

Some Families of Estimators for Estimating the Population Variance under Random Non-Response

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ABSTRACT

The present paper focuses on the estimation of finite population variance in the presence of random non-response. In this paper, we have proposed some families of factor-type estimators of population variance in simple random sampling using the information on an auxiliary variable under random non-response. We have considered two different realistic conditions of random non-response, namely, (i) random non-response on both study and auxiliary variables and (ii) random non-response on study variable only. The expressions for the biases and mean square errors have been derived up to the first order of approximation. The properties of the optimum estimators have also been discussed. The theoretical results have been illustrated through some real data sets and a simulation study.

Keywords: Factor-type estimators, population variance, auxiliary information, random non-response, simulation analysis.

Mathematics Subject Classification: 62D05, 62D99

International Journal of Applied Mathematics and Statistics:

1. INTRODUCTION

It is well known fact that the presence of variation is a natural phenomenon in our day to day life. There are various fields such as agriculture, medical, industry and business where the study of estimating the population variance has great significance. For instance, a physician is very keen about the full understanding of variations in the human pulse rate, blood pressure and body temperature for adequate treatment. An agriculturist needs an adequate indulgent of the variations in climatic behaviours, especially from place to place for planning of planting his/her crop. A manufacturer needs adequate knowledge about the degree of variations in response of consumers to his/her product so that he/she could be able to reduce or increase prices, or improve the quality of the product. Das and Tripathi (1978) were first who developed the variance estimation technique using auxiliary information. It was further extended by Srivastava and Jhaji (1980), Isaki (1983), Upadhyaya and Singh (1983), Tripathi et al. (1988), Ahamed et al. (2003) and Kadilar and Cingi (2007).

The non-response is a common phenomenon in most of the sample surveys. It is speculated that the information would be collected from all the selected units in the sample, but practically, it is not realizable because of the presence of non-response. The presence of non-response reduces the prescribed sample on one hand and the efficiency of the estimator on the other hand. Sometimes, it is seen that the presence of non-response is stochastic in nature. Statisticians have advocated that the ignorance of the random non-response can also damage the actual results. Singh and Joarder (1998) suggested several estimators of finite population variance under two different situations of random non-response. Singh et al. (2012) have suggested some families of estimators for estimating the population variance under random non-response in survey sampling. Bandyopadhyay and Singh (2015) have proposed the estimators of ratio of population variances under two different realistic situations of complete response and random non-response.

In this paper, we have proposed some families of factor-type estimators for estimating the finite population variance using the information on an auxiliary variable under two different realistic situations of random non-response. The proposed families of estimators can generate a number of well known existing estimators and other useful estimators of population variance. The optimum estimators of the suggested families have been pioneered out. An empirical study along with the simulation analysis has also been carried out to strengthen the theoretical results.

2. SAMPLING STRATEGY AND PROPOSED FAMILIES

Suppose a population $P(U_1, U_2, \dots, U_N)$ consists of N units from which a random sample of size n is selected by the method of simple random sampling without replacement (SRSWOR) scheme. Let X_0 and X_1 be the study and auxiliary variables with respective population means $\bar{X}_0$ and $\bar{X}_1$. Let x_{0i} and x_{1i} ($i = 1, 2, \dots, N$) be the observations on the i^{th} unit in the population for the study variable and auxiliary variable respectively. Now, our aim is to estimate the population variance of X_0 i. e.

$S_0^2 = (N-1)^{-1} \sum_{i=1}^N (x_{0i} - \bar{X}_0)^2$ under the situation where population variance of the auxiliary variable X_1

i. e. $S_1^2 = (N-1)^{-1} \sum_{i=1}^N (x_{1i} - \bar{X}_1)^2$ is known. Isaki (1983) has considered the ratio estimator of population

variance S_0^2 as

$$s_{0t}^2 = s_0^2 \frac{S_1^2}{s_1^2} \quad (1)$$

where $s_0^2 = (n-1)^{-1} \sum_i^n (x_{0i} - \bar{x}_0)^2$ and $s_1^2 = (n-1)^{-1} \sum_i^n (x_{1i} - \bar{x}_1)^2$. $\bar{x}_0$ and $\bar{x}_1$ are the sample means

