

Consumer Surplus and Producer Surplus of the Linear Demand and Supply Functions using Interval-Valued Triangular Fuzzy Numbers

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ABSTRACT

In this paper, we fuzzified the quantity of the functions of linear demand and supply by using the interval-valued triangular fuzzy numbers. Then we calculated consumer surplus and producer surplus. We apply the signed distance defuzzification method to get the crisp results.

Keywords: consumer surplus and producer surplus, interval-valued triangular fuzzy numbers, signed distance defuzzification method.

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1 Introduction

Fuzzy numbers are brought in the literature by Chang and Zadeh (Chang and Zadeh, 1972) and can be applied all areas where the uncertainty exists. Contrary to traditional logic, it provides consideration of uncertainty. In particular, imprecise expressions such as "less, more" laid the foundations of fuzzy logic.

There are many fuzzy numbers available. Examples of these are triangular, trapezoidal, and exponentially fuzzy numbers. It is seen that triangular and trapezoidal fuzzy numbers are largely preferred in economics since it deals with too many coefficients. There are many fuzzy number studies in the field of economics. The most of them focus on microeconomics. For example, Yao and Wu (Yao and Wu, 1999) calculate consumer and producer surpluses. Wu (Wu, 1999) also calculates the surpluses but when the supply and demand functions are quadratic. Optimal revenue and optimal profit in fuzzy cases are estimated by Chang and Yao (Chang and Yao, 2000; Yao and Chang, 2001). Signed distance defuzzification method of Yao and Wu (Yao and Wu, 2000) is frequently used in such economic studies.

In case of uncertainty, one of the methods that can be used is interval-valued fuzzy numbers. Interval-valued fuzzy numbers are introduced by Zadeh (Zadeh, 1975) and especially preferred when classical fuzzy numbers are inadequate.

Interval-valued fuzzy numbers was preferred in this study, as the literature confirms that it is more appropriate for real life. In this paper, we use the linear demand $p = a - bx$ of function $0 \leq x \leq a/b$ and linear supply $p = e + gx$ ($x \geq 0$) of function to calculate the consumer surplus and producer surplus where a, b, e, g are positive constants and $e < a$. Then we fuzzify the quantity x by using interval-valued triangular fuzzy number. Yao and Wu (Wu, 1999; Yao and Wu, 1999) use the centroid method for defuzzification. Here we use the signed distance defuzzification method defined in (Yao and Wu, 2000). In addition, we would like to point out that we have used the techniques similar to that used by Yao and Wu (Yao and Wu, 1999).

2 Preliminaries

The concept of interval-valued fuzzy numbers was first defined by Zadeh (Zadeh, 1975). Now we define this concept using Gorzalcany's (Gorzalcany, 1987) notations as follows:

An interval-valued fuzzy set $\tilde{A}$ (an IV fuzzy set) defined on the set of real numbers is given by

$$\tilde{A} \doteq \left\{ \left(x, \left[\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x) \right] \right) \right\}, \quad x \in \mathbb{R}, \quad (2.1)$$

$$\underline{\mu}_{\tilde{A}}, \overline{\mu}_{\tilde{A}} : \mathbb{R} \rightarrow [0, 1]; \text{ for all } x \in \mathbb{R}, \underline{\mu}_{\tilde{A}}(x) \leq \overline{\mu}_{\tilde{A}}(x) \quad (2.2)$$

Let

$$\tilde{\mu}_{\tilde{A}}(x) = \left[\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x) \right], x \in \mathbb{R}$$

and let $D[0, 1]$ be the family of all closed intervals contained in the interval $[0, 1]$, i.e.

$$a_1 \leq a_2 \text{ and } [a_1, a_2] \in D[0, 1], \text{ for all } a_1, a_2 \in [0, 1].$$

Hence we have $\tilde{\mu}_{\tilde{A}}(x) \in D[0, 1]$ for each $x \in \mathbb{R}$. Therefore, the expressions (2.1) and (2.2) may be rewritten in a more concise form as

$$\tilde{A} \doteq \left\{ \left(x, \tilde{\mu}_{\tilde{A}}(x) \right) \right\}, \quad \tilde{\mu}_{\tilde{A}} : \mathbb{R} \rightarrow D[0, 1], \quad x \in \mathbb{R}.$$

It follows from definition that an arbitrary IV fuzzy set $\tilde{A}$ is unequivocally represented by the lower and upper limits $\underline{\mu}_{\tilde{A}}, \overline{\mu}_{\tilde{A}}$, respectively, of its generalized membership $\tilde{\mu}_{\tilde{A}}$ (Gorzalcany, 1987).

