

Regress Exponential Estimators for Estimating the Population Mean via Auxiliary Attribute

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ABSTRACT

This research manuscript is an attempt to propose regress exponential estimators to estimate the population mean of study character utilizing auxiliary attribute by adopting the strategies of Kadilar and Cingi (2004). The bias (B) and mean square error (MSE) of the proposed estimators are obtained up to the first order of approximation. Conditions for getting the minimum mean square error of the proposed estimators are derived and hence the minimum value of mean square errors is obtained. Comparisons of suggested estimators are made theoretically with all related estimators and are supported by the empirical study based on real data sets.

Keywords: Population mean, bias, mean square error, auxiliary attributes.

Mathematics Subject Classification 2010: 62D05

1. INTRODUCTION

It is a well recognized fact that efficiency of the estimators to estimate the parameters can be enhanced by effective use of the characteristics of auxiliary character, especially when it is highly correlated with study character. If the characteristic of auxiliary character is qualitative in nature and associated with the study character then using the known parameters of auxiliary character, estimators can be formed to estimate the parameters of character under study.

Let y_i and ψ_i denote the observation for i^{th} unit of study and auxiliary character possessing an attribute (ψ) of the population ξ_N having N units. Let the population is dichotomy associated with absence or presence of an attribute ψ and $\psi_i = 1$ or 0 , if the i^{th} unit of the population possesses or does not possess the attribute ψ . Suppose a sample ξ_n of size n is drawn using simple random sampling without replacement (SRS_{WOR}) method and the total units possessing the attribute ψ in the population and sample are $A = \sum_{\xi_N} \psi_i$ and $a = \sum_{\xi_n} \psi_i$ respectively. Let the proportion and mean of the population and sample of study character is defined by $(P = A/N, \bar{Y} = \frac{1}{N} \sum_{\xi_N} y_i)$ and $(p = a/n, \bar{y} = \frac{1}{n} \sum_{\xi_n} y_i)$.

Naik and Gupta (1996) observed the coefficient of point biserial correlation between study and auxiliary attribute and suggested conventional estimators to estimate $\bar{Y}$ using known value of P as

$$t_{NG}^r = \bar{y} \left(\frac{p}{P} \right) \quad [\text{Ratio Estimator}], \quad (1)$$

$$t_{NG}^p = \bar{y} \left(\frac{p}{P} \right) \quad [Product\ Estimator] \quad (2)$$

and
$$t_{NG}^{reg} = \bar{y} + \hat{\beta}(p - P). \quad [Regression\ Estimator] \quad (3)$$

The mean square errors of t_{NG}^r , t_{NG}^p and t_{NG}^{reg} up to the first order of approximation $[O(n^{-1})]$ are given by

$$MSE(t_{NG}^r) = \lambda \left\{ S_y^2 + \left(\frac{\bar{y}}{P} \right)^2 S_\psi^2 - 2 \frac{\bar{y}}{P} \rho_{y\psi} S_y S_\psi \right\}, \quad (4)$$

$$MSE(t_{NG}^p) = \lambda \left\{ S_y^2 + \left(\frac{\bar{y}}{P} \right)^2 S_\psi^2 + 2 \frac{\bar{y}}{P} \rho_{y\psi} S_y S_\psi \right\} \quad (5)$$

and
$$MSE(t_{NG}^{reg}) = V(\bar{y}) - \lambda \rho_{y\psi}^2 S_y^2, \quad (6)$$

where
$$V(\bar{y}) = \lambda S_y^2, \quad (7)$$

$$\lambda = \frac{1}{n} - \frac{1}{N}, \quad S_y^2 = \frac{1}{N-1} \sum_{i=1}^N (y_i - \bar{Y})^2, \quad S_\psi^2 = \frac{1}{N-1} \sum_{i=1}^N (\psi_i - P)^2$$

and $\rho_{y\psi}$ is point biserial correlation coefficient between y and ψ and is defined by

$$\rho_{y\psi} = \frac{S_{y\psi}}{S_y S_\psi}, \quad \text{where } S_{y\psi} = \frac{1}{N-1} \sum_{i=1}^N (y_i - \bar{Y})(\psi_i - P).$$

Jhaji et al. (2006) proposed a class of estimator for estimating $\bar{Y}$ under some regularity conditions as

$$t_f^c = f(\bar{y}, u); \quad u = \frac{p}{P} \quad (8)$$

and its minimum MSE up to the first order of approximation on the basis of optimum value of first order partial derivative of the function $f(\bar{y}, u)$ with respect to u is given by-

$$[MSE(t_f^c)]_{min} = \lambda S_y^2 (1 - \rho_{y\psi}^2). \quad (9)$$

Further, modifications for the better efficiency of existing estimators to estimate the population mean using alike information on auxiliary attribute/variable have been achieved by Abd-Elfattah et al. (2010), Khare and Sinha (2012), Sinha and Kumar (2013), Singh (2014), Sinha (2014, 16), Pal and Singh (2016), Saini and Kumar (2015), Bhushan et al. (2017), Sinha and Kumar (2017), Zaman (2018), Haq and Shabbir (2018), Zaman and Sozen (2018), Singh and Usman (2018), Singh et al. (2019), Zaman (2020) and Sinha et al. (2020). Some exponential estimators under study suggested by Singh et al. (2007), Grover and Kaur (2011), Solanki and Singh (2013) and Yadav and Adewara (2013) are given by

$$t_{SEA}^r = \bar{y} \exp \left(\frac{p-p}{(p+p)} \right), \quad [Singh\ et\ al.\ (2007)] \quad (10)$$

$$t_{SEA}^p = \bar{y} \exp \left(\frac{p-p}{(p+p)} \right), \quad [Singh\ et\ al.\ (2007)] \quad (11)$$

$$t_{SEA}^g = \bar{y} \left[\alpha \exp \left(\frac{p-p}{(p+p)} \right) + (1 - \alpha) \exp \left(\frac{p-p}{(p+p)} \right) \right], \quad [Singh\ et\ al.\ (2007)] \quad (12)$$

$$t_{GK} = [\alpha \bar{y} + \beta (P - p)] \exp \left(\frac{p-p}{(p+p)} \right), \quad [Grover\ and\ Kaur\ (2011)] \quad (13)$$

$$t_{SS} = \bar{y} \exp \left[\frac{\alpha_1 (p-p)}{(p+p)} \right] \quad [Solanki\ and\ Singh\ (2013)] \quad (14)$$

and
$$t_{YA} = k \bar{y} \exp \left(\frac{p-p}{(p+p)} \right). \quad [Yadav\ and\ Adewara\ (2013)] \quad (15)$$

