

Monthly Payment Formula in Auto Leasing Industry and Its Analytical Properties

Guoping Zeng

4522 Oak Shores Dr, Plano TX 75024, USA
gzeng314159@gmail.com

ABSTRACT

A simple mathematical formula is used to calculate monthly payment in Auto Leasing industry. However, the literature about this formula is limited. Analytical properties are ignored. Moreover, the origin of this formula has not been explored. In this paper, we systematically study this monthly payment formula and its monotonic properties. We explore the origin of this simple formula by deriving the exact formula for monthly payment. We obtain an alternate, more complicated but more accurate monthly payment formula. We show that the monthly payment by the simple formula is generally smaller than and closed to that by the exact formula.

Keywords: Monthly payment, auto leasing, loan amount, interest rate, money factor, residual

Mathematics Subject Classification: 00A69, 91G50

1. INTRODUCTION

Auto leasing has been popular over the last few decades (Johnson et al. 2014). In Auto Leasing, financial institutions use credit scoring models, mainly via logistic regression (Aboulaich, et al. 2013; Zeng, 2013; Zeng, 2014a; Zeng, 2014b; Zeng, 2017a; Zeng, 2017b; Zeng, 2017c; Zeng, 2019a; Zeng, 2019b; Zeng, 2020), to make loan granting decisions. Once the loan is approved, a simple formula is used to calculate the monthly payment with a fixed value for the leased car.

However, the literature about this simple formula is limited. The analytical properties are ignored. Moreover, the origin of this simple formula has not been explored. In a one-page paper (Milanfar, 1996), Milanfar noticed that a simplified formula for monthly payment of a loan using an average interest method was originally from the Persian folk method by Iranian merchants at least a century ago. Burgess and Lindsey (2002) simply extended the Persian folk method to Auto Leasing without any theoretical justification.

In this paper, we systematically study the monthly payment formula in Auto Leasing. We explore the origin of the simple formula by deriving the exact formula for monthly payment in Auto Leasing. The remaining of the paper is organized as follows. In Section 2, we briefly introduce the monthly payment formula in Auto Leasing and discuss its monotonic properties. In Section 3, we first review the Persian Folk method of calculating monthly payment in Auto Loan and Auto Leasing. We then derive the exact monthly payment formula. Next, we give a rough derivation of the simple monthly payment formula, followed by a refined derivation. In Section 4, numerical results are presented to justify our theoretical results. The paper is concluded in Section 5. Throughout the paper, we use both brackets and nested parentheses for complex math expressions.

2. SIMPLE MONTHLY PAYMENT FORMULA IN AUTO LEASING

The following simple formula is widely used in Auto Leasing to calculate the monthly payment P

$$P = \frac{C - R}{n} + (C + R)F, \quad (2.1)$$

where C is the loan amount, R is the residual (the remaining value of the vehicle at lease end), n is the term in months, and F is so-called Money Factor.

The first part $\frac{C-R}{n}$ of the right-hand side of (2.1) is called Monthly Depreciation, whereas the second part $(C + R)F$ is called Monthly Rent or Monthly Interest.

It follows from simple algebraic manipulation that (2.1) can be expressed into another form

$$P = \left(\frac{1}{n} + F\right)C - \left(\frac{1}{n} - F\right)R. \quad (2.2)$$

Money Factor is also called Lease Factor. It is derived from the annual interest rate. If the annual interest rate without percentage is a , then *Money Factor* = $a/2400$. For instance, if the annual interest rate is 5%, then $F = \frac{5}{2400} = 0.0025$. If we denote the monthly interest rate by r , then $r = \frac{a}{100} \times \frac{1}{12} = \frac{a}{1200} = 2F$, or equivalently, $\frac{r}{2} = F$.

Term n is usually 12, 24, 36, 42, 48, 54, 60, 72. In general, $F < \frac{1}{n}$ or equivalently $\frac{1}{n} - F > 0$. If F is large enough, $\frac{1}{n} - F$ could be non-positive. For instance, if the annual interest is 34% and $n = 72$, then $\frac{1}{n} - F = \frac{1}{72} - \frac{34}{2400} < 0$. However, an annual interest of 34% is almost impossible in Auto Leasing. As long as the annual interest is less than 33%,

$$\frac{1}{n} - F \geq \frac{1}{72} - \frac{33}{2400} > 0. \quad (2.3)$$

Hence, without loss of generality, we assume $F < \frac{1}{n}$, that is $\frac{1}{n} - F > 0$.