If we take the fuzzy sets of $\underline{\mu}_{\tilde{A}}$ and $\overline{\mu}_{\tilde{A}}$ as triangular fuzzy numbers, then we get the definition of an interval-valued triangular fuzzy number (Wang, 2019). For the sake of convenience, we will use interval-valued fuzzy number Chiao (Chiao, 2011) uses as interval-valued general triangular fuzzy number, that is if we choose the upper and lower membership functions as

$$\overline{\mu}_{\tilde{A}}(y) = \begin{cases} \frac{y-a}{m-a} & , a \leq y \leq m \\ \frac{y-d}{m-d} & , m \leq y \leq d \\ 0 & , \text{otherwise} \end{cases},$$

$$\underline{\mu}_{\tilde{A}}(y) = \begin{cases} \frac{y-b}{m-b} & , b \leq y \leq m \\ \frac{y-c}{m-c} & , m \leq y \leq c \\ 0 & , \text{otherwise} \end{cases}$$

(where $a \leq b \leq m \leq c \leq d$) in (2.1) and (2.2), respectively, then $\tilde{A}$ is called an interval-valued triangular (briefly, IVT) fuzzy number and is denoted as $\tilde{A} = ([a, b], m, [c, d])$ (see Figure 1). We denote the set of all IVT fuzzy numbers by $\mathcal{F}(IVT)$ (Chiao, 2011). Now let us briefly review

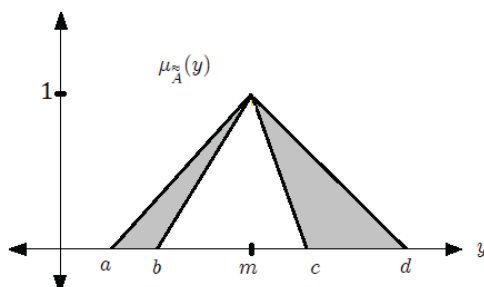


Figure 1: An IVT fuzzy number

the operations of *summation* and *scalar multiplication* on the set $\mathcal{F}(IVT)$ (Chiao, 2011). For $\tilde{x}_1 = ([a, b], m_1, [c, d])$, $\tilde{x}_2 = ([e, f], m_2, [g, h]) \in \mathcal{F}(IVT)$, summation, subtraction and scalar multiplication are defined as

$$\begin{aligned}\tilde{x}_1 + \tilde{x}_2 &= ([a + e, b + f], m_1 + m_2, [c + g, d + h]), \\ \tilde{x}_1 - \tilde{x}_2 &= \tilde{x}_1 + (-\tilde{x}_2) = ([a - h, b - g], m_1 - m_2, [c - f, d - e]), \\ k\tilde{x}_1 &= k \cdot \tilde{x}_1 = \begin{cases} ([ka, kb], km_1, [kc, kd]) & , k > 0 \\ ([kd, kc], km_1, [kb, ka]) & , k < 0 \end{cases}.\end{aligned}$$

The operations that enable converting an IV fuzzy numbers into an triangular fuzzy numbers and maintaining, at least partially, pieces of information stored in the former one, are frequently needed. This kind of operation is called type reduction by Karnik and Mendel (Karnik and Mendel, 1998; Karnik, Mendel and Liang, 1999) and Mendel (Mendel, 2017). In 2006 Niewiadomski et al. (Niewiadomski, Ochelska and Szczepaniak, 2006) have defined a different approaches for this processes as follows.