The MSE of all the estimators $\bar{y}_{SEA}^r$, $\bar{y}_{SEA}^p$, $\bar{y}_{SEA}^g$, $\bar{y}_{GK}$ and $\bar{y}_{YA}$ can be defined as

$$MSE(t_{SEA}^r) = \lambda \bar{Y}^2 \left(C_y^2 - \rho_{y\psi} C_y C_\psi + \frac{C_\psi^2}{4} \right), \quad (16)$$

$$MSE(t_{SEA}^p) = \lambda \bar{Y}^2 \left(C_y^2 + \rho_{y\psi} C_y C_\psi + \frac{C_\psi^2}{4} \right), \quad (17)$$

$$[MSE(t_{SEA}^g)]_{min} = \lambda \bar{Y}^2 C_y^2 (1 - \rho_{y\psi}^2) \text{ at } \alpha = \rho_{y\psi} \frac{C_y}{C_\psi} + \frac{1}{2}, \quad (18)$$

$$[MSE(t_{GK})]_{min} = \frac{\lambda \bar{Y}^2 C_y^2 (1 - \rho_{y\psi}^2)}{1 + \lambda C_y^2 (1 - \rho_{y\psi}^2)} - \frac{\lambda^2 \bar{Y}^2 C_\psi^2 \left\{ 4 C_y^2 (1 - \rho_{y\psi}^2) + \frac{C_\psi^2}{4} \right\}}{16 [1 + \lambda C_y^2 (1 - \rho_{y\psi}^2)]}, \quad (19)$$

$$[MSE(t_{SS})]_{min} = \lambda \bar{Y}^2 C_y^2 (1 - \rho_{y\psi}^2) \text{ at } \alpha_1 = 2 \rho_{y\psi} \frac{C_y}{C_\psi} \quad (20)$$

$$\text{and} \quad [MSE(t_{YA})]_{min} \cong \bar{Y}^2 \left\{ 1 - \frac{A_1^2}{4B_1} \right\} \text{ when } k_{opt} = \frac{A_1}{2B_1}, \quad (21)$$

$$\text{where } A_1 = \lambda \left(\frac{C_\psi^2}{4} - \rho_{y\psi} C_y C_\psi \right) + 2, \quad B_1 = \lambda C_y^2 + 2\lambda \left(\frac{C_\psi^2}{4} - \rho_{y\psi} C_y C_\psi \right) + 1.$$

REMARK 1: Estimators t_{NG}^{reg} , t_{SS} and t_{SEA}^g are the members of t_f^c , therefore their minimum MSE s are equal to the $[MSE(t_f^c)]_{min}$.

The present manuscript is intended to propose regress exponential estimators for estimating population mean using auxiliary attribute, which do not belong to the class of estimators t_f^c although more efficient than all the relevant existing estimators. The properties of suggested estimators are studied and their efficiency is theoretically and empirically discussed.

2. THE PROPOSED ESTIMATORS

Following Naik and Gupta (1996), Solanki and Singh (2013) and Kadilar and Cingi (2006), the proposed regress exponential ratio and product estimators for estimating the population mean using auxiliary attribute are as follows-

$$t_{SB}^r = A_1 \bar{y} + B_1 (P - p) \exp \left\{ \frac{(\eta P - L) - (\eta p - L)}{(\eta P - L) + (\eta p - L)} \right\} \quad (22)$$

$$\text{and} \quad t_{SB}^p = A_2 \bar{y} + B_2 (p - P) \exp \left\{ \frac{(\eta p - L) - (\eta P - L)}{(\eta p - L) + (\eta P - L)} \right\}, \quad (23)$$

where η and L are suitably chosen known constants while A_1 , B_1 , A_2 and B_2 are unknown constants to be optimized to obtain optimum value of MSE of t_{SB}^r and t_{SB}^p .

Various members of proposed regress exponential ratio (t_{SB}^r) and product (t_{SB}^p) estimators are formed by taking different values of η and L [see Kadilar and Cingi (2006)], which are as follows:

Constants		Ratio type Estimators	Product type Estimators
η	L		
1	ρ	$t_{SB}^{r(1)} = A\bar{y} + B(P - p) \exp \left\{ \frac{(P - p)}{p + p - 2\rho} \right\}$	$t_{SB}^{p(1)} = A\bar{y} + B(p - P) \exp \left\{ \frac{(p - P)}{p + P - 2\rho} \right\}$
β_2	ρ	$t_{SB}^{r(2)} = A\bar{y} + B(P - p) \exp \left\{ \frac{\beta_2(P - p)}{\beta_2(P + p) - 2\rho} \right\}$	$t_{SB}^{p(2)} = A\bar{y} + B(p - P) \exp \left\{ \frac{\beta_2(p - P)}{\beta_2(p + P) - 2\rho} \right\}$
β_2	C_y	$t_{SB}^{r(3)} = A\bar{y} + B(P - p) \exp \left\{ \frac{\beta_2(P - p)}{\beta_2(P + p) - 2C_y} \right\}$	$t_{SB}^{p(3)} = A\bar{y} + B(p - P) \exp \left\{ \frac{\beta_2(p - P)}{\beta_2(p + P) - 2C_y} \right\}$
C_ψ	C_y	$t_{SB}^{r(4)} = A\bar{y} + B(P - p) \exp \left\{ \frac{C_\psi(P - p)}{C_\psi(P + p) - 2C_y} \right\}$	$t_{SB}^{p(4)} = A\bar{y} + B(p - P) \exp \left\{ \frac{C_\psi(p - P)}{C_\psi(p + P) - 2C_y} \right\}$
ρ	C_y	$t_{SB}^{r(5)} = A\bar{y} + B(P - p) \exp \left\{ \frac{\rho(P - p)}{\rho(P + p) - 2C_y} \right\}$	$t_{SB}^{p(5)} = A\bar{y} + B(p - P) \exp \left\{ \frac{\rho(p - P)}{\rho(p + P) - 2C_y} \right\}$
β_2	C_ψ	$t_{SB}^{r(6)} = A\bar{y} + B(P - p) \exp \left\{ \frac{\beta_2(P - p)}{\beta_2(P + p) - 2C_\psi} \right\}$	$t_{SB}^{p(6)} = A\bar{y} + B(p - P) \exp \left\{ \frac{\beta_2(p - P)}{\beta_2(p + P) - 2C_\psi} \right\}$

