

Modified Techniques for Solutions of Generalized Fractional Burgers Model

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ABSTRACT

We implement the fractional Hirota bilinear technique to compute analytical solution for hyperbolic generalized space–time fractional Burgers model. We construct double soliton wave for the desired fractional differential model under study. These calculations are performed by the symbolic computation such as maple, which has been used more recently by researchers which proves that the Hirota bilinear method is a promising and straightforward procedure for treatment nonlinear differential models of integer and fractional orders. We checked whether the results demonstrate that the process is easy to calculate, effective, and direct to address a wide range of engineering and physics models. The arbitrary and random selection of the fractional orders enables us and allow to build richer structures. The changes in the soliton based on the changes of fractional order allows more applications in applied sciences. The dynamical behavior of the obtained solutions have been depicted and underlined in 3D and 2D graphical drawings with different chooses of the fractional order.

Keywords: Nonlinear fractional models, conformable fractional derivative, Burgers equation

Mathematics Subject Classification: 35R01, 35R15, 35R29

Journal of Economic Literature (JEL) Classification : C00, C02

1. INTRODUCTION

Nonlinear fractional dynamical systems can describe a wide range of scientific problems. Many researchers have conducted many studies on these systems to understand the features of these problems in real applications see Iqbal et al 2022, Hashemi et al 2016, Chu et al 2021, He et al 2022. Nonlinear and linear fractional models show essential roles in applied science. In the last two decades, several scientists and researchers have been interested in seeking a numerical and

analytical solution to fractional differential equations (FDEs). Examples include, Mathanaranjan 2022, Park 2022, Arnous and Mirzazadeh 2016, Yusuf et al 2022, Atangana et al 2015, Samko et al 1993, Lei et al 2021, Xiu and Shou 2022, Zhou et al 2016, Akbulut et al 2023, Hosseini et al 2022, Mathanaranjan 2023, Mia 2023, Siddique 2021, Iqbal 2023, Mathanaranjan 2022, Az-Zobi 2022, Abdel-Salam and Yousif 2013, Abdel-Salam and Hassan 2016, Abdel-Salam et al 2016, Yousif 2018, Abdel-Salam and Mourad 2018, Nouh and Abdel-Salam 2018, Abdel-Salam and Nouh 2020.

Interestingly, and also naturally, the soliton wave exhibit waves that have permanent shape. Since soliton is defined from localized wave, that is the solution behavior either decomposes exponentially to zero like solitons of KdV model, or it approaches to a constant at infinity like solitons of sine-Gordon model. In this manuscript we are going to discuss the double soliton and solitary wave, we will point out the interaction of these waves.

Fractional and non-integer calculus will become the calculus of the recent century and a generalization of previous mathematics results. The developments and applications of fractional calculus have become an exciting and distinctive research topics. Different real-life models are accurately precisely represented on the basis of fractional differential models. Multiple definitions of fractional derivatives have been discovered and founded. For example the Riemann–Liouville, Chen’s fractal defenations, Kolwankar–Gangal, Cresson’s, Caputo and modified Riemann–Liouville. Recently, the conformable fractional derivative (CFD) have been presented based on the limits as

$$D^{\alpha} M(s) = \lim_{\varepsilon \rightarrow 0} \frac{M(s + \varepsilon s^{1-\alpha}) - M(s)}{\varepsilon} \quad \forall s > 0, \quad \alpha \in (0, 1], \quad (1)$$

$$M^{(\alpha)}(0) = \lim_{s \rightarrow 0^+} M^{(\alpha)}(s). \quad (2)$$

Putting $\alpha = 1$ in the last equations, the noninteger differential become the well-known integer differential. The CFD satisfied the following axioms:

$$D^{\alpha} s^n = n s^{n-\alpha}, \quad n \in \mathbb{R}, \quad D^{\alpha} a = 0, \quad \forall M(s) = a, \quad (3)$$

