrow B_{I\alpha int}(D) \subseteq B_{I\alpha int}(C)$.

(i) $D \subseteq C \Rightarrow B_{I\alpha cl}(D) \subseteq B_{I\alpha cl}(C)$.

(j) $B_{I\alpha cl}(D \cup C) = B_{I\alpha cl}(D) \cup B_{I\alpha cl}(C)$.

(k) $B_{I\alpha int}(D \cap C) = B_{I\alpha int}(D) \cap B_{I\alpha int}(C)$.

(l) $B_{I\alpha cl}(D \cap C) = B_{I\alpha cl}(D) \cap B_{I\alpha cl}(C)$.

(m) $B_{I\alpha int}(D \cup C) = B_{I\alpha int}(D) \cup B_{I\alpha int}(C)$.

Proof. By the definitions of $Bl\alpha OS$, closure and interior definitions and by using the usual operators the theorem is proved. $\square$

Conclusion

Further applications of the binary intuitionistic open sets, introduced in this article on fuzzy space, fine space, which would bring more applications in the areas of digital lines, computer networking, data analysis, decision making problems, medicine etc., and also this article serves as the base for future applications of *BIOS* in binary intuitionistic fine topology and binary intuitionistic fuzzy topology.

Reference

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