β_1	ρ	$t_{SB}^{r(7)} = A\bar{y} + B(P-p)\exp\left\{\frac{\beta_1(P-p)}{\beta_1(P+p)-2\rho}\right\}$	$t_{SB}^{p(7)} = A\bar{y} + B(p-P)\exp\left\{\frac{\beta_1(p-P)}{\beta_1(p+P)-2\rho}\right\}$
β_1	C_ψ	$t_{SB}^{r(8)} = A\bar{y} + B(P-p)\exp\left\{\frac{\beta_1(P-p)}{\beta_1(P+p)-2C_\psi}\right\}$	$t_{SB}^{p(8)} = A\bar{y} + B(p-P)\exp\left\{\frac{\beta_1(p-P)}{\beta_1(p+P)-2C_\psi}\right\}$
β_1	C_y	$t_{SB}^{r(9)} = A\bar{y} + B(P-p)\exp\left\{\frac{\beta_1(P-p)}{\beta_1(P+p)-2C_y}\right\}$	$t_{SB}^{p(9)} = A\bar{y} + B(p-P)\exp\left\{\frac{\beta_1(p-P)}{\beta_1(p+P)-2C_y}\right\}$

To derive bias and mean square error (*MSE*) of suggested estimators, following terms are assumed under large sample approximation as-

$$\bar{y} = \bar{Y}(1 + \epsilon_0), \quad p = P(1 + \epsilon_1)$$

$$\text{such that } E(\epsilon_0) = E(\epsilon_1) = 0, \quad E(\epsilon_0^2) = \lambda C_y^2, \quad E(\epsilon_1^2) = \lambda C_\psi^2, \quad E(\epsilon_0 \epsilon_1) = \lambda \rho_{y\psi} C_y C_\psi,$$

where $\lambda = \frac{1}{n} - \frac{1}{N}$, $C_y = \frac{S_y}{\bar{Y}}$, $C_\psi = \frac{S_\psi}{P}$ and $\rho_{y\psi}$ is the coefficient of point bi-serial correlation between y and ψ .

Using these approximations and neglecting second and higher order terms of ϵ_i ; $i = 0, 1$, the proposed estimators t_{SB}^r and t_{SB}^p can be expressed as-

$$t_{SB}^r = A_1 \bar{Y}(1 + \epsilon_0) - B_1 P \left(\epsilon_1 - \frac{\eta P}{2(\eta P - L)} \epsilon_1^2 \right) \quad (24)$$

$$\text{and } t_{SB}^p = A_2 \bar{Y}(1 + \epsilon_0) + B_2 P \left(\epsilon_1 + \frac{\eta P}{2(\eta P - L)} \epsilon_1^2 \right). \quad (25)$$

On taking expectation both sides and subtracting $\bar{Y}$, the bias of the estimators t_{SB}^r and t_{SB}^p up to the $O(n^{-1})$ is given by-

$$B(t_{SB}^r) = (A_1 - 1)\bar{Y} + B_1 P \theta \lambda C_\psi^2 \quad (26)$$

$$\text{and } B(t_{SB}^p) = (A_2 - 1)\bar{Y} + B_2 P \theta \lambda C_\psi^2, \quad (27)$$

$$\text{where } \theta = \frac{\eta P}{2(\eta P - L)}.$$

Further the mean square error of estimators t_{SB}^r and t_{SB}^p is given by

$$MSE(t_{SB}^r) = E[t_{SB}^r - \bar{Y}]^2$$

$$\text{and } MSE(t_{SB}^p) = E[t_{SB}^p - \bar{Y}]^2.$$

Substituting the value of t_{SB}^r and t_{SB}^p from (24) and (25), we have

$$MSE(t_{SB}^r) = E \left[\left\{ A_1 \bar{Y}(1 + \epsilon_0) - B_1 P \left(\epsilon_1 - \frac{\eta P}{2(\eta P - L)} \epsilon_1^2 \right) \right\} - \bar{Y} \right]^2$$

$$\text{and } MSE(t_{SB}^p) = E \left[\left\{ A_2 \bar{Y}(1 + \epsilon_0) + B_2 P \left(\epsilon_1 + \frac{\eta P}{2(\eta P - L)} \epsilon_1^2 \right) \right\} - \bar{Y} \right]^2.$$

After solving, the *MSE* of estimators t_{SB}^r and t_{SB}^p up to the $O(n^{-1})$ is given by

$$MSE(t_{SB}^r) = (A_1 - 1)^2 \bar{Y}^2 + \lambda A_1^2 \bar{Y}^2 C_y^2 + \lambda B_1^2 P^2 C_\psi^2 + 2\lambda A_1 B_1 P \bar{Y} \theta C_\psi^2 - 2\lambda B_1 P \bar{Y} \theta C_\psi^2 - 2\lambda A_1 B_1 P \bar{Y} \rho_{y\psi} C_y C_\psi \quad (28)$$

$$\text{and } MSE(t_{SB}^p) = (A_2 - 1)^2 \bar{Y}^2 + \lambda A_2^2 \bar{Y}^2 C_y^2 + \lambda B_2^2 P^2 C_\psi^2 + 2\lambda A_2 B_2 P \bar{Y} \theta C_\psi^2 - 2\lambda B_2 P \bar{Y} \theta C_\psi^2 + 2\lambda A_2 B_2 P \bar{Y} \rho_{y\psi} C_y C_\psi. \quad (29)$$

Partially differentiate (28) with respect to (A_1, B_1) and (29) with respect to (A_2, B_2) and equating them to zero, we get

$$A_{1opt} = \frac{1 - \lambda\theta^2 C_{\psi}^2 + \lambda\theta \rho_{y\psi} C_y C_{\psi}}{1 - \lambda\theta^2 C_{\psi}^2 + 2\lambda\theta \rho_{y\psi} C_y C_{\psi} + \lambda C_y^2 - \lambda \rho_{y\psi}^2 C_y^2}, \quad (30)$$

$$B_{1opt} = \frac{\bar{Y} C_y (\rho_{y\psi} + \lambda\theta C_y C_{\psi})}{P C_p (1 - \lambda\theta^2 C_{\psi}^2 + 2\lambda\theta \rho_{y\psi} C_y C_{\psi} + \lambda C_y^2 - \lambda \rho_{y\psi}^2 C_y^2)}, \quad (31)$$

$$A_{2opt} = \frac{1 - \lambda\theta^2 C_{\psi}^2 - \lambda\theta \rho_{y\psi} C_y C_{\psi}}{1 - \lambda\theta^2 C_{\psi}^2 - 2\lambda\theta \rho_{y\psi} C_y C_{\psi} + \lambda C_y^2 - \lambda \rho_{y\psi}^2 C_y^2} \quad (32)$$

and

$$B_{2opt} = \frac{\bar{Y} C_y (-\rho_{y\psi} + \lambda\theta C_y C_{\psi})}{P C_p (1 - \lambda\theta^2 C_{\psi}^2 - 2\lambda\theta \rho_{y\psi} C_y C_{\psi} + \lambda C_y^2 - \lambda \rho_{y\psi}^2 C_y^2)}. \quad (33)$$