The loan amount C is the sales value of the car plus various fees and minus down payment, discount and trade-in, etc. Theoretically, it is possible that $C \leq R$. In Auto Leasing, it is requested that $C > R$.

The following monotonic properties of the simple Monthly Payment Formula are straight forward with assumptions $C > R$ and $\frac{1}{n} - F > 0$ and the 2 equivalent expressions (2.1) and (2.2).

Property 1. The monthly payment P is a strictly increasing function of loan amount C for fixed F, R and n .

Property 2. The monthly payment P is a strictly decreasing function of Residual R for fixed C, n and F .

Property 3. The monthly payment P is a strictly decreasing sequence of term n for fixed C, R and F .

Property 4. The monthly payment P is a strictly increasing function of Money Factor F for fixed C, R and n .

3. THE ORIGIN OF THE SIMPLE Monthly

According to Burgess and Lindsey (2002), the banking industry has simplified calculation of the monthly payment of a loan for at least a century by using an average interest method, which was borrowed from the Persian folk method by Iranian merchants:

$$\text{Monthly Payment} = \frac{\text{Principal} + \text{Interest}}{\text{Number of Manths}}, \quad (3.1)$$

where

$$\begin{aligned} \text{interest} &= \frac{1}{2} \text{Principal} \times \text{Number of years} \times \text{Annual interest rate} \\ &= \frac{1}{2} \text{Principal} \times \text{Number of months} \times \text{monthly interest rate.} \end{aligned}$$

Since Principal Payment in (3.1) is same as

$$\frac{\text{Principal} - 0 \text{ residual}}{\text{Number of Manths}}$$

and Interest Payment is same as

$$\frac{\frac{1}{2}(\text{Principal} + 0 \text{ residual}) \times \text{Number of Months} \times \text{Monthly interest rate}}{\text{Number of Manths}},$$

Burgess and Lindsey (2002) claimed that Persian folk method yields the simple monthly payment in auto leasing:

$$\frac{\text{Principal} - \text{Residual}}{\text{Number of Months}} + (\text{Principal} + \text{Residual}) \times \text{Money Factor}$$

as money factor represents one-half monthly interest.

However, the above analogy or parallel by Burgess and Lindsey (2002) has no theoretical justification. In what follows, we will explore the origin of the simple formula (2.1) in Auto Leasing.

3.1. Exact monthly payment formula

In loan lending, amortization refers to spreading payments over multiple periods. In mortgage loan, payments are fully amortized by dividing the whole amount into equal amount for the life of the loan. Loans in Auto Leasing is similar to mortgage. Yet, payments are not fully amortized in that the remaining balance at the end of the loan is the residual value R rather than 0.

Assume the monthly payment in Auto Leasing is a fixed amount x . Denote the monthly interest rate by r . Let us also use the following notation:

- I_i : Interest paid in month i ;
- P_i : Principal paid in month i ;
- B_i : Remaining balance at the end of month i .

For convenience, we denote C by B_0 . Then, we have the following 3 recursive formulas for $i = 1, 2, \dots, N$:

- $I_i = B_{i-1} \times r$
- $P_i = x - I_i$
- $B_i = B_{i-1} - P_i$.

For the first month $i = 1$, we have

- $I_1 = C \times r$
- $P_1 = x - I_1 = x - C \times r$
- $B_1 = C - P_1 = C - (x - C \times r) = C(1 + r) - x$.

For the second month $i = 2$, we have

- $I_2 = B_1 \times r = (C(1 + r) - x)r$
- $P_2 = x - I_2 = x - (C(1 + r) - x)r$
- $B_2 = B_1 - P_2 = C(1 + r) - x - x + (C(1 + r) - x)r = C(1 + r)^2 - x(1 + r) - x$.

