

Non-Separable Raw Material Inventory Model with Time Dependent Deterioration: Hybrid Algorithm

Nabajyoti Bhattacharjee¹ and Nabendu Sen²

¹ Assam University Silchar, Assam PIN: 788011(India)
nbgg2017@gmail.com

² Assam University Schar, Assam PIN:788011(India)
nsen08@yahoo.com

ABSTRACT

In our present work we propose a non-separable inventory model to predict the shortage period of the multi-item raw material inventory with time dependent deterioration. Further we study two cases; economic ordering quantity and fixed initial investment, when cost function is non-separable. In order to solve the model we provide a hybrid algorithm developed from GA and FPA. The inventory model is illustrated by numerically. Analyze the average total cost under fluctuations of requirement functions is performed and results presented graphically.

Key words: Raw material inventory, Time dependent deterioration, Multi-item inventory, Inventory Model, Optimization, Random Optimization Algorithm, Benchmark optimization problems, Threshold Number, Selection, Crossover, Transition.

Mathematics Subject Classification: 90C26, 90C27, 90C30

Journal of Economic Literature (JEL) Classification: C44, C63

1. INTRODUCTION

In industrial sector, a systematic inventory control plays a very important role in managing raw material inventory, inventory of finished goods or inventory of manpower. Industry hails inventory practitioner to build mathematical model that fits the parameters and constraints to serve some specific objectives according to their demand and requirements. In some situation the objective function obtained may not be convex or difficult to establish convexity due to (1) presence of too many variables (2) highly non-linear (3) non-separable equation etc; in those cases it is difficult to get a global optimal solution or a more appropriate solution analytically. In order to address those cases we apply computational algorithms to get an appropriate solution of desired accuracy. The aim of our present work is to develop a raw material inventory model with non-separable cost function and a computational hybrid algorithm motivated from Genetic Algorithm and Flower Pollination Algorithm. Further we analyse the change in average total cost during the fluctuation of requirement function and represent graphically. The development and application of algorithm to address optimization problem on inventory control, logistic, supply chain and lot sizing etc is an active area of research. Dorigo et al., introduced Ant Colony optimization together with some of its remarkable applications. We refer Chambers (2001), Haupt & Haupt (2004) and Mitchal (1995), for a brief account on Genetic Algorithm. Bansal et al. (2014) mimic the social behavior of spider monkey and introduced spider monkey algorithm. Bhunia et al. (2014) proposed an extended elitist NSGA-II to solve a multi-objective inventory problem based on tea industry. Ahmed et al. (2019), a brief study had been made to

analyse the shortage period of raw materials in garment industry using EOQ model and obtain the optimal solution using Multi Objective PSO. Elkhechan et al. (2018), use firefly algorithm to solve lot sizing optimization in supply chain. Panday et al. (2019), have used PSO and GA to solve marble industry inventory problem. Dishan et al. (2018), develop a new hybrid optimization algorithm using particle swarm optimization and harmony search algorithm to address lot sizing problem. Dounias & Vassiliadis (2014), presents a survey on algorithms and methods like PSO, ACO, Artificial Bee Colonies, Artificial Immune System etc. for solving supply chain and logistic optimization problem.

2. NOTATIONS AND ASSUMPTIONS

2.1 Notations

- (1) M , holding capacity of warehouse.
- (2) h_i , holding cost per unit item per unit time.
- (3) T , planning horizon
- (4) C_{oi} , ordering cost
- (5) tr_i , transportation cost
- (6) I_{oi} , initial inventory of the raw material i .
- (7) C_i , cost of per unit item of the raw material i .
- (8) C_{si} , shortage cost per unit of the raw material i .
- (9) sp , selling price per unit.
- (10) $\theta_i t$, deterioration at time t where $0 < \theta_i < 1$ is a constant, of the raw material i .
- (11) $I_i(t)$, inventory at time t of the raw material i .
- (12) $I_{si}(t)$, inventory at time t during the shortage period of the raw material i .
- (13) t_{si} , shortage period of the raw material i .