Substituting the value of A_{1opt} and B_{1opt} from (30) and (31) in (28) and A_{2opt} and B_{2opt} from (32) and (33) in (29), we get the minimum MSE of t_{SB}^r and t_{SB}^p as

$$[MSE(t_{SB}^r)]_{min} = \frac{\bar{Y}^2 [\lambda C_y^2 (1 - \rho_{y\psi}^2) - \lambda^2 \theta^2 C_{\psi}^2 C_y^2]}{1 - \lambda\theta^2 C_{\psi}^2 + 2\lambda\theta \rho_{y\psi} C_y C_{\psi} + \lambda C_y^2 - \lambda \rho_{y\psi}^2 C_y^2} \quad (34)$$

and

$$[MSE(t_{SB}^p)]_{min} = \frac{\bar{Y}^2 [\lambda C_y^2 (1 - \rho_{y\psi}^2) - \lambda^2 \theta^2 C_{\psi}^2 C_y^2]}{1 - \lambda\theta^2 C_{\psi}^2 - 2\lambda\theta \rho_{y\psi} C_y C_{\psi} + \lambda C_y^2 - \lambda \rho_{y\psi}^2 C_y^2}, \quad (35)$$

where $\theta = \frac{\eta^P}{2(\eta^P - L)}$.

REMARK 2: The proposed estimators t_{SB}^r and t_{SB}^p will attain minimum mean square error if corresponding value of the constants A_{iopt} and B_{iopt} , $i = 1, 2$ are calculated by (30)-(33). The optimum values of constants mentioned in (30)-(33) involve some unknown parameters, therefore past data or the estimated values of the unknown parameters may be utilized to obtain their values. Srivastava and Jhaji (1983) have shown that such values do not influence the (MSE) of the estimator up-to $O(n^{-1})$ while Reddy (1978) has shown that such values are stable overtime and region.

3. EFFICIENCY COMPARISONS

If we compare the mean square errors of proposed estimators t_{SB}^r and t_{SB}^p with the relevant estimators, we find for

(a) Ratio type estimator

(i) from (34) and (7)

$$[MSE(t_{SB}^r)]_{min} \leq MSE(\bar{y})$$

$$\text{if } \lambda C_y^2 - \lambda \rho_{y\psi}^2 C_y^2 + \rho_{y\psi}^2 + 2\lambda\theta \rho_{y\psi} C_y C_{\psi} \geq 0$$

(ii) from (34) and (4)

$$[MSE(t_{SB}^r)]_{min} \leq MSE(t_{NG}^r)$$

$$\text{if } (C_{\psi} - \rho_{y\psi} C_y)^2 - \lambda\theta^2 C_{\psi}^3 (C_{\psi} - 2\rho_{y\psi} C_y) + 2\lambda\theta \rho_{y\psi} C_y C_{\psi} G^- + \lambda C_y^2 (1 - \rho_{y\psi}^2) G^- \geq 0$$

$$\text{where } G^- = C_y^2 + C_{\psi}^2 - 2\rho_{y\psi} C_y C_{\psi}$$

(iii) from (34) and (5)

$$[MSE(t_{SB}^r)]_{min} \leq MSE(t_{NG}^p)$$

$$\text{if } (C_{\psi} + \rho_{y\psi} C_y)^2 - \lambda\theta^2 C_{\psi}^3 (C_{\psi} + 2\rho_{y\psi} C_y) + 2\lambda\theta \rho_{y\psi} C_y C_{\psi} G^+ + \lambda C_y^2 (1 - \rho_{y\psi}^2) G^+ \geq 0$$

where $G^+ = C_y^2 + C_\psi^2 + 2\rho_{y\psi} C_y C_\psi$

(iv) from (34) and (6)

$$[MSE(t_{SB}^r)]_{min} \leq MSE(t_{NG}^{reg})$$

$$\text{if } \lambda C_y^2 (1 - \rho_{y\psi}^2)^2 + \lambda \theta^2 C_\psi^2 + \lambda \theta C_\psi (1 - \rho_{y\psi}^2) (2\rho_{y\psi} C_y - \theta C_\psi) \geq 0$$

(v) from (34) and (9)

$$[MSE(t_{SB}^r)]_{min} \leq MSE(t_f^c)$$

$$\text{if } \lambda C_y^2 (1 - \rho_{y\psi}^2)^2 + \lambda \theta^2 C_\psi^2 + \lambda \theta C_\psi (1 - \rho_{y\psi}^2) (2\rho_{y\psi} C_y - \theta C_\psi) \geq 0$$

(vi) from (34) and (16)

$$[MSE(t_{SB}^r)]_{min} \leq MSE(t_{SEA}^r)$$

$$\text{if } \left(\frac{C_\psi}{2} - \rho_{y\psi} C_y \right)^2 - \lambda \theta^2 C_\psi^3 \left(\frac{C_\psi}{4} - \rho_{y\psi} C_y \right) + 2\lambda \theta \rho_{y\psi} C_y C_\psi \mathcal{F}^- + \lambda C_y^2 (1 - \rho_{y\psi}^2) \mathcal{F}^- \geq 0$$

$$\text{where } \mathcal{F}^- = \left(C_y^2 + \frac{C_\psi^2}{4} - \rho_{y\psi} C_y C_\psi \right)$$

(vii) from (34) and (17)

$$[MSE(t_{SB}^r)]_{min} \leq MSE(t_{SEA}^p)$$

$$\text{if } \left(\frac{C_\psi}{2} + \rho_{y\psi} C_y \right)^2 - \lambda \theta^2 C_\psi^3 \left(\frac{C_\psi}{4} + \rho_{y\psi} C_y \right) + 2\lambda \theta \rho_{y\psi} C_y C_\psi \mathcal{F}^+ + \lambda C_y^2 (1 - \rho_{y\psi}^2) \mathcal{F}^+ \geq 0$$

$$\text{where } \mathcal{F}^+ = \left(C_y^2 + \frac{C_\psi^2}{4} + \rho_{y\psi} C_y C_\psi \right)$$