$$D^{\alpha} (aM + bN) = aD^{\alpha} M + bD^{\alpha} N, \quad \forall a, b \in \mathbb{R}, \quad (4)$$

$$D^{\alpha} (MN) = MD^{\alpha} N + ND^{\alpha} M, \quad (5)$$

$$D^{\alpha} \left(\frac{M}{N} \right) = \frac{MD^{\alpha} N - ND^{\alpha} M}{N^2}, \quad (6)$$

$$D^{\alpha} M(N) = \frac{dM}{dN} D^{\alpha} N, \quad D^{\alpha} M(s) = s^{1-\alpha} \frac{dM}{ds}, \quad (7)$$

where M, N are two α -differentiable functions of a dependent variable s and a is an random constant. The CFD of some functions

$$\begin{aligned} D_s^\alpha e^{cs} &= c s^{1-\alpha} e^{cs}, \quad D_s^\alpha \sin(cs) = c s^{1-\alpha} \cos(cs), \quad D_s^\alpha \cos(cs) = -c s^{1-\alpha} \sin(cs), \\ D_s^\alpha e^{cs^\alpha} &= c \alpha e^{cs^\alpha}, \quad D_s^\alpha \sin(cs^\alpha) = c \alpha \cos(cs^\alpha), \quad D_s^\alpha \cos(cs^\alpha) = -c \alpha \sin(cs^\alpha). \end{aligned} \quad (8)$$

Currently the calculus of fractional order is the generalization of integer differentiation and integration or naturally calculus to non-integer order. Recently, nonlinear fractional models are become a hot point of research that has attracted many researchers such as physicists and mathematicians, as well as astronomers and engineers. The fractional models have a large number of applications in different branches of sciences examples include plasma, condensed matter, physics, biomathematics, chemistry, biology, communication, astronomy. The calculus of fractional order has many fundamental applications in engineering and physics, for example fractal wave propagations, particle physics, electrical systems, wave mechanics, etc.

The simple form of the fractional model that describes the combination of dissipative effects and nonlinear propagation is the fractional Burgers equation. There are many applications employing fractional Burgers models: hydrodynamics, fluids, wave propagation in a thermo elastic medium, acoustic transmission, plasma physics, traffic, magneto hydrodynamics, shock waves, supersonic, around airfoils, waves under the influence of diffusion, liquid dynamics, and information sciences.

In this manuscript we introduce the explicit characterization of fronts wave interacting for hyperbolic generalized fractional Burgers model (GFBM). This mathematical model can be considered as modeling system of fractional Navier-Stokes system with hyperbolic modification. In the past few years generalized fractional Burgers model (GBM) has been discussed extensively in research papers, most of which have been studied by numerical methods. In this current work, we are looking forward to find an explicit characterization of the double soliton solution for the desired model.

This research paper has been arranged as follows. Section two introduce an essential formulae of hyperbolic generalized fractional Burgers model, also we investigate its solutions based on Hirota's technique. In section 3, we investigate double soliton solution by using classical and modified Hirota procedure. Finally we conclude the manuscript and give some prospects and discussion.

2 Explanation of the model and method of solution

We introduce the following hyperbolic GFBM in the form

$$\tau D_t^{\alpha\alpha} u + D_t^\alpha u + u D_x^\alpha u - \kappa D_x^{\alpha\alpha} u = \sum_{n=0}^3 a_n u^n, \quad (9)$$

with $u = u(t, x)$ real function of time t and space x , since τ, κ are arbitrary positive constants, a_n are random constants and $D_t^{\alpha\alpha} = D_t^\alpha (D_t^\alpha)$ is the twice fractional differentiation regarding t . When $\alpha = 1$ equation (9) become the well-known hyperbolic GBM studied in [31] as

$$\tau u_{tt} + u_t + u u_x - \kappa u_{xx} = \sum_{n=0}^3 a_n u^n.$$

Since the polynomial on the right-hand side of equation (9) has three real roots, so we can rewrite it in this equivalent form,

$$\tau D_t^{\alpha\alpha} u - \kappa D_x^{\alpha\alpha} u + D_t^\alpha u + u D_x^\alpha u + B D_x^\alpha u = \lambda u(s - u)(q - u), \quad (10)$$

with s, q, B, λ are arbitrary constants. In the next, using equation (10) to express hyperbolic GFBM.