2.2 Assumptions

- (1) A deterministic raw material inventory with two items.
- (2) Requirement function $R(t) = \begin{cases} At + B, & \text{for item 1} \\ at^2 + bt + c, & \text{for item 2} \end{cases}$
- (3) Shortages are allowed.
- (4) Planning horizon is infinite.
- (5) Lead time is zero.
- (6) No replenishment during the shortage period.
- (7) Capacity of warehouse is finite and planning horizon for both the raw material is same.
- (8) Production starts when the inventory is full for one cycle.
- (9) Collection, holding and maintenance of two raw materials are assumed to be independent.
- (10) Transportation cost of raw materials from source to warehouse is constant.
- (11) Raw material with constant ordering cost.
- (12) There is a fixed initial investment for every planning horizon.
- (13) Holding capacity of warehouse is infinite

3. MATHEMATICAL MODEL

Let us define the objective function for our present study;

$$\min Z = TC(t_{s1}, t_{s2}, T)$$

Subject to, $0.5 < t_{s1} < 1.2$, $0.5 < t_{s2} < 1.2$ and $T \leq 1.2$

Where $0.1 = 1 \text{ month}$

For Raw Material 1:

The differential equation representing the situation is

$$\frac{dI_1}{dt} + \theta_1 t I_1 = -(at^2 + bt + c)$$

$$\text{With } I_1(0) = I_{01}$$

The solution of the equation is given by;

$$I_1 = \sum_{n=0}^{\infty} c_n t^n$$

$$\text{Where } c_0 = I_{01}, c_1 = -c, c_2 = -\frac{(\theta_1 I_{01} + b)}{2}, c_3 = \frac{(\theta_1 c - a)}{3},$$

$$c_{n+2} = -\frac{\theta_1 c_n}{(n+2)} \quad \forall n \geq 4$$

The differential equation representing the change of inventory during the shortage period $[t_{s1}, T]$,

$$\frac{dI_{s1}}{dt} = -(at^2 + bt + c)$$

$$\text{With } I_{s1}(t_{s1}) = 0$$

The inventory level during the shortage period is given by,

$$I_{s1} = \frac{a}{3}(t^3_{s1} - t^3) + \frac{b}{2}(t^2_{s1} - t^2) + c(t_{s1} - t)$$

Total average cost to maintain inventory of raw material 1 is given by;

$$TC1 = C_{01} + tr_1 + \sum_{n=0}^{\infty} \frac{c_n h_1}{n+1} t^{n+1} - C_{s1} \left[\frac{a}{3} \left(t^3_{s1} T - \frac{T^4}{4} \right) + \frac{b}{2} \left(t^2_{s1} T - \frac{T^3}{3} \right) + c \left(t_{s1} T - \frac{T^2}{2} \right) \right] \\ + C_{s1} \left[\frac{a}{4} t_{s1}^4 + \frac{b}{3} t_{s1}^3 + \frac{c}{2} t_{s1}^2 \right] + C_1 \left[I_{01} - \left(\frac{a}{3} t^3_{s1} + \frac{b}{2} t^2_{s1} + c t_{s1} \right) \right]$$

$$\text{Where, } c_0 = I_{01}, c_1 = -c, c_2 = -\frac{(\theta_1 I_{01} + b)}{2}, c_3 = \frac{(\theta_1 c - a)}{3},$$

$$c_{n+2} = -\frac{\theta_1 c_n}{(n+2)} \quad \forall n \geq 4$$

For Raw Material 2

The differential equation representing the situation is given by,

$$\frac{dI_2}{dt} + \theta_2 t I_2 = -(At + B)$$

$$\text{With } I_2(0) = I_{02}$$

The solution of the equation is given by;

$$I_2 = \sum_{n=0}^{\infty} b_n t^n$$

$$\text{Where } b_0 = I_{02}, b_1 = -B, b_2 = -\frac{(\theta_2 I_{02} + A)}{2},$$

$$b_{n+2} = -\frac{\theta_2 b_n}{(n+2)} \quad \forall n \geq 3$$

The inventory level of raw material 2 during the shortage period $[t_{s2}, T]$.