In general, for month i , we have

- $I_i = [C(1 + r)^{i-1} - x(1 + r)^{i-2} - \dots - x(1 + r) - x]r$
- $P_i = x - [C(1 + r)^{i-1} - x(1 + r)^{i-2} - \dots - x(1 + r) - x]r$
- $B_i = C(1 + r)^i - x(1 + r)^{i-1} - \dots - x(1 + r) - x$.

As time goes on, I_i will be smaller and smaller and P_i will be larger and larger.

We can solve x by letting the remaining balance at lease end B_n be the residual value R :

$$x = \frac{C(1 + r)^n - R}{1 + (1 + r) + \dots + (1 + r)^{n-1}} = \frac{Cr(1 + r)^n - R \times r}{(1 + r)^n - 1}. \quad (3.2)$$

In particular, if $R = 0$, it reduces to the classic mortgage formula: $\frac{Cr(1+r)^n}{(1+r)^n - 1}$.

Now, the interest in month i can be found as follows:

$$\begin{aligned}
 I_i &= [C(1+r)^{i-1} - x(1+r)^{i-2} - \dots - x(1+r) - x]r \\
 &= \left[C(1+r)^{i-1} - x \frac{(1+r)^{i-1} - 1}{r} \right] r \\
 &= Cr(1+r)^{i-1} - x((1+r)^{i-1} - 1) \\
 &= x - (1+r)^{i-1}(x - Cr) \\
 &= \frac{Cr(1+r)^n - R \times r}{(1+r)^n - 1} - (1+r)^{i-1} \left(\frac{Cr(1+r)^n - R \times r}{(1+r)^n - 1} - Cr \right).
 \end{aligned} \tag{3.3}$$

The principal in month i can be found as follows:

$$P_i = x - I_i = (1+r)^{i-1} \left(\frac{Cr(1+r)^n - R \times r}{(1+r)^n - 1} - Cr \right). \tag{3.4}$$

The total interest is

$$\begin{aligned}
 \sum_{i=1}^n I_i &= \sum_{i=1}^n (x - P_i) \\
 &= nx - \sum_{i=1}^n P_i \\
 &= nx - (C - R) \\
 &= n \frac{Cr(1+r)^n - R \times r}{(1+r)^n - 1} - C + R.
 \end{aligned} \tag{3.5}$$

The total principal is

$$\begin{aligned}
 \sum_{i=1}^n P_i &= \sum_{i=1}^n (x - I_i) \\
 &= nx - \sum_{i=1}^n I_i \\
 &= nx - [nx - (C - R)] \\
 &= C - R.
 \end{aligned} \tag{3.6}$$

The exact monthly payment x in (3.2) also has the following monotonically Properties. The first 2 properties are straightforward, whereas the last 2 are trickier.

Property 5. The exact monthly payment x is a strictly increasing function of loan amount C for fixed F, R and n .

Property 6. The exact monthly payment x is a strictly decreasing function of Residual R for fixed C, n and F .

Property 7. The exact monthly payment x is a strictly decreasing sequence of term n for fixed C, R and F .

Proof. Rewrite the exact monthly payment x in (3.2) as follows:

$$x = \frac{Cr(1+r)^n - R \times r}{(1+r)^n - 1} = \frac{Cr(1+r)^n - Cr + Cr - R \times r}{(1+r)^n - 1} = Cr + \frac{(C-R)r}{(1+r)^n - 1}.$$

Since $C - R > 0$, the exact monthly payment x is a strictly decreasing sequence of term n for fixed C, R and F . ■

Property 8. The exact monthly payment x is an increasing function of monthly interest rate r for fixed C, R and n .

Proof. Denote x by function $f(r) = \frac{Cr(1+r)^n - R \times r}{(1+r)^n - 1}$. Then

$$f'(r) = \frac{C(1+r)^{n-1}((1+r)^{n+1} - nr - r - 1)}{[(1+r)^n - 1]^2} - R \frac{(1+r)^n - 1 - nr(1+r)^{n-1}}{[(1+r)^n - 1]^2}.$$

Let us denote the numerator of $f'(r)$ after combining the 2 fractions by $g(r)$, that is,

$$g(r) = C(1+r)^{n-1}((1+r)^{n+1} - nr - r - 1) - R((1+r)^n - 1 - nr(1+r)^{n-1}).$$