Let $\tilde{A}$ be an IVT fuzzy number and $\underline{\mu}_{\tilde{A}}$ and $\overline{\mu}_{\tilde{A}}$ be its lower and upper membership functions, respectively. The following operations, which transform $\tilde{A}$ into a triangular fuzzy number are type-reductions

$$TI_I(\tilde{A}) = \overline{\mu}_{\tilde{A}}(y), y \in \mathbb{R} \quad (2.3)$$

$$TI_K(\tilde{A}) = \underline{\mu}_{\tilde{A}}(y), y \in \mathbb{R} \quad (2.4)$$

$$TI_O(\tilde{A}) = \frac{\underline{\mu}_{\tilde{A}}(y) + \overline{\mu}_{\tilde{A}}(y)}{2}, y \in \mathbb{R} \quad (2.5)$$

Here, lower indices mean: I – optimistic, K – pessimistic, O – realistic. "Optimistic" means that in the triangular fuzzy number which is the result of (2.3), membership degrees for y 's, are taken as the highest (the most optimistic) degrees in $\tilde{A}$ and (2.4), (2.5) are defined accordingly. It is also possible to modify (2.5) with a weighted average of $\underline{\mu}_{\tilde{A}}(y)$ and $\overline{\mu}_{\tilde{A}}(y)$, if there are premises

that one of membership levels could be more influential on a description of phenomenon than the other. In that case, (2.5) is in the form of

$$TI_{AO}(\tilde{A}) = w_1 \underline{\mu}_{\tilde{A}}(y) + w_2 \overline{\mu}_{\tilde{A}}(y), y \in \mathbb{R} \quad (2.6)$$

provided that $w_1 + w_2 = 1$ and $w_1, w_2 \geq 0$. Formula (2.6) is a generalization of the equality (2.5) (Niewiadomski et al., 2006).

Yao and Wu (Yao and Wu, 2000) introduced the concept of *signed distance* in the fuzzy sense as follows: If $\tilde{A} \in \mathcal{F}(T)$ then the signed distance of $\tilde{A} = (a_1, a_2, a_3)$ measure from as

$$d^*(\tilde{A}, 0_1) = \frac{1}{4} (a_1 + 2a_2 + a_3).$$

3 Results

We consider the demand function $p = a - b\bar{x}$, $0 \leq \bar{x} \leq \frac{a}{b}$ and the supply function $p = e + g\bar{x}$, $\bar{x} \geq 0$ where $a > e > 0$, $b > 0$, $e > 0$ and $g > 0$ are fixed numbers and p is the unit price with respect to quantity $\bar{x}$. In the real situation, for a fixed price p , the demand and supply quantity $\bar{x}$ will have small changes. We characterize this ambiguity using IVT fuzzy numbers. For a fixed unit price p we fuzzify the quantity $\bar{x}$ to an IVT fuzzy number

$$\tilde{X}_D = ([\bar{x} - \delta_{11}, \bar{x} - \delta_{21}], \bar{x}, [\bar{x} + \delta_{22}, \bar{x} + \delta_{12}]), \quad (3.1)$$

where $0 \leq \delta_{21} \leq \delta_{11} < \bar{x}$ and $0 \leq \delta_{22} \leq \delta_{12}$. The upper and lower membership functions of $\tilde{X}_D$ demand IVF fuzzy number are

$$\overline{\mu}_{\tilde{X}_D}(x) = \begin{cases} \frac{x - \bar{x} + \delta_{11}}{\delta_{11}} & , \bar{x} - \delta_{11} \leq x \leq \bar{x} \\ \frac{x - \bar{x} - \delta_{12}}{-\delta_{12}} & , \bar{x} \leq x \leq \bar{x} + \delta_{12} \\ 0 & , \text{otherwise} \end{cases}$$

and

$$\underline{\mu}_{\tilde{X}_D}(x) = \begin{cases} \frac{x - \bar{x} + \delta_{21}}{\delta_{21}} & , \bar{x} - \delta_{21} \leq x \leq \bar{x} \\ \frac{x - \bar{x} - \delta_{22}}{-\delta_{22}} & , \bar{x} \leq x \leq \bar{x} + \delta_{22} \\ 0 & , \text{otherwise} \end{cases}$$

respectively.