$$\frac{dI_{s2}}{dt} = -(At + B)$$

$$\text{With } I_{s2}(t_{s2}) = 0$$

The inventory level during the shortage period is given by,

$$I_{s2} = \frac{A}{2}(t_{s2}^2 - t^2) + B(t_{s2} - t)$$

The total average cost to maintain raw material 2 is given by;

$$TC_2 = C_{02} + tr_2 + \sum_{n=0}^{\infty} \frac{b_n h_2}{n+1} t_{s2}^{n+1} - C_{s2} \left[\frac{A}{2} \left(t_{s2}^2 T - \frac{T^3}{3} \right) + B \left(t_{s2} T - \frac{T^2}{2} \right) \right] \\ + C_{s2} \left[\frac{A}{3} t_{s2}^3 + \frac{B}{2} t_{s2}^2 \right] + C_2 \left[I_{02} - \left(\frac{A}{2} t_{s2}^2 + B t_{s2} \right) \right]$$

$$\text{Where, } b_0 = I_{02}, \quad b_1 = -B, \quad b_2 = -\frac{(\theta_2 I_{02} + A)}{2},$$

$$b_{n+2} = -\frac{\theta_2 b_n}{(n+2)} \quad \forall n \geq 3$$

Therefore the total average cost to maintain the inventory of raw material which is collected, stocked and maintained independently in a warehouse with infinite holding capacity is given by;

$$TC = \frac{TC_1 + TC_2}{T}$$

Subject to $0.5 \leq t_{s1}, t_{s2} \leq 1.2$ and $1.2 \geq T$ where $0.1 = 1$ month

Average total cost TC is the function of three variables t_{s1}, t_{s2}, T . Furthermore, the equation is highly non-linear and non-separable due to the presence of an infinite series and product of decision variables. Therefore, it is difficult to establish convexity analytically and so we turn to algorithmic approach for a local minimum of desired accuracy satisfying the above constraints.

4. HYBRID ALGORITHM

The algorithm begins with a set of female population and a single male. In the successive iterations we optimise the fitness of male which is the solution of our problem, using the set of females. We design the algorithm on the basis of three rules;

- (1) **Selection:** The selection of fittest female to generate offspring is done by Roulette's wheel method.
- (2) **Crossover:** The crossover operations are defined by;

$$z = (1 - k)x + \beta ky \quad (1.2)$$

Where, $k \in (0, 1)$ and $\beta \in [-1, 1]$ are random numbers.

- (3) **Transition:** The invasion of foreign male can be express using Levy flight or by convex combination. Levy distribution is given by;

$$L(s, l) \approx \frac{l \Gamma(l) \sin\left(\frac{l\pi}{2}\right)}{s^{1+l}} \quad (1.3)$$

Where, $s > 0$ is a sufficiently large number and l is a predefined parameter.

The transition operations to introduce the foreign male w are defined by;

$$w = (1 - q)c_{best} + q(r g_{best} + (1 - r)y_{bf}) \quad (1.4(a))$$

$$w = \alpha c_{best} + rL(g_{best}) \quad (1.4(b))$$

Where, α is the acceleration coefficients, g_{best} is the global best solution, c_{best} is the current best solution, y_{bf} is the female with best fitness, $r \in (0,1)$ and $q \in [-1, 1]$ are random numbers,

Rule (1) and (2) is motivated from GA and Rule (3) is motivated from FPA. In order to regulate the exchange of current male by a foreign male, we assign a threshold number Δ_0 and a threshold parameter p . When the threshold parameter p satisfied the condition set for threshold number Δ_0 then we replace the current male by a foreign male using equation 1.4(a) or 1.4(b). Threshold parameter and number can be connected in the following ways;

1. Number of iterations
2. Random numbers
3. Probability distributions

Table 1: Pseudo Program

THE COMPUTATIONAL STEPS OF THE ALGORITHM
do co If(Fitness(co)>Fitness(ck)) for all k=1,2,...,n co=ck else c=cj for j whose fitness is least ck = selection of female by Roulette's method; c=crossover(co, ck) if(Fitness(co)>Fitness(c)) co = c Else c = cj for j ∈ {1,2,...,n} whose fitness is least count=count+1 // "count" is to calculate number of iterations while(stopping criteria)

INTRODUCTION OF FOREIGN MALE
if($p < \Delta_0$) co = expression of foreign male Else Choose $ck = \text{random}\{c_1, c_2, \dots, c_n\}$ Continue
SEARCH FOR GLOBAL OPTIMA
c(global)=co co=c(foreign) Run the algorithm for local optimum co1=c(local) if ($co1 < c(\text{global})$) c(global)=co1 co1=c(foreign) Run the algorithm for local optimum co2=c(local) else co1=c(foreign) Run the algorithm for local optimum co2=c(local) if ($co2 < c(\text{global})$) continue as above.

5. EXPERIMENTAL VERIFICATION OF ALGORITHM

In order to verify its ability to search the optimal solution under a given absolute error we solve few benchmark optimization problems using the proposed algorithm. All the programs are designed in Turbo C++ in C language. The solution is accepted after a successful run with an accuracy $1.0E-05$. Optimization problems are adopted from Jamil et al. (2013).