As we talked in the beginning, the primary and main objective of this manuscript is to construct explicit explanation of the one and double soliton, based on Hirota's technique. Based on the use of this method, we will obtain a complex and very difficult scheme of nonlinear algebraic system, which rather cannot be solved simply without any extra information of parameters in the model. The technique used to solve such systems of algebraic equations that appear when using Hirota's procedure gets easier if we just look for solutions, possession some known features. Therefore, we will consider the double soliton tends to the corresponding travelling wave when x go to $\pm\infty$. For this purpose we will start with the analytical explanation of travelling wave solutions, which play the main role of asymptotic regimes. To get it, we put

$$u = \frac{D_x^\alpha f}{f}, \quad (11)$$

we assume that

$$f = 1 + \varepsilon \varphi(t^\alpha, x^\alpha), \quad (12)$$

with $\varphi(t^\alpha, x^\alpha)$, is function will be calculated, ε is parameter. Therefore, by inserting equation (11) into (10), multiplying the results by ε^3 . We have a three order series of ε . Putting each coefficient of ε by zero we can get system of ordinary fractional differential in $\varphi(t^\alpha, x^\alpha)$, for example from the coefficient of ε we have

$$\tau D_t^{\alpha\alpha} (D_x^\alpha \varphi) + D_t^\alpha (D_x^\alpha \varphi) + B D_x^{\alpha\alpha} \varphi - \kappa D_x^{\alpha\alpha\alpha} \varphi + q s \lambda D_x^\alpha \varphi = 0, \quad (13)$$

assuming the solution of fractional differential equation (13) in the form

$$\varphi = e^{(ax^\alpha - vt^\alpha + c)/\alpha}, \quad (14)$$

where a, v, c are constants will be computed. Using equation (14) transformed the system of fractional differential equations to algebraic system as

$$aB - v - a^2\kappa + qs\lambda + v^2\tau = 0 \quad (15)$$

$$-v + a^2(1 + \kappa) + 2qs\lambda + a[B + (s - q)\lambda] - v^2\tau = 0, \quad (16)$$

$$\lambda(a + q)(a - s) = 0. \quad (17)$$

From equation (17), since $\lambda \neq 0$, so the parameter a is either equal to s or $-q$.

Firstly, the case when $a = s$, we have

$$\varphi_1 = e^{(sx^\alpha - v_1 t^\alpha + c_1)/\alpha}, \quad (18)$$

$$v_1 = \frac{s[s(\lambda + 1) + 2B + 2\lambda q]}{2}. \quad (19)$$

Using equations (19), (18), we have kink wave solution

$$u_1 = \frac{s e^{(sx^\alpha - v_1 t^\alpha + c_1)/\alpha}}{1 + e^{(sx^\alpha - v_1 t^\alpha + c_1)/\alpha}}, \quad v_1 = \frac{s[s(\lambda + 1) + 2B + 2\lambda q]}{2}, \quad (20)$$

with the condition $\kappa = \kappa_1 = \frac{\tau[\lambda(2q + s) + 2B + s]^2 - 2(\lambda + 1)}{4}$.