based on n units for X_0 and X_1 respectively. The expressions for bias and mean square error (MSE) of the estimator s_{0I}^2 are respectively given as

$$\text{Bias}(s_{0I}^2) = \left(\frac{1}{n} - \frac{1}{N} \right) S_0^2 [\lambda_{04} - \lambda_{22}] \quad (2)$$

$$\text{MSE}(s_{0I}^2) = \left(\frac{1}{n} - \frac{1}{N} \right) S_0^4 [\lambda_{40} + \lambda_{04} - 2\lambda_{22}] \quad (3)$$

where $\lambda_{ls} = \frac{\mu_{ls}}{\mu_{20}^{1/2} \mu_{02}^{s/2}}$ and $\mu_{ls} = (N-1)^{-1} \sum_{i=1}^N (x_{0i} - \bar{X}_0)^l (x_{1i} - \bar{X}_1)^s$.

Further, due to random non-response, let $r[r=0,1,2,\dots,(n-2)]$ be the number of sample units in a random sample of size n , on which information could not be obtained and hence the remaining $(n-r)$ units can be treated as the units drawn by SRSWOR scheme from the population of N units. In order to find the unbiased estimate of the population variance, it is assumed that r should be less than $(n-1)$ i. e. $0 \leq r \leq (n-2)$. Let p be the probability of a non-response among $(n-2)$ cases of non-response. Thus, the probability distribution of r is given by

$$P(r) = \frac{(n-r)}{(nq+2p)} \binom{n-2}{r} p^r q^{n-2-r} \quad (4)$$

where $q = (1-p)$, $r = 0,1,2,\dots,(n-2)$ and $\binom{n-2}{r}$ represents the total number of ways of r non-responses out of total possible non-responses. Let

$$s_0^{*2} = S_0^2(1+e_0), \quad s_1^{*2} = S_1^2(1+e_1), \quad s_1^2 = S_1^2(1+e_2)$$

where

$$s_0^{*2} = \frac{1}{n-r-1} \sum_i^{n-r} (x_{0i} - \bar{x}_0^*)^2, \quad s_1^{*2} = \frac{1}{n-r-1} \sum_i^{n-r} (x_{1i} - \bar{x}_1^*)^2, \quad \bar{x}_0^* = \frac{1}{n-r} \sum_i^{n-r} x_{0i} \text{ and } \bar{x}_1^* = \frac{1}{n-r} \sum_i^{n-r} x_{1i}.$$

Thus, we have

$$E(e_0) = E(e_1) = E(e_2) = 0, \quad E(e_0^2) = \left(\frac{1}{nq+2p} - \frac{1}{N} \right) (\lambda_{40} - 1), \quad E(e_1^2) = \left(\frac{1}{nq+2p} - \frac{1}{N} \right) (\lambda_{04} - 1),$$

$$E(e_2^2) = \left(\frac{1}{n} - \frac{1}{N} \right) (\lambda_{04} - 1), \quad E(e_0 e_1) = \left(\frac{1}{nq+2p} - \frac{1}{N} \right) (\lambda_{22} - 1), \quad E(e_0 e_2) = \left(\frac{1}{n} - \frac{1}{N} \right) (\lambda_{22} - 1)$$

$$\text{and } E(e_1 e_2) = \left(\frac{1}{n} - \frac{1}{N} \right) (\lambda_{04} - 1).$$

2.1 Random non-response on both study and auxiliary variables

In this section, we have considered the situation where the random non-response occurs on both study and auxiliary variables. Motivated by Singh and Shukla (1987), we now propose a family of factor-type estimators for estimating the population variance S_0^2 as

$$T_\alpha^* = s_0^{*2} \left[\frac{(A+C)S_1^2 + fBs_1^{*2}}{(A+fB)S_1^2 + Cs_1^{*2}} \right] \quad (5)$$

where $A = (\alpha-1)(\alpha-2)$, $B = (\alpha-1)(\alpha-4)$, $C = (\alpha-2)(\alpha-3)(\alpha-4)$ for $\alpha > 0$ and $f = \frac{n}{N}$.