(viii) from (34) and (18)

$$[MSE(t_{SB}^r)]_{min} \leq MSE(t_{SEA}^g)$$

$$\text{if } \lambda C_y^2 (1 - \rho_{y\psi}^2)^2 + \lambda \theta^2 C_\psi^2 + \lambda \theta C_\psi (1 - \rho_{y\psi}^2) (2\rho_{y\psi} C_y - \theta C_\psi) \geq 0$$

(ix) from (34) and (19)

$$[MSE(t_{SB}^r)]_{min} \leq [MSE(t_{GK})]_{min}$$

$$\text{if } \left(\lambda C_y^2 C_\psi^2 \left(16\lambda \theta^2 C_y^2 - 4 - 4\lambda \theta^2 C_\psi^2 - \frac{\lambda C_\psi^2}{4} \right) + 8\lambda \theta \rho_{y\psi} C_y^3 C_\psi (4 - \lambda C_\psi^2) \right) (1 - \rho_{y\psi}^2) \\ + \lambda \theta^2 C_\psi^2 \left(16\rho_{y\psi}^2 C_y^2 + \frac{C_\psi^4}{4} \right) - \frac{\lambda^2 \theta \rho_{y\psi} C_y C_\psi^5}{2} - \frac{\lambda C_\psi^4}{4} - 4\lambda^2 C_y^4 C_\psi^2 (1 - \rho_{y\psi}^2)^2 \geq 0$$

(x) from (34) and (20)

$$[MSE(t_{SB}^r)]_{min} \leq [MSE(t_{SS})]_{min}$$

$$\text{if } \lambda C_y^2 (1 - \rho_{y\psi}^2)^2 + \lambda \theta^2 C_\psi^2 + \lambda \theta C_\psi (1 - \rho_{y\psi}^2) (2\rho_{y\psi} C_y - \theta C_\psi) \geq 0$$

(xi) from (34) and (21)

$$[MSE(t_{SB}^r)]_{min} \leq [MSE(t_{YA})]_{min}$$

$$\text{if } \theta^2 \left[\lambda^2 C_y^2 C_\psi^2 (2 + 4C_y^2 + \rho_{y\psi}^2 C_\psi^2) + \lambda \rho_{y\psi} C_y C_\psi^3 \left(4 - 8\lambda C_y^2 - \frac{\lambda C_\psi^2}{2} \right) - \lambda C_\psi^4 \left(1 - \frac{\lambda C_\psi^2}{16} \right) \right] \\ + \theta \left[\lambda \rho_{y\psi}^2 C_y^2 C_\psi^2 (C_y^2 - 8) + \lambda \rho_{y\psi} C_y C_\psi \left(8C_y^2 + 2C_\psi^2 - \frac{\lambda C_\psi^4}{8} - 2\lambda \rho_{y\psi}^2 C_y^2 C_\psi^2 \right) \right] \\ + \left[\rho_{y\psi}^2 C_y^2 (4 - \lambda C_\psi^2) + C_\psi^2 \left(1 - \frac{\lambda C_\psi^2}{16} \right) - \rho_{y\psi} C_y C_\psi \left(4 - \frac{\lambda C_\psi^2}{2} \right) \right]$$

$$+ \left\{ \lambda \rho_{y\psi} C_y^3 C_\psi \left(4 + \frac{\lambda C_\psi^2}{2} \right) - \lambda C_y^2 C_\psi^2 \left(1 + \frac{\lambda C_\psi^2}{16} + \lambda \rho_{y\psi}^2 C_y^2 \right) \right\} \{ 1 - \rho_{y\psi}^2 \} \geq 0$$

(b) Product type estimator

(i) from (35) and (7)

$$[MSE(t_{SB}^p)]_{min} \leq MSE(\bar{y})$$

$$\text{if } \lambda C_y^2 - \lambda \rho_{y\psi}^2 C_y^2 + \rho_{y\psi}^2 - 2\lambda\theta \rho_{y\psi} C_y C_\psi \geq 0$$

(ii) from (35) and (4)

$$[MSE(t_{SB}^p)]_{min} \leq MSE(t_{NG}^r)$$

$$\text{if } (C_\psi - \rho_{y\psi} C_y)^2 - \lambda\theta^2 C_\psi^3 (C_\psi - 2\rho_{y\psi} C_y) - 2\lambda\theta \rho_{y\psi} C_y C_\psi G^- + \lambda C_y^2 (1 - \rho_{y\psi}^2) G^- \geq 0$$

$$\text{where } G^- = C_y^2 + C_\psi^2 - 2\rho_{y\psi} C_y C_\psi$$

(iii) from (35) and (5)

$$[MSE(t_{SB}^p)]_{min} \leq MSE(t_{NG}^p)$$

$$\text{if } (C_\psi + \rho_{y\psi} C_y)^2 - \lambda\theta^2 C_\psi^3 (C_\psi + 2\rho_{y\psi} C_y) - 2\lambda\theta \rho_{y\psi} C_y C_\psi G^+ + \lambda C_y^2 (1 - \rho_{y\psi}^2) G^+ \geq 0$$

$$\text{where } G^+ = C_y^2 + C_\psi^2 + 2\rho_{y\psi} C_y C_\psi$$

(iv) from (35) and (6)

$$[MSE(t_{SB}^p)]_{min} \leq MSE(t_{NG}^{reg})$$

$$\text{if } \lambda C_y^2 (1 - \rho_{y\psi}^2)^2 + \lambda\theta^2 C_\psi^2 + \lambda\theta C_\psi (1 - \rho_{y\psi}^2) (-2\rho_{y\psi} C_y - \theta C_\psi) \geq 0$$

(v) from (35) and (9)

$$[MSE(t_{SB}^p)]_{min} \leq MSE(t_f^c)$$

$$\text{if } \lambda C_y^2 (1 - \rho_{y\psi}^2)^2 + \lambda\theta^2 C_\psi^2 + \lambda\theta C_\psi (1 - \rho_{y\psi}^2) (-2\rho_{y\psi} C_y - \theta C_\psi) \geq 0$$

(vi) from (35) and (16)

$$[MSE(t_{SB}^p)]_{min} \leq MSE(t_{SEA}^r)$$

$$\text{if } \left(\frac{C_\psi}{2} - \rho_{y\psi} C_y \right)^2 - \lambda\theta^2 C_\psi^3 \left(\frac{C_\psi}{4} - \rho_{y\psi} C_y \right) - 2\lambda\theta \rho_{y\psi} C_y C_\psi \mathcal{F}^- + \lambda C_y^2 (1 - \rho_{y\psi}^2) \mathcal{F}^- \geq 0$$

$$\text{where } \mathcal{F}^- = \left(C_y^2 + \frac{C_\psi^2}{4} - \rho_{y\psi} C_y C_\psi \right)$$