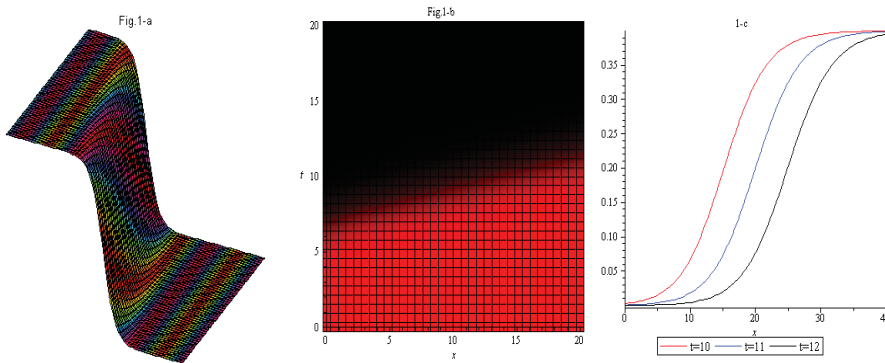


Fig. 1. 3-D plot, density plot, and 2-D plot of one kink soliton solution given by u_1 with the selection

$$\alpha = 0.9, s = 0.4, q = 0.5, B = 3, c_1 = 10; \lambda = 2.$$

secondly, the case when $a = -q$, we have

$$\varphi_2 = e^{(-qx^\alpha - v_2 t^\alpha + c_2)/\alpha}, \quad (21)$$

$$v_2 = \frac{-q[2B - q(\lambda + 1) - 2\lambda s]}{2}. \quad (22)$$

Using equations (21), (22), we have kink wave solution

$$u_2 = \frac{-qe^{(-qx^\alpha - v_2 t^\alpha + c_2)/\alpha}}{1 + e^{(-qx^\alpha - v_2 t^\alpha + c_2)/\alpha}}, \quad v_2 = \frac{-q[2B - q(\lambda + 1) - 2\lambda s]}{2}, \quad (23)$$

with the condition $\kappa = \kappa_2 = \frac{\tau[\lambda(2q + s) + q - 2B]^2 - 2(1 + \lambda)}{4}$.

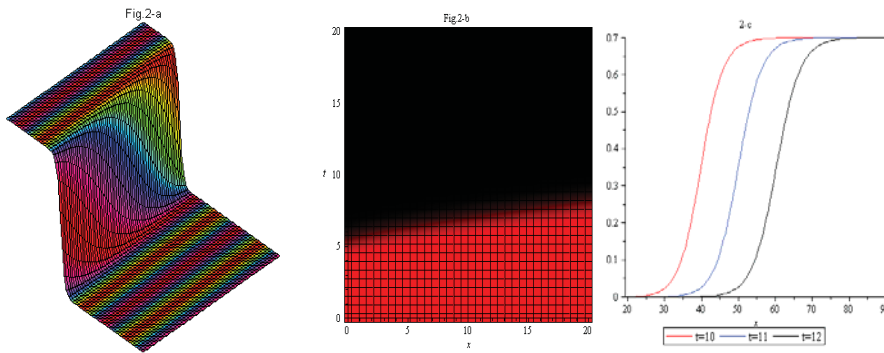


Fig. 2. 3-D plot, density plot, and 2-D plot of one kink soliton solution given by u_2 with the selection

$$\alpha = 0.8, s = -0.6, q = -0.7, B = 3, c_2 = 20; \lambda = 4.$$

The dynamical behavior of the one kink soliton solution are introduced here via 2D and 3D graphics in figures 1 and 2. The parameter selections for figure 1, we have $\alpha = 0.9, s = 0.4, q = 0.5, B = 3, c_1 = 10; \lambda = 2$. But for figure 2, we take $\alpha = 0.8, s = -0.6, q = -0.7, B = 3, c_2 = 20; \lambda = 4$.

3 Double soliton of the model

We search for the solution in the form

$$u(t^\alpha, x^\alpha) = f(\xi_1, \xi_2), \quad \xi_1 = (sx^\alpha - v_1 t^\alpha + c_1)/\alpha, \quad \xi_2 = (-qx^\alpha - v_2 t^\alpha + c_2)/\alpha, \quad (24)$$

equation (26) represented the interactions of many travelling wave when the two travelling waves ξ_1 and ξ_2 do not have a proportional relationship.