The proposed family can generate a number of well known existing estimators and some other estimators of population variance under random non-response.

Case (i): If $\alpha = 1$ then we have

$$T_1^* = s_0^{*2} \left[\frac{S_1^2}{s_1^{*2}} \right]$$

which is ratio-type estimator under random non-response.

Case (ii): If $\alpha = 2$ then we have

$$T_2^* = s_0^{*2} \left[\frac{s_1^{*2}}{S_1^2} \right]$$

which is product-type estimator under random non-response.

Case (iii): If $\alpha = 3$ then we have

$$T_3^* = s_0^{*2} \left[\frac{S_1^2 - fs_1^{*2}}{(1-f)S_1^2} \right]$$

which is dual to ratio-type estimator under random non-response. Srivenkataramana (1980) has proposed dual to ratio estimator for estimating the population mean.

Case (iv): If $\alpha = 4$ then we have

$$T_4^* = s_0^{*2}$$

which is usual variance estimator under random non-response.

On simplifying the equation (5), we have

$$T_\alpha^* = s_0^{*2} \left[\frac{1 + \frac{fB}{A+fB+C} \left(\frac{s_1^{*2}}{S_1^2} - 1 \right)}{1 + \frac{C}{A+fB+C} \left(\frac{s_1^{*2}}{S_1^2} - 1 \right)} \right] \quad (6)$$

Expressing the equation (6) in terms of e_0 and e_1 , we get

$$T_\alpha^* = S_0^2 (1 + e_0) [1 + \phi_1(\alpha)e_1] [1 - \phi_2(\alpha)e_1 + \phi_2^2(\alpha)e_1^2 - \dots] \quad (7)$$

where $\phi_1(\alpha) = \frac{fB}{(A + fB + C)}$ and $\phi_2(\alpha) = \frac{C}{(A + fB + C)}$

From the equation (7), we get

$$T_{\alpha}^* - S_0^2 = S_0^2 [e_0 - \phi(\alpha)e_1 + \phi_1(\alpha)\phi_2(\alpha)e_1^2 - \phi(\alpha)e_0e_1 + \dots] \quad (8)$$

where $\phi(\alpha) = \phi_2(\alpha) - \phi_1(\alpha)$.

Taking expectation on both the sides of equation (8) by ignoring the terms having powers of e_0 and e_1 greater than two, we get the expression for bias of T_{α}^* up to the first order of approximation as

$$\text{Bias}(T_{\alpha}^*) = S_0^2 \left(\frac{1}{nq + 2p} - \frac{1}{N} \right) [(\lambda_{04} - 1)\phi_1(\alpha)\phi_2(\alpha) - (\lambda_{22} - 1)\phi(\alpha)] \quad (9)$$

Squaring both the sides of equation (8) and then taking expectation on neglecting the terms having powers of e_0 and e_1 higher than two, we get the expression for mean square error (MSE) of T_{α}^* as

$$\text{MSE}(T_{\alpha}^*) = S_0^4 \left(\frac{1}{nq + 2p} - \frac{1}{N} \right)^2 [(\lambda_{40} - 1) + \phi^2(\alpha)(\lambda_{04} - 1) - 2\phi(\alpha)(\lambda_{22} - 1)] \quad (10)$$

In order to get the minimum MSE of T_{α}^* , we differentiate $\text{MSE}(T_{\alpha}^*)$ with respect to α and equate the derivative to zero. Thus, we have

$$\phi(\alpha) = \frac{(\lambda_{22} - 1)}{(\lambda_{04} - 1)} = V \quad (11)$$

The above equation is the cubic equation in α and provides three real roots of α for which the MSE of T_{α}^* would attain its minimum. Thus, the expression for minimum MSE of T_{α}^* is given as

$$\text{MSE}(T_{\alpha}^*)_{\min} = S_0^4 \left(\frac{1}{nq + 2p} - \frac{1}{N} \right)^2 \left[(\lambda_{40} - 1) - \frac{(\lambda_{22} - 1)^2}{(\lambda_{04} - 1)} \right] \quad (12)$$