Similarly, for a fixed unit price p , we fuzzify the supply quantity $\bar{x}$ to an IVT fuzzy number

$$\tilde{X}_S = ([\bar{x} - \delta_{31}, \bar{x} - \delta_{41}], \bar{x}, [\bar{x} + \delta_{42}, \bar{x} + \delta_{32}]), \quad (3.2)$$

where $0 \leq \delta_{41} \leq \delta_{31} < \bar{x}$ and $0 \leq \delta_{42} \leq \delta_{32}$. The upper and lower membership function of $\tilde{X}_S$ supply IVT fuzzy number are

$$\overline{\mu}_{\tilde{X}_S}(x) = \begin{cases} \frac{x - \bar{x} + \delta_{31}}{\delta_{31}} & , \bar{x} - \delta_{31} \leq x \leq \bar{x} \\ \frac{x - \bar{x} - \delta_{32}}{-\delta_{32}} & , \bar{x} \leq x \leq \bar{x} + \delta_{32} \\ 0 & , \text{otherwise} \end{cases}$$

and

$$\underline{\mu}_{\tilde{X}_S}(x) = \begin{cases} \frac{x - \bar{x} + \delta_{41}}{\delta_{41}} & , \bar{x} - \delta_{41} \leq x \leq \bar{x} \\ \frac{x - \bar{x} - \delta_{42}}{-\delta_{42}} & , \bar{x} \leq x \leq \bar{x} + \delta_{42} \\ 0 & , \text{otherwise} \end{cases}$$

respectively.

Hence the IVT demand function is

$$\tilde{P}_D = ([a - b(\bar{x} + \delta_{12}), a - b(\bar{x} + \delta_{22})], a - b\bar{x}, [a - b(\bar{x} - \delta_{21}), a - b(\bar{x} - \delta_{11})]).$$

We have the triangular fuzzy numbers using the type reductions as

$$\begin{aligned} TI_I(\tilde{P}_D) &= (a - b(\bar{x} + \delta_{12}), a - b\bar{x}, a - b(\bar{x} - \delta_{11})) \\ TI_K(\tilde{P}_D) &= (a - b(\bar{x} + \delta_{22}), a - b\bar{x}, a - b(\bar{x} - \delta_{21})) \\ TI_{AO}(\tilde{P}_D) &= ((1 - w_1)(a - b(\bar{x} + \delta_{12})) + w_1(a - b(\bar{x} + \delta_{22})), \\ &\quad a - b\bar{x}, \\ &\quad (1 - w_1)(a - b(\bar{x} - \delta_{21})) + w_1(a - b(\bar{x} - \delta_{11}))) \\ TI_O(\tilde{P}_D) &= \left(\frac{a - b(\bar{x} + \delta_{12}) + a - b(\bar{x} + \delta_{22})}{2}, \frac{a - b\bar{x} + a - b\bar{x}}{2}, \right. \\ &\quad \left. \frac{a - b(\bar{x} - \delta_{21}) + a - b(\bar{x} - \delta_{11})}{2} \right). \end{aligned} \quad (3.3)$$

where $0 \leq w_1 \leq 1$. The signed distance of triangular fuzzy number $TI_O(\tilde{P}_D)$ is

$$\begin{aligned} E_1(\bar{x}) &= d^*(TI_O(\tilde{P}_D), 0_1) = \frac{1}{4} \left(\frac{a - b(\bar{x} + \delta_{12}) + a - b(\bar{x} + \delta_{22})}{2} \right. \\ &\quad \left. + 2 \left(\frac{a - b\bar{x} + a - b\bar{x}}{2} \right) + \frac{a - b(\bar{x} - \delta_{21}) + a - b(\bar{x} - \delta_{11})}{2} \right) \\ &= a - b\bar{x} + \frac{1}{8}b(\delta_{11} + \delta_{21}) - \frac{1}{8}b(\delta_{22} + \delta_{12}) \end{aligned} \quad (3.4)$$

where E_1 denotes the interval-valued fuzzy demand quantity.