Attempts to investigate double soliton wave based on Hirota's procedure, we are faced with the solution of extremely complex system of nonlinear algebraic relations. To make the system solvable, we must put some hypotheses on the determined parameters. Therefore, considering that

the solution we want to find is represented asymptotically fronts of one or two travelling wave which are depending on the kind of interaction, we select the parameters v_1 and v_2 according to formulas (19) and (22). When looking for a simple and easy way to investigate a double soliton wave, it is useful to assume the function f as a functions superposition matching to travelling wave in the form

$$f = 1 + \varepsilon(e^{\xi_1} + e^{\xi_2}) + R \varepsilon^2 e^{\xi_1 + \xi_2}, \quad (25)$$

Substituting (25) with equation (11) into hyperbolic GFBM (10), we obtain six order series in the parameter ε . Equating each coefficient of ε by zero we have six nonlinear algebraic equations (denoted by $E_1, E_2, \dots, E_6$) in the determined constants and the products of $e^{m\xi_1} \cdot e^{n\xi_1}$. In the next, we use the abbreviation $X^m = e^{m\xi_1}$ and $Y^n = e^{n\xi_1}$. Therefore, the system of nonlinear algebraic equations E_k expressed by the abbreviation X^m and Y^n in the form

$$\sum_{n=0}^k a_n^k X^n Y^{k-n} = 0, \quad k = 1, 2, \dots, 6, \quad (26)$$

with a_n^k containing the determined constants $\tau, B, \kappa, \lambda, s, q$ and R .

Since we will not take into account the case in which the waves ξ_1 and ξ_2 are proportional, therefore we treat the variables X and Y as independent and unrelated variables. Thus system (20) have been satisfied iff $a_n^k = 0$. Based on the symbolic software "Maple" or "Mathematica", we solve the algebraic system $a_n^k = 0$, $0 \leq n \leq k \leq 6$. Footnote that, we do not have into account solutions given by $s = 0$, $q = 0$, $q = -s$ and $\lambda = -1/3$.

Now we begin to explain the procedure which enabling us to obtain the double soliton based ansatz (11). Firstly, since $a_0^1 = a_1^1 = 0$, we have

$$\begin{aligned} & 2\lambda[2qs\tau - 1 + s^2\tau + 2B\tau(2q + s)] - 2 - 4\kappa \\ & + 4B^2\tau + 4Bs\tau + (2q + s)^2\lambda^2\tau = 0, \end{aligned} \quad (27)$$

$$\begin{aligned} & \lambda[2q^2\tau - 2 + 4qs\tau - 4B\tau(2s + q)] - 2 - 4\kappa \\ & + 4B^2\tau - 4Bq\tau + q^2\tau + (q + 2s)^2\lambda^2\tau = 0, \end{aligned} \quad (28)$$

By subtracting (28) from (27), we have

$$\tau[(s - q)(1 - \lambda) + 4B](1 + 3\lambda)(q + s) = 0. \quad (29)$$

According to what we agreed upon previously, we reject the cases when $q = -s$ and $\lambda = -1/3$, so equation (29) satisfied only when $\tau = 0$ or the null value of expression in the brackets. It is clear that the first choice cannot be neglected in some steps, so we choose it from the very beginning. Using this choice in expression (27), we get this formula

$$\kappa = -\frac{1+\lambda}{2}. \quad (30)$$

Secondly, we consider $a_3^6 = 0$, so

$$(s-q)q s R^3 = 0,$$

which gives either $R = 0$ or $s = q$. Obviously we are forced into the first selection, because when using the second selection, we get $R = 0$ in the calculations. Cases $R = 0$, $\tau = 0$ and $\kappa = -(1+\lambda)/2$ verifies that the system of algebraic equations $E_1, E_2, \dots, E_6$ take the zero value. The direct analysis, which can be considered somewhat lengthy analysis of some other possibilities investigate the algebraic system $a_n^k = 0$ shows that the introduced cases are being the general ones and many other non-trivial double soliton solutions, which can be obtained from solving the algebraic system in different techniques, that could be everything obtained from this one, while non-trivial solutions of this kind correspond to non-zero τ is not present.