2.2 Random non-response on study variables only

We now propose a family of factor-type estimators for estimating the population variance S_0^2 under the situation in which study variable suffers from random non-response and auxiliary variable is free from random non-response as

$$T_{\alpha}^{**} = S_0^{*2} \left[\frac{(A + C)S_1^2 + fBS_1^2}{(A + fB)S_1^2 + CS_1^2} \right] \quad (13)$$

Remark: It is to be remembered that the proposed family in equation (13) reduces to ratio-type estimator, product-type estimator, dual to ratio-type estimator and usual variance estimator under random non-response for $\alpha = 1$, $\alpha = 2$, $\alpha = 3$ and $\alpha = 4$ respectively. Besides it, the proposed family can generate a number of other estimators of population variance under random non-response for the suitable choices of α .

Now, we express the equation (13) in terms of e_0 and e_2 as

$$T_{\alpha}^{**} = S_0^2 (1 + e_0) [1 + \phi_1(\alpha)e_2] [1 - \phi_2(\alpha)e_2 + \phi_2^2(\alpha)e_2^2 - \dots] \quad (14)$$

The equation (14) reduces to

$$T_{\alpha}^{**} - S_0^2 = S_0^2 [e_0 - \phi(\alpha)e_2 + \phi_1(\alpha)\phi_2(\alpha)e_2^2 - \phi(\alpha)e_0e_2 + \dots] \quad (15)$$

Taking expectation on both the sides of equation (15) by ignoring the terms having powers of e_0 and e_2 greater than two, we get the expression for bias of T_{α}^{**} up to the first order of approximation as

$$\text{Bias}(T_{\alpha}^{**}) = S_0^2 \left(\frac{1}{n} - \frac{1}{N} \right) [(\lambda_{04} - 1)\phi_1(\alpha)\phi_2(\alpha) - (\lambda_{22} - 1)\phi(\alpha)] \quad (16)$$

Squaring both the sides of equation (15) and then taking expectation on neglecting the terms having powers of e_0 and e_2 greater than two, we get the expression for MSE of T_{α}^{**} up to the first order of approximation as

$$\text{MSE}(T_{\alpha}^{**}) = S_0^4 \left[\left(\frac{1}{nq + 2p} - \frac{1}{N} \right) (\lambda_{40} - 1) + \left(\frac{1}{n} - \frac{1}{N} \right) \{ \phi^2(\alpha)(\lambda_{04} - 1) - 2\phi(\alpha)(\lambda_{22} - 1) \} \right] \quad (17)$$

To get the minimum MSE of T_{α}^{**} , we differentiate the $\text{MSE}(T_{\alpha}^{**})$ with respect to α and equate the derivative to zero. Thus, the optimum value of α is obtained by the expression

$$\phi(\alpha) = \frac{(\lambda_{22} - 1)}{(\lambda_{04} - 1)} = V$$

which is a cubic equation in α and provides three real roots of α for which we get the minimum MSE of T_{α}^{**} . Substituting the value of $\phi(\alpha)$ from the above expression into equation (17), the expression for minimum MSE of T_{α}^{**} is given by

$$\text{MSE}(T_{\alpha}^{**})_{\min} = S_0^4 \left[\left(\frac{1}{nq + 2p} - \frac{1}{N} \right) (\lambda_{40} - 1) - \left(\frac{1}{n} - \frac{1}{N} \right) \frac{(\lambda_{22} - 1)^2}{(\lambda_{04} - 1)} \right] \quad (18)$$

3. EMPIRICAL STUDY

In order to demonstrate the theoretical results obtained in this paper and to observe the behaviour of the proposed families, it is essential to illustrate whatever has been discussed in the previous sections with some numerical data. To execute the assignment, we have considered two different aspects of data analysis: (i) analysis of real data and (ii) simulation analysis.