(vii) from (35) and (17)

$$[MSE(t_{SB}^p)]_{min} \leq MSE(t_{SEA}^p)$$

$$\text{if } \left(\frac{C_\psi}{2} + \rho_{y\psi} C_y \right)^2 - \lambda\theta^2 C_\psi^3 \left(\frac{C_\psi}{4} + \rho_{y\psi} C_y \right) - 2\lambda\theta \rho_{y\psi} C_y C_\psi \mathcal{F}^+ + \lambda C_y^2 (1 - \rho_{y\psi}^2) \mathcal{F}^+ \geq 0$$

$$\text{where } \mathcal{F}^+ = \left(C_y^2 + \frac{C_\psi^2}{4} + \rho_{y\psi} C_y C_\psi \right)$$

(viii) from (35) and (18)

$$[MSE(t_{SB}^p)]_{min} \leq [MSE(t_{SEA}^g)]_{min}$$

$$\text{if } \lambda C_y^2 (1 - \rho_{y\psi}^2)^2 + \lambda\theta^2 C_\psi^2 + \lambda\theta C_\psi (1 - \rho_{y\psi}^2) (-2\rho_{y\psi} C_y - \theta C_\psi) \geq 0$$

(ix) from (35) and (19)

$$[MSE(t_{SB}^p)]_{min} \leq [MSE(t_{GK})]_{min}$$

$$\text{if } \left(\lambda C_y^2 C_\psi^2 \left(16\lambda\theta^2 C_y^2 - 4 - 4\lambda\theta^2 C_\psi^2 - \frac{\lambda C_\psi^2}{4} \right) - 8\lambda\theta \rho_{y\psi} C_y^3 C_\psi (4 - \lambda C_\psi^2) \right) (1 - \rho_{y\psi}^2) \\ + \lambda\theta^2 C_\psi^2 \left(16\rho_{y\psi}^2 C_y^2 + \frac{C_\psi^4}{4} \right) + \frac{\lambda^2 \theta \rho_{y\psi} C_y C_\psi^5}{2} - \frac{\lambda C_\psi^4}{4} - 4\lambda^2 C_y^4 C_\psi^2 (1 - \rho_{y\psi}^2)^2 \geq 0$$

(x) from (35) and (20)

$$[MSE(t_{SB}^p)]_{min} \leq [MSE(t_{SS})]_{min}$$

$$\text{if } \lambda C_y^2 (1 - \rho_{y\psi}^2)^2 + \lambda\theta^2 C_\psi^2 + \lambda\theta C_\psi (1 - \rho_{y\psi}^2) (-2\rho_{y\psi} C_y - \theta C_\psi) \geq 0$$

(xi) from (35) and (21)

$$[MSE(t_{SB}^p)]_{min} \leq [MSE(t_{YA})]_{min}$$

$$\text{if } \theta^2 \left[\lambda^2 C_y^2 C_\psi^2 (2 + 4C_y^2 + \rho_{y\psi}^2 C_\psi^2) + \lambda\rho_{y\psi} C_y C_\psi^3 \left(4 - 8\lambda C_y^2 - \frac{\lambda C_\psi^2}{2} \right) - \lambda C_\psi^4 \left(1 - \frac{\lambda C_\psi^2}{16} \right) \right] \\ - \theta \left[\lambda\rho_{y\psi}^2 C_y^2 C_\psi^2 (\lambda C_\psi^2 - 8) + \lambda\rho_{y\psi} C_y C_\psi \left(8C_y^2 + 2C_\psi^2 - \frac{\lambda C_\psi^4}{8} - 2\lambda\rho_{y\psi}^2 C_y^2 C_\psi^2 \right) \right] \\ + \left[\rho_{y\psi}^2 C_y^2 (4 - \lambda C_\psi^2) + C_\psi^2 \left(1 - \frac{\lambda C_\psi^2}{16} \right) - \rho_{y\psi} C_y C_\psi \left(4 - \frac{\lambda C_\psi^2}{2} \right) \right] \\ + \left\{ \lambda\rho_{y\psi} C_y^3 C_\psi \left(4 + \frac{\lambda C_\psi^2}{2} \right) - \lambda C_y^2 C_\psi^2 \left(1 + \frac{\lambda C_\psi^2}{16} + \lambda\rho_{y\psi}^2 C_y^2 \right) \right\} \{1 - \rho_{y\psi}^2\} \geq 0$$

4. EMPIRICAL STUDY

To illustrate the performance of the proposed regress exponential ratio and product estimators along with the relevant estimators, two data sets on number of workers, fixed capital and output for 80 factories in a region are considered [Source: Murthy (1967), pg-228].

Data set I

y : output x : number of workers

ψ : number of workers greater ≥ 85

Parameters:

$$N = 80 \quad \bar{Y} = 5182.6 \quad P = 0.763 \quad \rho_{y\psi} = 0.675 \quad \beta_1 = 1.522$$

$$n = 20 \quad S_y = 1835.66 \quad S_\psi = 0.42824 \quad \beta_2 = 2.522$$

Data set II

y : output x : fixed capital

ψ : fixed capital > 510

Parameters:

$$N = 80 \quad \bar{Y} = 5182.6 \quad P = 0.75 \quad \rho_{y\psi} = 0.689 \quad \beta_1 = 1.333$$

$$n = 20 \quad S_y = 1835.66 \quad S_\psi = 0.436 \quad \beta_2 = 2.333$$

To show the efficiency of the proposed estimators, minimum mean square errors (MSE) and percentage relative efficiency (PRE) are calculated. The minimum MSE and PRE of proposed and existing estimators for data set I and II are respectively shown in tables 1 and 2. Percentage relative efficiency

(PRE) of the existing (T_{ex}) and proposed (T_{pro}) estimators with respect to conventional mean per unit estimator ($\bar{y}$) is calculated by formula $PRE(T_{ex/pro}) = \frac{V(\bar{y})}{[MSE(T_{ex/pro})]_{min}} \times 100$.