Then $g(0) = 0$. The derivative of $g(r)$ can be found as

$$\begin{aligned} g'(r) &= C(n-1)(1+r)^{n-2}[(1+r)^{n+1} - nr - r - 1] \\ &\quad + C(1+r)^{n-1}[(n+1)(1+r)^n - (n+1)] \\ &\quad - R[n(1+r)^{n-1} - n(1+r)^{n-1} - n(n-1)r(1+r)^{n-2}] \\ &= C(n-1)(1+r)^{n-2}[(1+r)^{n+1} - nr - r - 1] \\ &\quad + C(1+r)^{n-1}[(n+1)((1+r)^n - 1)] + Rn(n-1)r. \end{aligned}$$

By Bernoulli's inequality, $(1+r)^{n+1} \geq 1 + (n+1)r = 1 + nr + r$. It follows from $(1+r)^n - 1 > 0$ that

$g'(r) > 0$. Hence, $g(r) > g(0) = 0$ for $r > 0$. So, $f'(r) = \frac{g(r)}{[(1+r)^n - 1]^2} > 0$, that is, $f(r)$ is a strictly

increasing function of r . ■

3.2. Rough derivation of simple monthly payment formula

From Section 2.1, the average principle per month is $(C - R)/n$. The interest of the first month is $C \times r$. The interest of the last month is $I_n = B_{n-1} \times r \approx B_n \times r = R \times r$. Hence, the average interest per month is approximately $\frac{C \times r + R \times r}{2} = \frac{C+R}{2}r$. So, the monthly payment is approximately $\frac{C-R}{n} + \frac{C+R}{2}r = \frac{C-R}{n} + (C+R)F$.

3.3. Refined derivation of simple monthly payment formula

Let us now refine the derivation of the approximate formula. To start with, let us denote the monthly payment by function f , that is,

$$f(r) = \frac{Cr(1+r)^n - R \times r}{(1+r)^n - 1}. \quad (3.7)$$

Although $f(r)$ is not continuous at $r = 0$, its limit exists by L'Hospital's Rule:

$$\lim_{r \rightarrow 0} f(r) = \lim_{r \rightarrow 0} \frac{C(1+r)^n + nCr(1+r)^{n-1} - R}{n(1+r)^{n-1}} = \frac{C - R}{n}. \quad (3.8)$$

Therefore, we can extend function f to make it continuous when $r \geq 0$ by defining its value at 0 to be $\lim_{r \rightarrow 0} f(r)$. For simplicity, we continue to use f to denote the extended function. Next, we evaluate $f'(0)$ as follows:

$$\begin{aligned}
 & f'(0) \\
 &= \lim_{r \rightarrow 0} \frac{f(r) - f(0)}{r} \\
 &= \lim_{r \rightarrow 0} \frac{\frac{Cr(1+r)^n - R \times r}{(1+r)^n - 1} - \frac{C - R}{n}}{r} \\
 &= \lim_{r \rightarrow 0} \frac{\frac{[C(1+r)^n + nCr(1+r)^{n-1} - R] \times [(1+r)^n - 1] - n(1+r)^{n-1}[Cr(1+r)^n - Rr]}{[(1+r)^n - 1]^2}}{1} \\
 &= \lim_{r \rightarrow 0} \frac{C(1+r)^{2n} - R(1+r)^n - C(1+r)^n - nCr(1+r)^{n-1} + R + nRr(1+r)^{n-1}}{[(1+r)^n - 1]^2} \\
 &= \lim_{r \rightarrow 0} \left\{ \frac{2nc(1+r)^{2n-1} - nR(1+r)^{n-1} - nC(1+r)^{n-1} - nC(1+r)^{n-1}}{2[(1+r)^n - 1]n(1+r)^{n-1}} \right. \\
 &\quad \left. + \frac{-n(n-1)Cr(1+r)^{n-2} + nR(1+r)^{n-1} + n(n-1)Rr(1+r)^{n-2}}{2[(1+r)^n - 1]n(1+r)^{n-1}} \right\} \\
 &= \lim_{r \rightarrow 0} \frac{2nc(1+r)^{2n-1} - 2nC(1+r)^{n-1} - n(n-1)Cr(1+r)^{n-2} + n(n-1)Rr(1+r)^{n-2}}{2[(1+r)^n - 1]n(1+r)^{n-1}} \\
 &= \lim_{r \rightarrow 0} \left\{ \frac{2n(2n-1)C(1+r)^{2n-2} - 2n(n-1)C(1+r)^{n-2} - n(n-1)(n-2)Cr(1+r)^{n-3}}{2n(1+r)^{n-1}n(1+r)^{n-1} + 2[(1+r)^n - 1]n(n-1)(1+r)^{n-2}} \right. \\
 &\quad \left. + \frac{-n(n-1)C(1+r)^{n-2} + n(n-1)R(1+r)^{n-2} + n(n-1)(n-2)Rr(1+r)^{n-3}}{2n(1+r)^{n-1}n(1+r)^{n-1} + 2[(1+r)^n - 1]n(n-1)(1+r)^{n-2}} \right\} \\
 &= \frac{2n(2n-1)C - 2n(n-1)C - n(n-1)C + n(n-1)R}{2n^2} \\
 &= \frac{(C+R)n + C - R}{2n} \\
 &= \frac{C+R}{2} + \frac{C-R}{2n}.
 \end{aligned}$$