Table 2: Experimental Verification

<i>Optimization Problem</i>	<i>Dimension</i>	<i>Optimal Value</i>	<i>Iterations</i>
Six – Hump Camel Back Problem	2	−1.031594	6805
Hosaki Problem	2	−2.3458	9872
Rosenbrock Problem	10	0	9
Bartels Con Function	2	1	2000
Aluffi – Pentini’s Function	2	−0.352308	10317068
Ackley 2 Function	2	−200	15
Adijman Function	2	−2.02182	131494983

6. NUMERICAL ILLUSTRATION

For numerical computation we assume $a = 1, b = 2, c = 3$ and $A = 1, B = 2$

Table 3: The values of all the parameters for the two materials with their appropriate units are taken as follows;

Table 3: Values of the parameter

Item(i)	$C_{0i}(\text{\$})$	$tr_i(\text{\$})$	$I_{0i}(\text{units})$	$C_{si}(\text{\$})$	$C_i(\text{\$})$	θ_i	$h_i(\text{\$})$
1	100	120	100	10	8	0.001	0.5
2	80	150	150	8	8	0.003	0.7

Crossover

The crossover operation defined for the algorithm in this problem is;

$$c_{new} = (1 - k)c_{00} + kc_r$$

Where, $k = \text{random}(0,1)$ and c_r = female chosen at random for breeding.

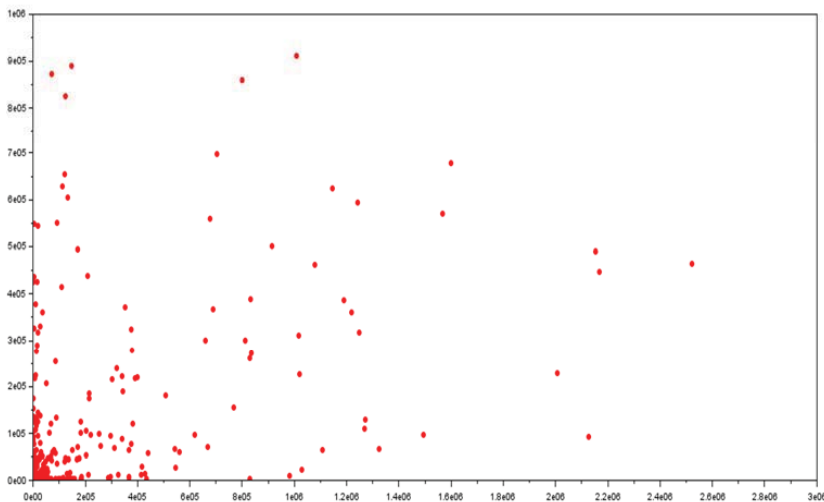


Figure 1: Representation of Shortage Periods after Crossover

Transition

The expression for the introduction of new individual is;

$$c_{foreign} = (1 - r)c_{00} + r(lc_{new} + (1 - l)c_{bf})$$

Where, N = iteration, $r = \text{random}[-1,1]$, $l = \text{random}(0,1)$ and c_{bf} = Female with best fitness.

Effect of Transition Operation on Global Search

We run the algorithm for $M = 100$ iterations and define $\Delta_0 = \text{mod}(M, k)$, whenever $\Delta_0 = 0$ transition operation is performed. The results obtained see figure (2)-(5);

Table 5: Effect of transition operation

k	Number of times transition operation used	Number of improvements in Global Optimum	Best Solution Obtained	Improvement shows till the iteration
20	5	6	2255.11	6
40	2	8	2234.87	82
60	1	11	2288.58	89
80	1	8	2286.40	93
100	0	4	2281.83	61