Table 1: Minimum MSE and PRE of proposed and existing estimators for data set I

Estimators	Data set I		
	Constants	MSEs	PRE
$\bar{y}$		126362.0	(100%)
$t_{NG}^r = \bar{y} \left(\frac{P}{p} \right)$		173574.0	(72.8%)
$t_{NG}^p = \bar{y} \left(\frac{p}{P} \right)$		714555.0	(17.7%)
$t_{NG}^{reg} = \bar{y} + \beta(p - P)$	$\hat{\beta}_{opt} = 2893.4$	68788.1	(183.7%)
$t_f^c = f \left(\bar{y}, \frac{p}{P} \right)$	$f^{(2)}_{opt} = -2206.22$	68788.1	(183.7%)
$t_{SEA}^r = \bar{y} \exp \left(\frac{(P - p)}{(P + p)} \right)$		70542.1	(179.1%)
$t_{SEA}^p = \bar{y} \exp \left(\frac{(p - P)}{(p + P)} \right)$		341033.0	(37.0%)
$t_{SEA}^g = \bar{y} \left[\alpha \exp \left(\frac{(P - p)}{(P + p)} \right) + (1 - \alpha) \exp \left(\frac{(p - P)}{(p + P)} \right) \right]$	$\alpha_{opt} = 0.9257$	68788.1	(183.7%)
$t_{GK} = [\alpha \bar{y} + \beta(P - p)] \exp \left(\frac{(P - p)}{(P + p)} \right)$	$\alpha_{opt} = 0.9960$ $\beta_{opt} = -489.30$	68350.9	(184.9%)
$t_{SS} = \bar{y} \exp \left[\frac{\alpha_1(P - p)}{(P + p)} \right]$	$\alpha_{1opt} = 0.8514$	68788.1	(183.7%)
$t_{YA} = k \bar{y} \exp \left(\frac{(P - p)}{(P + p)} \right)$	$k_{opt} = 0.9984$	70474.5	(179.3%)
$t_{SB}^{r(1)} = A \bar{y} + B(P - p) \exp \left[\frac{(P - p)}{P + p - 2\rho} \right]$	$A = 0.9702,$ $B = 3689.96$	49171.2	(257.0%)
$t_{SB}^{r(2)} = A \bar{y} + B(P - p) \exp \left[\frac{\beta_2(P - p)}{\beta_2(P + p) - 2\rho} \right]$	$A = 0.9936$ $B = 2908.44$	67677.7	(186.7%)
$t_{SB}^{r(3)} = A \bar{y} + B(P - p) \exp \left[\frac{\beta_2(P - p)}{\beta_2(P + p) - 2C_y} \right]$	$A = 0.9944$ $B = 2900.55$	67935.3	(186.0%)
$t_{SB}^{r(4)} = A \bar{y} + B(P - p) \exp \left[\frac{C_p(P - p)}{C_p(P + p) - 2C_y} \right]$	$A = 0.9818$ $B = 3197.16$	60438.2	(209.1%)
$t_{SB}^{r(5)} = A \bar{y} + B(P - p) \exp \left[\frac{\rho(P - p)}{\rho(P + p) - 2C_y} \right]$	$A = 0.9892$ $B = 2979.31$	65720.1	(192.3%)
$t_{SB}^{r(6)} = A \bar{y} + B(p - P) \exp \left[\frac{\beta_2(p - P)}{\beta_2(p + P) - 2C_p} \right]$	$A = 0.9939$ $B = 2905.03$	67786.4	(186.4%)
$t_{SB}^{r(7)} = A \bar{y} + B(p - P) \exp \left[\frac{\beta_1(p - P)}{\beta_1(p + P) - 2\rho} \right]$	$A = 0.9911$ $B = 2949.98$	64277.0	(196.6%)
$t_{SB}^{r(8)} = A \bar{y} + B(p - P) \exp \left[\frac{\beta_1(p - P)}{\beta_1(p + P) - 2C_p} \right]$	$A = 0.9924$ $B = 2931.51$	64814.4	(195.0%)
$t_{SB}^{r(9)} = A \bar{y} + B(p - P) \exp \left[\frac{\beta_1(p - P)}{\beta_1(p + P) - 2C_y} \right]$	$A = 0.9937$ $B = 2915.56$	65313.9	(193.5%)
$t_{SB}^{p(1)} = A \bar{y} + B(p - P) \exp \left[\frac{(p - P)}{p + P - 2\rho} \right]$	$A = 1.0264$ $B = -3751.50$	55049.1	(229.5%)
$t_{SB}^{p(2)} = A \bar{y} + B(p - P) \exp \left[\frac{\beta_2(p - P)}{\beta_2(p + P) - 2\rho} \right]$	$A = 1.0013$ $B = -2904.25$	68740.8	(183.8%)
$t_{SB}^{p(3)} = A \bar{y} + B(p - P) \exp \left[\frac{\beta_2(p - P)}{\beta_2(p + P) - 2C_y} \right]$	$A = 1.0005$ $B = -2897.13$	68780.6	(183.7%)

$t_{SB}^{p(4)} = A\bar{y} + B(p - P)\exp\left[\frac{C_p(p - P)}{C_p(p + P) - 2C_y}\right]$	$A = 1.0136$ $B = -3198.61$	64434.0	(196.1%)
$t_{SB}^{p(5)} = A\bar{y} + B(p - P)\exp\left[\frac{\rho(p - P)}{\rho(p + P) - 2C_y}\right]$	$A = 1.0058$ $B = -2972.71$	67937.4	(186.0%)
$t_{SB}^{p(6)} = A\bar{y} + B(p - P)\exp\left[\frac{\beta_2(p - P)}{\beta_2(p + P) - 2C_p}\right]$	$A = 1.0010$ $B = -2901.15$	68761.1	(183.8%)
$t_{SB}^{p(7)} = A\bar{y} + B(p - P)\exp\left[\frac{\beta_1(p - P)}{\beta_1(p + P) - 2\rho}\right]$	$A = 1.004$ $B = -2947.18$	65958.3	(191.6%)
$t_{SB}^{p(8)} = A\bar{y} + B(p - P)\exp\left[\frac{\beta_1(p - P)}{\beta_1(p + P) - 2C_p}\right]$	$A = 1.0027$ $B = -2928.86$	66177.8	(190.9%)
$t_{SB}^{p(9)} = A\bar{y} + B(p - P)\exp\left[\frac{\beta_1(p - P)}{\beta_1(p + P) - 2C_y}\right]$	$A = 1.0014$ $B = -2913.37$	66325.2	(190.5%)