Note that in the 1st, 2nd, 3rd, 4th, 5th, 6th, 7th, 8th, 9th, 10th, we have used definition of derivative, substitution, L'Hopital's Rule, algebraic manipulation, L'Hospital's Rule, algebraic manipulation, L'Hospital's Rule, evaluation at $r = 0$, algebraic manipulation, respectively.

Note that $r/2$ is Money factor. By the first order Taylor series expansion around $r = 0$,

$$\begin{aligned}
 f(r) &= f(0) + f'(0)r + \frac{f''(\zeta)}{2}r^2 \\
 &= \frac{C-R}{n} + \frac{C+R}{2}r + \frac{C-R}{2n}r + \frac{f''(\zeta)}{2}r^2 \\
 &= \frac{C-R}{n} + (C+R)F + \frac{C-R}{n}F + \frac{f''(\zeta)}{2}r^2,
 \end{aligned} \tag{3.9}$$

where ζ is a positive constant.

The 3rd term $\frac{C-R}{n}F$ in (3.9) means the interest on depreciation. It is small when r is small. If we ignore it, we have an approximated and simple formula

$$f(r) \approx \frac{C-R}{n} + (C+R)F. \tag{3.10}$$

Note that principal $\frac{C-R}{n}$ and interest $(C+R)F$ in (3.10) are both constants, where in the exact monthly payment formula (3.2) principal is larger and larger and interest is smaller and smaller from month to month.

If we include the 3rd term $\frac{C-R}{n}F$ in (3.9), we obtain an alternate monthly payment formula:

$$f(r) \approx \frac{C-R}{n} + (C+R)F + \frac{(C-R)F}{n}. \tag{3.11}$$

Since $C-R > 0$, the alternate formula (3.11) will yield a larger and more accurate monthly payment than the simple formula (2.1).

From the expression of $f'(r)$ from the proof of Property 8 we rewrite

$$f'(r)[(1+r)^n - 1]^2 = C(1+r)^{n-1}[(1+r)^{n+1} - nr - r - 1] - R[(1+r)^n - 1 - nr(1+r)^{n-1}]$$

Differentiating both sides yields

$$\begin{aligned}
 &f''(r)[(1+r)^n - 1]^2 + 2n(1+r)^{n-1}f'(r) \\
 &= C(n-1)(1+r)^{n-2}[(1+r)^{n+1} - nr - r - 1] + C(1+r)^{n-1}[(n+1)(1+r)^n - n - 1] \\
 &\quad - R[n(1+r)^{n-1} - n(1+r)^{n-1} - n(n-1)r(1+r)^{n-2}] \\
 &= C(n-1)(1+r)^{n-2}[(1+r)^{n+1} - nr - r - 1] + C(1+r)^{n-1}[(n+1)(1+r)^n - n - 1] \\
 &\quad + Rn(n-1)r(1+r)^{n-2}.
 \end{aligned}$$