3.1 Real data sets

Data set 1: We have used the data considered by Shukla and Thakur (2008). In this data set, a random sample of size 20 is selected from a population of size 200. The details are given below:

$N = 200$, $n = 20$, $\bar{X}_0 = 42.485$, $\bar{X}_1 = 18.515$, $S_0^2 = 199.0598$, $S_1^2 = 48.5375$, $\rho_{01} = 0.8652$, $\lambda_{22} = 2.47$, $\lambda_{04} = 3.74$, $\lambda_{40} = 2.56$

Table 1 shows the MSE and percentage relative efficiency (PRE) of the families T_{α}^* and T_{α}^{**} for $\alpha = 1, 2, 3, 4$ and α_{opt} at the different choices of non-response probability $p = 0.05(0.05)0.20$. PRE is computed with respect to T_4^* or T_4^{**} (It is remembered that $T_4^* = T_4^{**} = s_0^{*2}$).

Table 1: MSE and PRE of the families T_{α}^* and T_{α}^{**}

p	α	T_{α}^*		T_{α}^{**}	
		MSE	PRE	MSE	PRE
0.05	1	2552.00	114.71	2570.67	113.87
	2	13585.66	21.55	13055.40	22.42
	3	2377.79	123.11	2405.13	121.71
	4	2927.30	100.00	2927.30	100.00
	α_{opt}	1447.42	202.24	1521.04	192.45
0.10	1	2691.53	114.71	2730.72	113.06
	2	14328.42	21.55	13215.44	23.36
	3	2507.79	123.11	2565.17	120.36
	4	3087.34	100.00	3087.34	100.00
	α_{opt}	1526.55	202.24	1681.08	183.65
0.15	1	2845.56	114.71	2907.41	112.27
	2	15148.45	21.55	13392.13	24.37
	3	2651.31	123.11	2741.86	119.04
	4	3264.03	100.00	3264.03	100.00
	α_{opt}	1613.92	202.24	1857.78	175.70
0.20	1	3016.51	114.71	3103.49	111.49
	2	16058.48	21.55	13588.22	25.46
	3	2810.59	123.11	2937.95	117.77
	4	3460.12	100.00	3460.12	100.00
	α_{opt}	1710.87	202.24	2053.86	168.47

Data set 2: In this data set, we have used the data considered by Sukhatme (1970). The details are given below:

$N = 34$, $n = 12$, $\bar{X}_0 = 201.4118$, $\bar{X}_1 = 218.4118$, $S_0^2 = 23154.85561$, $S_1^2 = 28123.21925$,
 $\rho_{01} = 0.6203457435$, $\lambda_{22} = 2.433161$, $\lambda_{04} = 3.18507$, $\lambda_{40} = 3.4426$

Table 2 also shows the MSE and PRE of the families T_α^* and T_α^{**} for $\alpha = 1, 2, 3, 4$ and α_{opt} at $p = 0.05(0.05)0.20$.

Table 2: MSE and PRE of families T_α^* and T_α^{**}

p	α	T_α^*		T_α^{**}	
		MSE	PRE	MSE	PRE
0.05	1	54342762.65	138.68	55666136.58	135.38
	2	231209066.50	32.59	221396413.90	34.04
	3	47182103.71	159.72	48956333.95	153.94
	4	75361054.70	100.00	75361054.70	100.00
	α_{opt}	46359947.35	162.56	48185942.93	156.40
0.10	1	58075396.85	138.68	60842451.43	132.37
	2	247090093.20	32.59	226572728.70	35.55
	3	50422894.66	159.72	54132648.82	148.78
	4	80537369.56	100.00	80537369.56	100.00
	α_{opt}	49544266.97	162.56	53362257.83	150.93
0.15	1	62163520.01	138.68	66511748.67	129.61
	2	264483598.70	32.59	232242025.90	37.12
	3	53972332.38	159.72	59801946.06	144.15
	4	86206666.79	100.00	86206666.80	100.00
	α_{opt}	53031855.05	162.56	59031555.03	146.03
0.20	1	66660455.51	138.68	72747975.62	127.07
	2	283616454.80	32.59	238478252.90	38.76
	3	57876713.88	159.72	66038173.00	139.98
	4	92442893.76	100.00	92442893.74	100.00
	α_{opt}	58682020.30	157.53	65267782.03	141.64