This procedure we can convinced that it is impossible to get a double-soliton solution satisfying (10), using Hirota method. we have the following result.

$$\begin{aligned} u_{11} &= \frac{s e^{(s x^\alpha - v_1 t^\alpha + c_1)/\alpha} - q e^{(-q x^\alpha - v_2 t^\alpha + c_2)/\alpha}}{1 + e^{(s x^\alpha - v_1 t^\alpha + c_1)/\alpha} + e^{(-q x^\alpha - v_2 t^\alpha + c_2)/\alpha}}, \\ v_1 &= \frac{s[2B + s(\lambda + 1) + 2\lambda q]}{2}, \\ v_2 &= \frac{-q[2B - q(\lambda + 1) - 2\lambda s]}{2}, \end{aligned} \quad (31)$$

since the reminder of parameters are optional.

The dynamical behavior of the double-kink soliton solution of u_{11} is presented via 2D and 3D graphics in figures 3. The parameter selections are given by $\alpha = 0.9$, $s = 9$, $q = 2$, $B = 0.3$, $c_1 = 1$, $c_2 = 5$, $\lambda = 0.25$.

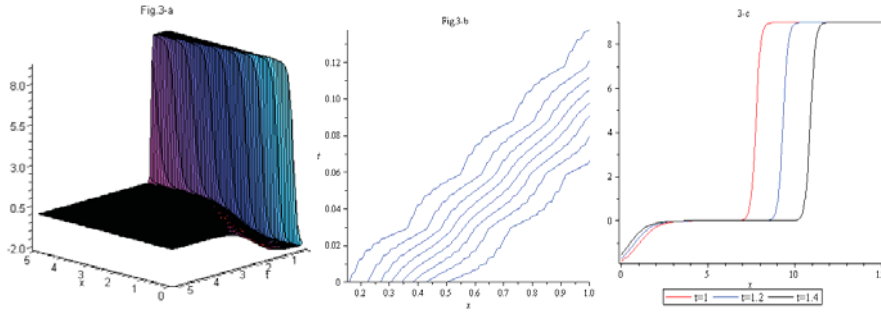


Fig. 3. 3-D plot, contour plot, and 2-D plot of double-soliton kink soliton given by u_{11} with the selection $\alpha = 0.9$, $s = 9$, $q = 2$, $B = 0.3$, $c_1 = 1$, $c_2 = 5$, $\lambda = 0.25$.

4 The technique realized by using Hirota's modified procedure

We can succeed in obtaining the double-soliton solutions of fractional model (10) by adding a simple modification to relation (11). Assuming that

$$u = \frac{g}{f}, \quad (32)$$

that is we have

$$f = 1 + \varepsilon(e^{\xi_1} + e^{\xi_2}) + R\varepsilon^2 e^{\xi_1 + \xi_2}, \quad (33)$$

$$g = \varepsilon(r_1 e^{\xi_1} + r_2 e^{\xi_2}) + A\varepsilon^2 e^{\xi_1 + \xi_2}, \quad (34)$$

it is clear from the hypothesis of the two functions f and g functions that we do not present a relationship between these two functions from the beginning which is in contrast to what followed in the procedure of Hirota's.

By making direct substitution using the relations (32)- (34) into hyperbolic GFBM (10), we obtain a series of degree six in ε . Equating each coefficient of ε by zero we have six nonlinear algebraic equations (denoted by $E_1, E_2, \dots, E_6$) in the determined constants and the products of $e^{m\xi_1} \cdot e^{n\xi_1}$. Using the same abbreviations above we can obtain fifteen nonlinear algebraic equations when $a_n^k = 0, 0 \leq n \leq k \leq 6$. Based on the symbolic software "Maple" or "Mathematica", we can solve this system $a_n^k = 0$. As we presented in the above section, we turn a blind eye to the cases of $q = 0, s = 0$, $q = -s$ and $\lambda = -1/3$. Moreover, we will not take into account such cases $r_2 = 0, R = 1$ and $A = r_1$, because those choices lead to solutions that describe single travelling waves. Obviously, we can notice the transformations

$$r_1 \rightarrow r_2, \quad r_2 \rightarrow r_1, \quad s \rightarrow -q, \quad q \rightarrow -s, \quad (35)$$

be an equivalence relationship in the group of double soliton, so we have one investigation for every pair.