Similar operations can be repeated easily by taking $TI_I(\tilde{P}_D)$ or $TI_K(\tilde{P}_D)$ or $TI_{AO}(\tilde{P}_D)$ instead of $TI_O(\tilde{P}_D)$. We omitted these operations here.

On the other hand, the IVT supply function is

$$\tilde{P}_S = ([e + g(\bar{x} - \delta_{31}), e + g(\bar{x} - \delta_{41})], e + g\bar{x}, [e + g(\bar{x} + \delta_{42}), e + g(\bar{x} + \delta_{32})]).$$

We have the triangular fuzzy numbers using the type reductions as

$$\begin{aligned} TI_I(\tilde{P}_S) &= (e + g(\bar{x} - \delta_{31}), e + g\bar{x}, e + g(\bar{x} + \delta_{32})) \\ TI_K(\tilde{P}_S) &= (e + g(\bar{x} - \delta_{41}), e + g\bar{x}, e + g(\bar{x} + \delta_{42})) \\ TI_{AO}(\tilde{P}_S) &= ((1 - w_1)(e + g(\bar{x} - \delta_{31})) + w_1(e + g(\bar{x} - \delta_{41})), \\ &\quad e + g\bar{x}, \\ &\quad (1 - w_1)(e + g(\bar{x} + \delta_{42})) + w_1(e + g(\bar{x} + \delta_{32}))) \\ TI_O(\tilde{P}_S) &= \left(\frac{e + g(\bar{x} - \delta_{31}) + e + g(\bar{x} - \delta_{41})}{2}, \frac{e + g\bar{x} + e + g\bar{x}}{2}, \right. \\ &\quad \left. \frac{e + g(\bar{x} + \delta_{42}) + e + g(\bar{x} + \delta_{32})}{2} \right). \end{aligned} \quad (3.6)$$

The signed distance of triangular fuzzy number $TI_O(\tilde{P}_S)$ is

$$\begin{aligned} E_2(\bar{x}) &= d^*(TI_O(\tilde{P}_S), 0_1) = \frac{1}{4} \left(\frac{e + g(\bar{x} - \delta_{31}) + e + g(\bar{x} - \delta_{41})}{2} \right. \\ &\quad \left. + 2 \left(\frac{e + g\bar{x} + e + g\bar{x}}{2} \right) + \frac{e + g(\bar{x} + \delta_{42}) + e + g(\bar{x} + \delta_{32})}{2} \right) \\ &= \frac{1}{4} (4e + 4g\bar{x} - \frac{1}{2}g(\delta_{31} + \delta_{41}) + \frac{1}{2}g(\delta_{42} + \delta_{32})) \\ &= e + g\bar{x} - \frac{1}{8}g(\delta_{31} + \delta_{41}) + \frac{1}{8}g(\delta_{42} + \delta_{32}) \end{aligned} \quad (3.7)$$

where E_2 denotes the interval-valued fuzzy supply demand quantity.

Similar operations can be repeated easily by taking $TI_I(\tilde{P}_S)$ or $TI_K(\tilde{P}_S)$ or $TI_{AO}(\tilde{P}_S)$ instead of $TI_O(\tilde{P}_S)$.

Now we recall the classical consumer surplus and producer surplus for the linear case as follows.

Proposition 3.1 ((Varian, 2014)). *If the demand equation is $p = a - bx$, and the supply function is $p = e + gx$ then the consumer surplus for the crisp case is*

$$CSC = \frac{1}{2}bx_*^2 \quad (3.8)$$

and the producer surplus for the crisp case is

$$PSC = \frac{1}{2}gx_*^2 \quad (3.9)$$

where x_* is the equilibrium quantity.