3.2 Simulation analysis

To carry out the simulation study, we first consider two random variables X_1 and X_2 of a normal population i. e. $X_1 \sim N(5,2)$ and $X_2 \sim N(5,2)$. Secondly, we define the study variable X_0 using the

transformation $X_0 = \rho X_1 + \sqrt{1-\rho^2} X_2$. We consider 2000 values for each of the variables X_0 and X_1 by generating the numbers of X_1 and X_2 using R software. Now, we select a random sample of size n from the population of 2000 units and compute estimate of MSE of the estimators. The procedure of selecting the random sample and computing the estimate of MSE of the estimators is repeated 500 times. Finally, we compute the average MSE on taking the average of the estimated MSEs. The analysis has been done for considering the different choices of sample size n and correlation coefficient ρ . Table 3 represents the average MSE of the families T_α^* and T_α^{**} for $\rho = 0.6, 0.7, 0.8$ and $n = 100, 150, 200$ at $p = 0.20$.

Table 3: Average MSE of families T_α^* and T_α^{**}

ρ	α	T_α^*			T_α^{**}		
		n			n		
		100	150	200	100	150	200
0.6	1	0.093573	0.083117	0.076139	0.095571	0.084324	0.076064
	2	0.322158	0.272354	0.232027	0.270768	0.224374	0.186507
	3	0.080162	0.063735	0.058674	0.085292	0.06998	0.06369
	4	0.102126	0.08776	0.07788	0.102126	0.08776	0.07788
	α_{opt}	0.071216	0.062908	0.056369	0.078435	0.069367	0.062057
0.7	1	0.07381	0.06571	0.060066	0.080369	0.071421	0.064556
	2	0.34086	0.289399	0.246724	0.285048	0.236969	0.1968
	3	0.071103	0.056257	0.047883	0.079361	0.064426	0.055925
	4	0.101895	0.087684	0.075468	0.101895	0.087684	0.075468
	α_{opt}	0.060113	0.053201	0.047645	0.069872	0.062164	0.055756
0.8	1	0.051318	0.045864	0.041822	0.063042	0.056709	0.051534
	2	0.361169	0.308385	0.263391	0.300526	0.250995	0.208512
	3	0.049331	0.043779	0.035713	0.060848	0.054127	0.047206
	4	0.101517	0.087589	0.075139	0.101517	0.087589	0.075139
	α_{opt}	0.045251	0.040092	0.035898	0.058393	0.052437	0.047337

4. CONCLUDING REMARKS

We have proposed the families of factor-type estimators for estimating the population variance under two different situations of random non-response. First, we have discussed the situation in which

random non-response occurs on both study and auxiliary variables. Secondly, the situation has been discussed where random non-response occurs only on study variable. The expressions for the biases and MSEs of the proposed families have been derived up to the first order of approximation under both the situations. In order to get the optimum estimators of the proposed families, the expressions for α_{opt} have also been derived. To support the theoretical results, an empirical study has been carried out through two different real data sets. A simulation study has also been done to check the robustness of the estimators. From Table 1 and Table 2, it is revealed that the optimum estimators provide better estimates than other existing estimators. It is also revealed that the MSE increases with increase in random non-response probability under both the situations. The same pattern of the results has also been seen under simulation study in the context of efficiency comparison (see Table 3). From Table 3, it is also seen that the average MSE decreases with increase in sample size and correlation coefficient. The results are intuitively anticipated.

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