At first, we select the status when $r_1 \neq 0, r_2 \neq 0$, from $E_1 = 0$ we have

$$B = \frac{(s-q)(\lambda-1)}{4}. \quad (36)$$

$$\kappa = \frac{\tau(1+3\lambda)^2(q+s)^2 - 8(1+\lambda)}{16}, \quad (37)$$

using (38) and (39) into $a_2^2 = a_0^2 = a_3^3 = a_0^3 = 0$, we obtain

$$\begin{aligned} r_1(s-r_1)[s+\lambda(s-q)] &= 0, \\ r_2(q+r_2)[q+\lambda(q-s)] &= 0, \\ r_1\lambda(s-r_1)(q+r_1) &= 0, \\ r_2\lambda(s-r_2)(q+r_2) &= 0. \end{aligned} \quad (38)$$

When solving the previous set of algebraic equations, we find that they are achieved if we give $r_1 = r_2 = s$ and

$$\lambda = \frac{q}{s-q}, \quad (39)$$

from $a_3^6 = 0$, we have

$$A(A+qR)(A-sR) = 0, \quad (40)$$

that is achieved for example when $A = -qR$, using this selection in $a_2^5 = 0$, we get

$$R^2(q+s)^3 = 0, \quad (41)$$

therefore, we must take $R = 0$ in this equation. The reminder equation $a_1^2 = 0$, so we have

$$s^2(q+s)^2[2(q-s)+\tau q(2q+s)^2] = 0, \quad (42)$$

that is gives

$$\tau = \frac{2(s-q)}{q(2q+s)^2}. \quad (43)$$

Thus, the hyperbolic GFBM (10) have the double soliton in the form

$$u_{22}(t^\alpha, x^\alpha) = \frac{s(e^{(sx^\alpha - v_1 t^\alpha + c_1)/\alpha} + e^{(-qx^\alpha - v_2 t^\alpha + c_2)/\alpha})}{1 + e^{(sx^\alpha - v_1 t^\alpha + c_1)/\alpha} + e^{(-qx^\alpha - v_2 t^\alpha + c_2)/\alpha}}, \quad (44)$$

$$v_1 = \frac{s(2q^2 + 3qs + s^2)}{4(s - q)}, \quad v_2 = \frac{q(2q^2 + 3qs + s^2)}{4(s - q)},$$

where $\tau = \frac{2(s - q)}{q(2q + s)^2}$, $\lambda = \frac{q}{s - q}$, $B = \frac{2q - s}{4}$ and $\kappa = \frac{s - q}{8q}$.

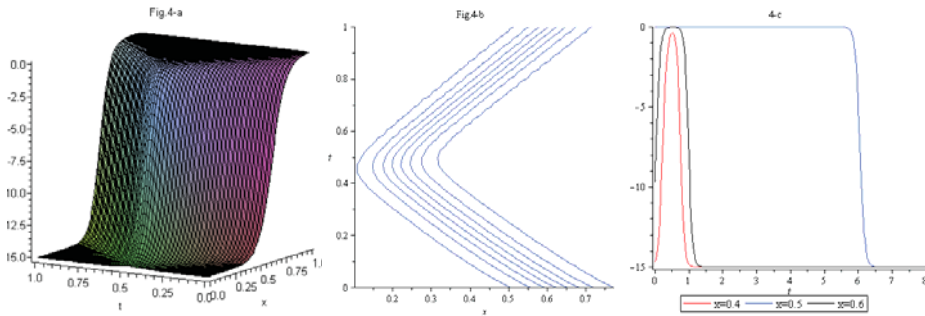


Fig. 4. 3-D plot, contour plot, and 2-D plot of double-soliton kink soliton given by u_{22} with the selection $\alpha = 0.9$, $s = -15$, $q = 20$, $c_1 = 10$, $c_2 = -5$.