Graphical Representation

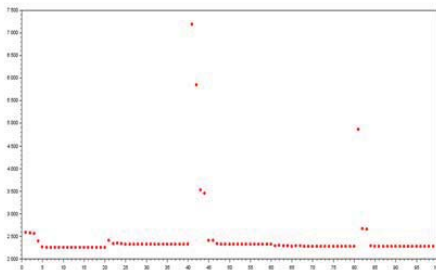


Figure 2: For 20 Transition Operations

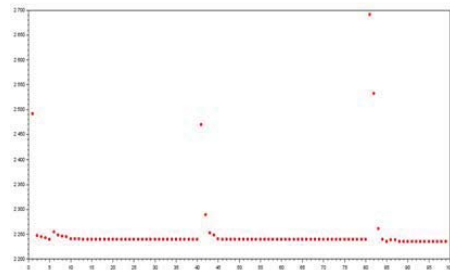


Figure 3: For 40 Transition Operations

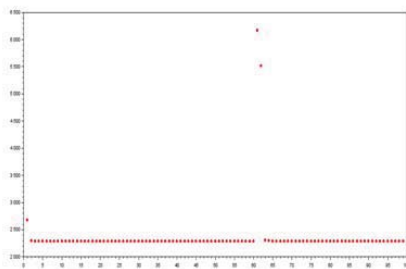


Figure 4: For 60 Transition Operations

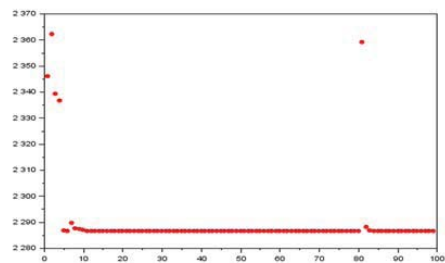


Figure 5: For 80 Transition Operations

Stopping Criteria: Variance of Finite Set of Global Optimum

We follow the stopping discussed by Bhandari et al. to find optimal solutions. Due the limited system capacity and power supply we consider small sets of global optimum to obtain variance. Let S be a finite set of global optima.

Table 6: Stopping Criteria

$ S $	TC	t_{s1}	t_{s2}	T	variance
5	2319.9072	0.6287887	0.6695093	1.1383116	0.76
10	2313.0873	0.59726	0.8218023	1.149607	0.008042
15	2347.9426	0.919793	0.9586097	1.1416902	1.136604
20	2324.2201	0.9233732	0.815082	1.1458198	5.009584

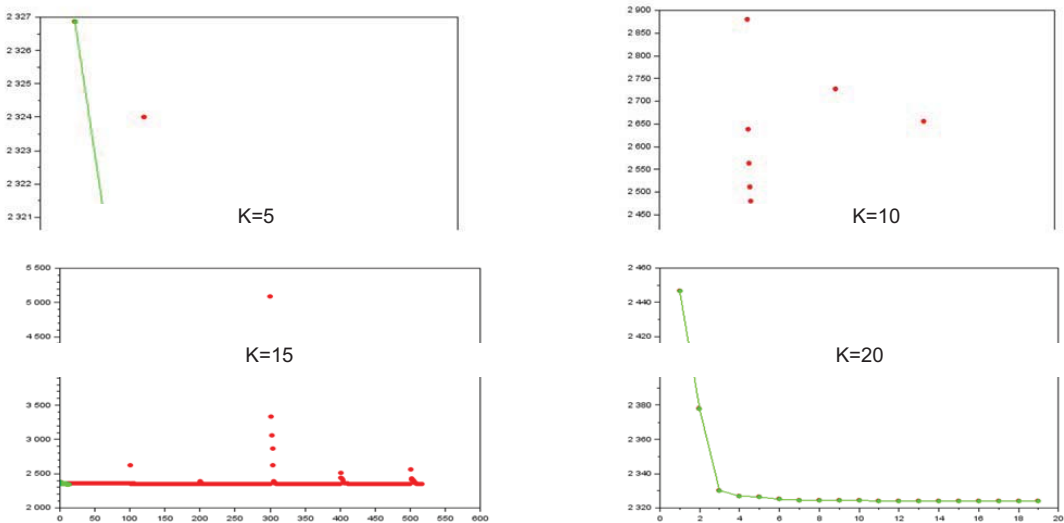


Figure 6: '-' and '.' represents global best and fitness of new born

The optimal solution obtained with so far satisfying all the constraints and with variance less than $1.0E - 02$ is given by;

Table 7: Optimal Solution

Optimal Value	Optimal solution			Variance
	t_{s1}^*	t_{s2}^*	T^*	
2283.835	0.6872104	0.510463	1.1486608	0.000005

7. SPECIAL CASE

7.1 When Holding Capacity of Warehouse is Finite

Let us change the Assumption (14) and consider that holding capacity is finite and equal to M of warehouse is finite. Then the objective function becomes;

Objective Function

$$\text{Min } Z = TC(I_{01}, I_{02}, t_{s1}, t_{s2}, T)$$

Subject to

$$0.5 \leq t_{s1}, t_{s2} \leq 1.2$$

$$1 \leq T \leq 1.2$$

$$I_{01} + I_{02} \leq M$$

$$I_{01}, I_{02} \geq 0$$

The objective function is highly non-linear, non-separable and due to the presence of too many variables it is not possible to establish convexity so consider algorithmic approach.