Proposition 3.2 (CSF and PSF). *If the demand equation is $\tilde{P}_D = a - b\tilde{x}$, and the supply function is $\tilde{P}_S = e + g\tilde{x}$ then the consumer surplus for the fuzzy case is*

$$CSF = \frac{1}{2}b \left(\frac{1}{b+g} \left[\begin{array}{l} a - e + \frac{1}{8}b(\delta_{11} + \delta_{21}) - \frac{1}{8}b(\delta_{22} + \delta_{12}) \\ + \frac{1}{8}g(\delta_{31} + \delta_{41}) - \frac{1}{8}g(\delta_{32} + \delta_{42}) \end{array} \right] \right)^2$$

and the producer surplus for the fuzzy case is

$$PSF = \frac{1}{2}g \left(\frac{1}{b+g} \left[\begin{array}{l} a - e + \frac{1}{8}b(\delta_{11} + \delta_{21}) - \frac{1}{8}b(\delta_{22} + \delta_{12}) \\ + \frac{1}{8}g(\delta_{31} + \delta_{41}) - \frac{1}{8}g(\delta_{32} + \delta_{42}) \end{array} \right] \right)^2.$$

Proof. First we should find the equilibrium quantity. We have

$$a - b\bar{x} + \frac{1}{8}b(\delta_{11} + \delta_{21}) - \frac{1}{8}b(\delta_{22} + \delta_{12}) = e + g\bar{x} - \frac{1}{8}g(\delta_{31} + \delta_{41}) + \frac{1}{8}g(\delta_{42} + \delta_{32}),$$

$$\bar{x} = \bar{x}_* = \frac{1}{b+g} \left[\begin{array}{l} a - e + \frac{1}{8}b(\delta_{11} + \delta_{21}) - \frac{1}{8}b(\delta_{22} + \delta_{12}) \\ + \frac{1}{8}g(\delta_{31} + \delta_{41}) - \frac{1}{8}g(\delta_{32} + \delta_{42}) \end{array} \right]$$

by using the equations (3.5) and (3.7). The equilibrium price can be determined by using E_1 or E_2 as:

$$p_* = a - b\bar{x}_* + \frac{1}{8}b(\delta_{11} + \delta_{21}) - \frac{1}{8}b(\delta_{22} + \delta_{12}).$$

Then we can calculate the consumer surplus:

$$\begin{aligned}
 CSF &= \int_0^{\bar{x}_*} [E_1(\bar{x}) - p_*] d\bar{x} \\
 &= \int_0^{\bar{x}_*} \left[a - b\bar{x} + \frac{1}{8}b(\delta_{11} + \delta_{21}) - \frac{1}{8}b(\delta_{22} + \delta_{12}) - p_* \right] d\bar{x} \\
 &= \frac{1}{2}b \left(\frac{1}{b+g} \left[a - e + \frac{1}{8}b(\delta_{11} + \delta_{21}) - \frac{1}{8}b(\delta_{22} + \delta_{12}) \right. \right. \\
 &\quad \left. \left. + \frac{1}{8}g(\delta_{31} + \delta_{41}) - \frac{1}{8}g(\delta_{32} + \delta_{42}) \right] \right)^2.
 \end{aligned}$$

Similarly, we get the produces surplus:

$$\begin{aligned}
 PSF &= \int_0^{\bar{x}_*} [p_* - E_2(\bar{x})] d\bar{x} \\
 &= \int_0^{\bar{x}_*} \left[p_* - \left[e + g\bar{x} - \frac{1}{8}g(\delta_{31} + \delta_{41}) + \frac{1}{8}g(\delta_{32} + \delta_{42}) \right] \right] d\bar{x} \\
 &= \frac{1}{2}g \left(\frac{1}{b+g} \left[a - e + \frac{1}{8}b(\delta_{11} + \delta_{21}) - \frac{1}{8}b(\delta_{22} + \delta_{12}) \right. \right. \\
 &\quad \left. \left. + \frac{1}{8}g(\delta_{31} + \delta_{41}) - \frac{1}{8}g(\delta_{32} + \delta_{42}) \right] \right)^2.
 \end{aligned}$$

□

Example 3.1. Define the demand function

$$p = 22 - 2x, \quad 0 \leq x \leq 11$$

and the supply function

$$p = 10 + 2x, \quad x \geq 0.$$

Here our coefficients are $a = 22$, $b = 2$, $e = 10$ and $g = 2$. We use (Yao and Wu, 1999, Example 2.1) in order to compare our results and Yao and Wu's results (Yao and Wu, 1999).