Currently, we suggest that $r_2 = 0$ and $r_1 \neq 0$, through this hypothesis the constant κ continuous can still be obtained from the expression $a_1^1 = 0$. Also, we can get from the expressions $a_2^2 = a_3^3 = a_5^5 = 0$, that $r_1 = s$ and $A = Rs$. From $a_2^3 = 0$ we have

$$\tau s q (R - 1)(q + s)^2 (1 - 3\lambda)[4B + q(\lambda - 1) + 2s(\lambda + 1)] = 0, \quad (45)$$

when $R = 1$ we do not have the double soliton solution but we have one soliton solution. Therefore, we must have

$$B = \frac{q(1 - \lambda) - 2s(\lambda + 1)}{4}, \quad (46)$$

using (46), we can write $a_1^3 = 0$ as

$$s q (R - 1)[q + s + \lambda(q + 2s)] = 0, \quad (47)$$

which gives

$$\lambda = -\frac{q+s}{q+2s}, \quad (48)$$

The reminder two equations $a_1^2 = 0$ and $a_2^4 = 0$, having the same factor

$$2q+4s+\tau(q+s)(2q+s)^2,$$

that is compute the parameter τ as

$$\tau = -\frac{2(q+2s)}{(q+s)(2q+s)^2}, \quad (49)$$

Thus, the hyperbolic GFBM (10) have the double soliton in the form

$$u = \frac{s e^{(s x^\alpha - v_1 t^\alpha + c_1)/\alpha} (1 + R e^{(-q x^\alpha - v_2 t^\alpha + c_2)/\alpha})}{1 + e^{(s x^\alpha - v_1 t^\alpha + c_1)/\alpha} + e^{(-q x^\alpha - v_2 t^\alpha + c_2)/\alpha} + R e^{(s x^\alpha - v_1 t^\alpha + c_1)/\alpha + (-q x^\alpha - v_2 t^\alpha + c_2)/\alpha}}, \quad (50)$$

$$v_1 = \frac{-s q (2q+s)}{4(q+2s)}, \quad v_2 = \frac{-q (2q+s)}{4},$$

with $B = \frac{2q-s}{4}, \lambda = -\frac{q+s}{q+2s}, \tau = -\frac{2(q+2s)}{(q+s)(2q+s)^2}, \kappa = -\frac{q+2s}{8(q+s)},$ and s, q, R, c_1, c_2 are

constants. According to the conditions, in application of the physical meaning of the constants τ, κ as well as the parameters s, q in above must investigate the following inequality $(q+2s)(q+s) < 0$. We should note that for the random selection of the parameter R involves that the solution (50) designates both regular and singular solutions when $R \geq 0$ and $R < 0$ respectively. When $\alpha = 1$ all the results in Vladimirov and Maczka 2007 are recovered.

5 conclusions and discussion

In this manuscript, We implement the fractional Hirota bilinear technique to compute analytical solution for hyperbolic generalized space–time fractional Burgers model. We construct one and double soliton wave for the desired fractional differential model under study. These calculations are performed by the symbolic computation such as maple, which has been used more recently by researchers which proves that the Hirota bilinear method is a promising and straightforward procedure for treatment nonlinear differential models of integer and fractional orders. We checked whether the results demonstrate that the process is easy to calculate, effective, and direct to address a wide range of engineering and physics models. The arbitrary and random selection of the fractional orders enables us and allow to build richer structures. The changes in the soliton based on the changes of fractional order allows more applications in applied sciences.

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