Let $M = 400$ units then the optimal solution is given by;

Table 8: Optimal Solution for Case A

Optimal Value	Optimal Solution					Variance
	t_{s1}^*	t_{s2}^*	T^*	I_{01}^*	I_{02}^*	
3564.5354	0.7435477	0.5368381	1.1044678	249.50952	150.49048	0.0061817

7.2 When Total Investment on Raw Material Maintenance is Finite

During the setup of a micro-industry the entrepreneur usually have a fixed amount to invest in every sector like raw material acquisition, maintenance, energy, packing, distribution, labour and administration. Although a buffer quota is also maintained to deal with shortage and crises. Due to this the initial investment to purchase and maintain raw material is fixed for the present cycle with a certain limit within which the actual cost and buffer quota is maintained. When these small scale industries earn profit by selling the produced good. They can increase the investment to purchase more raw materials for the next cycle and eventually raise the buffer quota to deal with adverse situations as compared to previous cycle. In this situation the total cost TC cannot exceed certain cost limit. Let $[L, M]$ be the cost limit of initial investment.

Objective Function

$$\text{Min } Z = TC(t_{s1}, t_{s2}, T)$$

Subject to

$$0.5 \leq t_{s1}, t_{s2} \leq 1.2$$

$$1 \leq T \leq 1.2$$

$$L \leq TC \leq M$$

Let $[L, M] = [2000, 2500]$ be the investment limit, then the optimal solution is given by;

Table 9: Optimal Solution for Case B

Optimal Value	Optimal solution			Variance
	t_{s1}^*	t_{s2}^*	T^*	
2381.523	0.5829991	0.6953567	1.1091857	0.0056843

8. ANALYSIS OF FLUCTUATIONS IN REQUIREMENT FUNCTION

On the basis of requirements of two items we analyse the average total cost (TC). We change the parameters involved in requirement function and observe the change in mean TC.

Table 10: Fluctuation of requirement function

Requirement		t_{s1}	t_{s2}	T	TC
Item 1	Item 2				
$At + B$	c	0.5829014	0.69533	1.1091827	2381.3
$At + B$	$bt + c$	0.6904525	0.6691816	1.1178676	2362.8
$At + B$	$at^2 + bt + c$	0.6872104	0.510463	1.1486608	2283.8
At	c	0.5	0.5961596	1.0983437	2395.4
At	$bt + c$	0.8483798	0.7527284	1.1178961	2384.03
At	$at^2 + bt + c$	0.640483	0.7111346	1.1198594	2370.12
B	c	0.9629259	0.8517906	1.1370229	2363.11
B	$bt + c$	1.0928429	0.7311666	1.1387779	2336.11
B	$at^2 + bt + c$	0.5296653	0.6085229	1.1516504	2285.1