Differentiating both sides again yields

$$\begin{aligned}
 & f'''(r)[(1+r)^n - 1]^2 + 2nf''(r)(1+r)^{n-1} + 2nf''(r)(1+r)^{n-1} + 2n(n-1)f'(r)(1+r)^{n-2} \\
 & = C(n-1)(n-2)(1+r)^{n-3}[(1+r)^{n+1} - nr - r - 1] + C(n-1)(1+r)^{n-2}[(n+1)(1+r)^n - n - 1] \\
 & \quad + C(n-1)(1+r)^{n-2}[(n+1)(1+r)^n - n - 1] + Cn(n+1)(1+r)^{2n-2} \\
 & \quad + n(n-1)R(1+r)^{n-2} + n(n-1)(n-2)Rr(1+r)^{n-3}.
 \end{aligned}$$

Evaluating both sides at $r = 0$, we obtain

$$f''(0) = \frac{(n+1)C + (n-1)R}{4} > 0. \quad (3.12)$$

So, when r is small, $f''(r) > 0$, the approximated payment is less than the exact payment.

4. NUMERICAL RESULTS

In Table 1, we calculate the monthly payment rounding to 2 decimal places (cents) by using the simple formula (2.1), alternate formula (3.11) and exact formula (3.2) for 36 leases. We observe that

- the monthly payment by the simple formula (2.1) is less than that by the exact formula (3.2);
- the monthly payment by the alternate formula (3.11) is between that by the simple formula (2.1) and that by the exact formula (3.2);
- the difference between the monthly payment by the exact formula (3.2) and that by the simple formula (2.1) is small (from \$0.81 to \$7.96) as can be seen from the last column.

Therefore, the alternate formula (3.11) is not necessary albeit more accurate. After all, the simple formula is accurate enough and provides a simple, quick, realistic, reasonable, reliable and easy to understand method of calculating the monthly payment in Auto Leasing.

5. CONCLUSIONS

In this paper, we have systematically studied the simple formula for monthly payment in Auto Leasing industry and its monotonic property. We explored the origin of this simple formula by deriving the exact monthly payment formula and its monotonic properties. We showed that the monthly payment by the simple formula is generally smaller than and closed to that by the exact formula. This monthly payment formula provides a simple, quick, realistic, reasonable, reliable, accurate and easy to understand method of calculating the monthly payment in Auto Leasing.

5. REFERENCES

- Aboulaich, R., Khamlichi, F. I. & Kaicer, M. 2013, Microcredit Scoring with Fuzzy Logic Model. *International Journal of Statistics and Economics*. **10**(1), 109-117.
- Johnson, J. P., Schneider, H. S. & Waldman, M. 2014, The Role and Growth of New-Car Leasing: Theory and Evidence. *The Journal of Law & Economics*. **57**(3), 665-698.
- Burgess, D. O. and Lindsey C. 2002, A Persian Folk Method of Calculating Automobile Lease Payments. *Commercial Lending Review*. **17**, 41-44.

- Milanfar, P. 1996, A Persian Folk Method of Figuring Interest. *Mathematics Magazine*, **69**(5), 376-376.
- Zeng, G. (2013) Metric divergence measures and information value in credit scoring. *Journal of Mathematics*, vol. 2013, Article ID 848271. doi: 10.1155/2013/848271.
- Zeng, G. (2014a) A rule of thumb for reject inference in credit scoring. *Mathematical Finance Letters*, vol. 2014, article 2, 1-13.
- Zeng, G. (2014b) A necessary condition for a good binning algorithm in credit scoring. *Applied Mathematical Sciences*. **8**(65), 3229–3243. doi: 10.12988/ams.2014.44300.
- Zeng, G. (2015) A unified definition of mutual information with applications in machine learning. *Mathematical Problems in Engineering*, vol. 2015, Article ID 201874. doi: 10.1155/2015/201874.
- Zeng, G. (2017a) A comparison Study of Computational Methods of Kolmogorov–Smirnov Statistic in Credit Scoring. *Communications in Statistics: Simulation and Computation*. **46**(10), 7744-7760. doi: 10.1080/03610918.2016.1249883.
- Zeng, G. (2017b) Invariant Properties of Logistic Regression Model in Credit Scoring under Monotonic Transformations. *Communications in Statistics: Theory and Methods*. **46**(17), 8791-8807. doi: 10.1080/03610926.2016.1193200.
- Zeng, G. (2017c) On the Existence of Maximum Likelihood Estimates for Weighted Logistic Regression. *Communications in Statistics: Theory and Methods*. **46**(22), 11194-11203. doi: 10.1080/03610926.2016.1260743.
- Zeng, G. and Zeng, E. (2019a) On the Three-way Equivalence of AUC in Credit Scoring with Tied Scores. *Communications in Statistics: Theory and Methods*. **48**(7), 1635-1650. doi: 10.1080/03610926.2018.1435814.
- Zeng, G. and Zeng, E. (2019b) On the Relationship between Multicollinearity and Separation in Logistic Regression. *Communications in Statistics: Simulation and Computation*. doi: 10.1080/03610918.2019.1589511.
- Zeng, G. (2020) On the Confusion Matrix in Credit Scoring and Its Analytical Properties. *Communications in Statistics: Theory and Methods*. **49**(9), 2080-2093. doi: 10.1080/03610926.2019.1568485