Case 1 (Crisp)((Yao and Wu, 1999)) From $22 - 2x = 10 + 2x$, the equilibrium quantity is $x_* = 3$ and equilibrium price is $p_* = 16$. Then we have the consumer surplus and producer surplus:

$$\begin{aligned}
 CSC &= \frac{1}{2}bx_*^2 = 9, \\
 PSC &= \frac{1}{2}gx_*^2 = 9.
 \end{aligned}$$

We note that in the rest of paper, the constants aren't chosen arbitrarily. In order words, the constants of the lower membership functions of IVT demand and supply fuzzy number are similar to that of Yao and Wu's triangular fuzzy number (Yao and Wu, 1999, Example 2.1).

Case 2 (IVT) Let the fuzzified coefficients $\delta_{11} = 6$, $\delta_{12} = 4$, $\delta_{22} = 3$, $\delta_{21} = 4$, $\delta_{31} = 4$, $\delta_{32} = 2$, $\delta_{41} = 2$, $\delta_{42} = 1$. Hence the equilibrium quantity is

$$\begin{aligned}
 \bar{x}_* &= \left(\frac{1}{b+g} \left[a - e + \frac{1}{8}b(\delta_{11} + \delta_{21}) - \frac{1}{8}b(\delta_{22} + \delta_{12}) \right. \right. \\
 &\quad \left. \left. + \frac{1}{8}g(\delta_{31} + \delta_{41}) - \frac{1}{8}g(\delta_{32} + \delta_{42}) \right] \right)^2 \\
 &= 3.375.
 \end{aligned}$$

Then the equilibrium price p_* can be calculated as

$$\begin{aligned} E_1(\bar{x}_*) &= a - b\bar{x}_* + \frac{1}{8}b(\delta_{11} + \delta_{21}) - \frac{1}{8}b(\delta_{22} + \delta_{12}) \\ &= 16 \end{aligned}$$

or

$$\begin{aligned} E_2(\bar{x}_*) &= e + g\bar{x}_* - \frac{1}{8}g(\delta_{31} + \delta_{41}) + \frac{1}{8}g(\delta_{32} + \delta_{42}) \\ &= 16. \end{aligned}$$

The consumer surplus and producer surplus estimate as

$$\begin{aligned} CSF &= \frac{1}{2}b\bar{x}_*^2 = 11.391, \\ PSF &= \frac{1}{2}g\bar{x}_*^2 = 11.391. \end{aligned}$$

The values for different cases are given in Table 1.

	$\bar{x}_*$	p_*	PSF/CSF
Crisp	3	16	9
Yao and Wu's (Yao and Wu, 1999), Case 1	3	16	9
Yao and Wu's (Yao and Wu, 1999), Case 2	3	15.33	9
Yao and Wu's (Yao and Wu, 1999), Case 3	2.667	15.999	7.113
Yao and Wu's (Yao and Wu, 1999), Case 4	3.333	16.001	11.109
IVT fuzzy number with Realistic type reduction	3.375	16	11.391
Table 1: PSF/CSF for the triangular and IVT fuzzy numbers			

4 Conclusion

One of the best fuzzifying techniques for a quantity is to use interval-valued fuzzy numbers. An interval-valued fuzzy numbers consists of lower and upper membership functions. If these membership functions of interval-valued fuzzy numbers are curved fuzzy numbers (such as Gaussian, exponential, etc), then in this case, arithmetic operations with these type of fuzzy numbers can become complex. On the other hand, we know that triangular fuzzy numbers are simple to arithmetic. In this context, we chose the lower and upper membership functions of interval-valued fuzzy numbers as triangular fuzzy numbers in order to avoid this complexity. Using these type of interval-valued fuzzy numbers, we fuzzify the demand and supply functions and calculate consumer surplus and producer surplus.

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