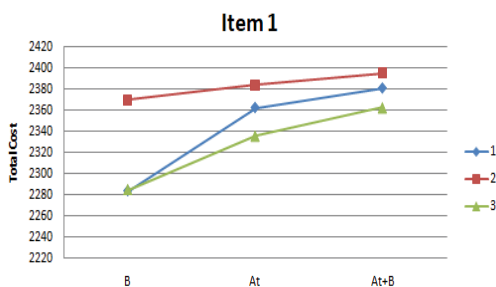


Figure 7: Change in total cost with respect to Item 1

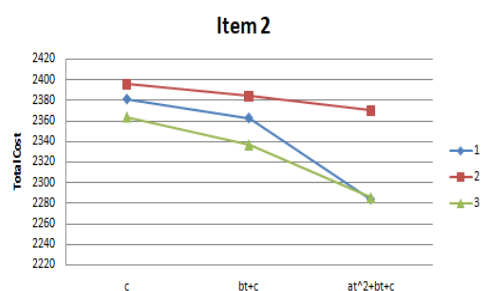


Figure 8: Change in total cost with respect to Item 2

9. DISCUSSION AND CONCLUSION

This is a two item inventory model for raw materials. The requirement function is linear for item 1 and quadratic for item 2 further the deterioration rate for both the items are linear function of time and holding cost is constant. The cost function is not convex and the proposed algorithm although optimize the cost function but often solution get stuck at a local optima which can be shifted from that origin using translation operation. From Table 6, we can see that the repeated use of translation operation after every small number of iterations results in less number of improvements therefore we suggest translation operation should be used wisely and the displacement of solution due to translation can be observed in Figure 2-5. So far the use of variance as stopping criterion in this algorithm works for finding better local optima. From Table 7 and Figure 6, we can make a remark that due to the random nature of algorithm, large number of improvements may not lead to local optima even if we take sufficiently large number of iterations. The optimal solution for our problem under the stopping criterion $variance < 0.01$ is $TC = 2283.835$ at the points $t_{s1}^* = 0.6872104$, $t_{s2}^* = 0.510463$ and $T^* = 1.1486608$. The variation in total cost and decision variables due to fluctuation of requirement function is analyses in Table 10 and we can conclude that in all the three cases average total cost raises when the raw material stays in the warehouse for longer period results are represented graphically in Figure 7 & 8. The algorithm and inventory model is developed to study the optimization of average total cost and predict the shortage period of a non-convex function. The model is studied under economic ordering quantity with finite holding capacity and fixed investment policy see Table 8 and Table 9. We have programmed the algorithm using Scilab 6.0.2 in windows 2010 environment, CPU consists of 4GB RAM, Intel Core 2 Duo 2.60 GHz processor and 64-bit Operating System. The proposed inventory model and algorithm are not reported in the previous research works as per author's knowledge is concern.

Scope of further research in this area is by allowing variable transportation cost, preservation cost. Performance tests of proposed algorithm under Swarm Intelligence criteria and its improvement is also an interesting area of research.

10. Reference

Ahmed, M., & Sarwar, F., Rahman, M 2019 Raw Material Inventory Control Model for RGM with Shortage Prediction using Nature Inspired Algorithm, *International Scientific Conference on Sustainability of Global Garment Industry*, **ICSG2i2019-PI-77**, 92-102

Bansal, J. C., Sharma, H., & Jadon, S. S., 2014. Spider Monkey Optimization algorithm for numerical optimization, *Memetic Computing*.

Bhandari D., Murthy, C. A., & Pal Sankar K., (2012). Variance as a Stopping Criterion for Genetic Algorithms with Elitist Model, *Fundamenta Informaticae*, **120**, 145-164.

Bhunia, A. K., Biswas, A., & Sen, N., 2014, An application of extended elitist non-dominated sorting Genetic Algorithm in multi-objective linear programming problem of tea industry with interval objectives, *Uncertain Supply Chain Management*, 245-256.

Chambers L., 2001. The Practical Handbook of Genetic Algorithms Application, *Chapman and Hall/CRC*.

Dorigo, M., Birattari, M., & Stützle, T., 2006 *Ant Colony Optimization*, IEEE Computational Intelligence Magazine.

Dounias G., Vassiliadis V., 2014. *Algorithms and Methods Inspired from Nature for Solving Supply Chain and Logistics Optimization Problems: Survey*, International Journal of Natural Computing Research. DOI:10.4018/ijncr.2014070102

Dishan, B. P., Sahithi, V. V. D., & Rao, C. S. P., 2018. A New Hybrid Optimization Algorithm to Solve Lot Sizing Problems, *International Journal for research in Applied Science & Engineering Technology*, **6**, , 2978-2987

Elkhechan, M., Benmamoun, Z., Hachimi, H., Amine, A., & Elkettani, Y., 2018, Firefly Algorithm for Supply Chain Optimization, *Lobachevskii Journal of Mathematics*, **39(3)**, 355-367.

Haupt, R. L., & Haupt, S. E., 2004. *Practical Genetic Algorithms*, Hoboken Wiley.

Jamil, Momin, & Yang, Xin-She, 2013. A Literature Survey of Benchmark Functions For Global Optimization Problems, *arxiv:1308-4008v1*.

Mitchal M., 1995. Genetic Algorithm: An Overview, *Complexity*, **1(1)**, 31-39.

Pandey, T., Yadav, A. S., & Malik, M., 2019. An Analysis Marble Industry Inventory Optimization Based on Genetic Algorithms and Particle swarm optimization, Vol. 7, **6S4**, 369-373.

,