Table 1. Monthly Payment by Different Formulas

C	R	r	F	n	By (2.1)	By (3.11)	By (3.2)	(3.2) – (2.1)
37117	16500	0.00398	0.00199	48	536.22	537.07	538.38	2.16
34700	11400	0.00398	0.00199	60	480.07	480.85	482.68	2.61
29000	11000	0.0044	0.0022	60	388.00	388.66	390.40	2.40

22000	9200	0.0059	0.00295	48	358.71	359.49	361.27	2.56
48000	15000	0.00498	0.00249	60	706.87	708.24	712.31	5.44
29200	11400	0.00418	0.00209	60	381.52	382.14	383.69	2.17
24016	8002	0.00334	0.00167	48	387.10	387.65	388.36	1.27
53421	20418	0.00418	0.00209	60	704.37	705.52	708.40	4.02
27704	13891	0.00482	0.00241	48	388.01	388.71	389.99	1.97
48263	24528	0.00684	0.00342	60	644.53	645.88	651.40	6.87
46033	13278	0.00498	0.00249	60	693.60	694.96	699.00	5.40
69036	39385	0.00584	0.00292	36	1140.23	1142.63	1145.65	5.43
81923	37686	0.00386	0.00193	36	1459.65	1462.02	1463.99	4.34
27922	9688	0.00568	0.00284	54	444.48	445.44	448.07	3.59
53297	15000	0.00568	0.00284	60	832.25	834.06	840.21	7.96
69036	39388	0.00584	0.00292	36	1140.15	1142.56	1145.58	5.42
81551	27826	0.00374	0.00187	60	1099.95	1101.63	1105.37	5.42
40799	15760	0.00494	0.00247	48	661.35	662.64	665.07	3.72
42593	10678	0.00334	0.00167	60	620.88	621.77	623.54	2.66
28945	8000	0.00594	0.00297	60	458.81	459.85	463.52	4.71
44315	15327	0.00314	0.00157	48	697.55	698.50	699.64	2.09
41055	24933	0.00524	0.00262	36	620.72	621.90	623.22	2.50
25164	12217	0.00378	0.00189	42	378.91	379.49	380.14	1.23
33309	25766	0.00378	0.00189	24	425.94	426.54	426.75	0.81
27436	17899	0.0056	0.0028	36	391.85	392.60	393.49	1.64
34760	16129	0.0074	0.0037	48	576.44	577.87	581.93	5.49
48475	17111	0.00524	0.00262	60	694.57	695.94	700.22	5.66
53980	26778	0.00378	0.00189	48	719.34	720.41	721.96	2.62
36965	17762	0.00378	0.00189	54	459.05	459.72	460.95	1.90
19715	7683	0.00458	0.00229	36	396.96	397.73	398.48	1.52
35899	16080	0.00478	0.00239	48	537.13	538.11	539.92	2.79
22449	3316	0.00498	0.00249	48	462.76	463.75	465.64	2.88
26048	9577	0.00738	0.00369	60	405.97	406.99	411.44	5.47
41208	11911	0.00398	0.00199	60	593.99	594.96	597.27	3.28
52442	42878	0.00314	0.00157	24	548.15	548.78	548.97	0.81
52662	43348	0.0044	0.0022	24	599.31	600.16	600.52	1.21