Table 2: Minimum MSE and PRE of proposed and existing estimators for data set II

Estimators	Data set II		
	Constants	MSEs	PRE
$\bar{y}$		126362.0	(100%)
$t_{NG}^r = \bar{y}\left(\frac{P}{p}\right)$		180728.0	(69.9%)
$t_{NG}^p = \bar{y}\left(\frac{p}{P}\right)$		751964.0	(16.8%)
$t_{NG}^{reg} = \bar{y} + \hat{\beta}(p - P)$	$\hat{\beta}_{opt} = 2902.58$	66375.1	(190.4%)
$t_f^f = f\left(\bar{y}, \frac{p}{P}\right)$	$f^{(2)}_{opt} = -2176.93$	66375.1	(190.4%)
$t_{SEA}^r = \bar{y}\exp\left(\frac{P - p}{P + p}\right)$		68548.5	(184.3%)
$t_{SEA}^p = \bar{y}\exp\left(\frac{p - P}{p + P}\right)$		354167.0	(35.7%)
$t_{SEA}^g = \bar{y}\left[\alpha\exp\left(\frac{P - p}{P + p}\right) + (1 - \alpha)\exp\left(\frac{p - P}{p + P}\right)\right]$	$\alpha_{opt} = 0.92$	66375.1	(190.4%)
$t_{GK} = [\alpha\bar{y} + \beta(P - p)]\exp\left(\frac{P - p}{P + p}\right)$	$\alpha_{opt} = 0.9960$ $\beta_{opt} = -536.29$	65934.9	(191.6%)
$t_{SS} = \bar{y}\exp\left[\frac{\alpha_1(P - p)}{(P + p)}\right]$	$\alpha_{1opt} = 0.84$	66375.1	(190.4%)
$t_{YA} = k\bar{y}\exp\left(\frac{P - p}{P + p}\right)$	$k_{opt} = 0.9985$	68490.0	(184.5%)
$t_{SB}^{r(1)} = A\bar{y} + B(P - p)\exp\left[\frac{(P - p)}{P + p - 2\rho}\right]$	$A = 0.9404$ $B = 5263.02$	10056.3	(1256.5%)
$t_{SB}^{r(2)} = A\bar{y} + B(P - p)\exp\left[\frac{\beta_2(P - p)}{\beta_2(P + p) - 2\rho}\right]$	$A = 0.9932$ $B = 2921.7$	65115.7	(194.1%)
$t_{SB}^{r(3)} = A\bar{y} + B(P - p)\exp\left[\frac{\beta_2(P - p)}{\beta_2(P + p) - 2C_y}\right]$	$A = 0.9942$ $B = 2910.83$	65474.1	(193.0%)
$t_{SB}^{r(4)} = A\bar{y} + B(P - p)\exp\left[\frac{C_p(P - p)}{C_p(P + p) - 2C_y}\right]$	$A = 0.9823$ $B = 3176.96$	58469.6	(216.1%)
$t_{SB}^{r(5)} = A\bar{y} + B(P - p)\exp\left[\frac{\rho(P - p)}{\rho(P + p) - 2C_y}\right]$	$A = 0.9889$ $B = 2991.74$	63136.0	(200.1%)
$t_{SB}^{r(6)} = A\bar{y} + B(p - P)\exp\left[\frac{\beta_2(p - P)}{\beta_2(p + P) - 2C_p}\right]$	$A = 0.9936$ $B = 2917.17$	65260.9	(186.4%)
$t_{SB}^{r(7)} = A\bar{y} + B(p - P)\exp\left[\frac{\beta_1(p - P)}{\beta_1(p + P) - 2\rho}\right]$	$A = 0.9888$ $B = 2994.33$	63067.3	(190.4%)

$t_{SB}^{r(8)} = A\bar{y} + B(p - P)\exp\left[\frac{\beta_1(p - P)}{\beta_1(p + P) - 2C_p}\right]$	$A = 0.9912$ $B = 2949.85$	64280.6	(195.0%)
$t_{SB}^{r(9)} = A\bar{y} + B(p - P)\exp\left[\frac{\beta_1(p - P)}{\beta_1(p + P) - 2C_y}\right]$	$A = 0.9934$ $B = 2918.64$	65213.3	(193.5%)
$t_{SB}^{p(1)} = A\bar{y} + B(p - P)\exp\left[\frac{(p - P)}{p + P - 2\rho}\right]$	$A = 1.0659$ $B = -5891.72$	12922.4	(977.8%)
$t_{SB}^{p(2)} = A\bar{y} + B(p - P)\exp\left[\frac{\beta_2(p - P)}{\beta_2(p + P) - 2\rho}\right]$	$A = 1.0019$ $B = -2919.29$	66275.2	(190.7%)
$t_{SB}^{p(3)} = A\bar{y} + B(p - P)\exp\left[\frac{\beta_2(p - P)}{\beta_2(p + P) - 2C_y}\right]$	$A = 1.0009$ $B = -2908.87$	66355.0	(190.4%)
$t_{SB}^{p(4)} = A\bar{y} + B(p - P)\exp\left[\frac{C_p(p - P)}{C_p(p + P) - 2C_y}\right]$	$A = 1.0132$ $B = -3184.64$	62217.6	(203.1%)
$t_{SB}^{p(5)} = A\bar{y} + B(p - P)\exp\left[\frac{\rho(p - P)}{\rho(p + P) - 2C_y}\right]$	$A = 1.0063$ $B = -2989.42$	65371.3	(193.3%)
$t_{SB}^{p(6)} = A\bar{y} + B(p - P)\exp\left[\frac{\beta_2(p - P)}{\beta_2(p + P) - 2C_p}\right]$	$A = 1.0015$ $B = -2914.92$	66313.1	(190.6%)
$t_{SB}^{p(7)} = A\bar{y} + B(p - P)\exp\left[\frac{\beta_1(p - P)}{\beta_1(p + P) - 2\rho}\right]$	$A = 1.0064$ $B = -2992.07$	65332.3	(193.4%)
$t_{SB}^{p(8)} = A\bar{y} + B(p - P)\exp\left[\frac{\beta_1(p - P)}{\beta_1(p + P) - 2C_p}\right]$	$A = 1.0040$ $B = -2947.05$	65959.9	(191.6%)
$t_{SB}^{p(9)} = A\bar{y} + B(p - P)\exp\left[\frac{\beta_1(p - P)}{\beta_1(p + P) - 2C_y}\right]$	$A = 1.0017$ $B = -2916.33$	66301.5	(190.6%)

5. CONCLUSION

Tables 1 and 2 exhibit that all the members of proposed regress exponential ratio and product estimators $t_{SB}^{r(i)}$ and $t_{SB}^{p(i)}$, ($i = 1, 2, \dots, 9$) are more efficient than the conventional as well as all the relevant estimators. Therefore, the proposed estimators $t_{SB}^{r(i)}$ and $t_{SB}^{p(i)}$ may be recommended upon the availability of the population parameters $(\rho, \beta_2, C_p, C_y, \beta_1)$.

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