

Investigating the Specification of the Distributional Assumption of the Innovations of Generalised Autoregressive Score Model with its Variants

O. T. Babatunde¹, O. S. Yaya² and A. V. Oladugba³

¹Department of Statistics, University of Nigeria, Nsukka, Nigeria
Email: babatundegbenge03@gmail.com

²Department of Statistics, University of Ibadan, Nigeria
Email: o.s.olaluwa@gmail.com

³Department of Statistics, University of Nigeria, Nsukka, Nigeria
Email: abimibola.oladugba@unn.edu.ng

ABSTRACT

This paper investigates the specification of the distributional assumption of the innovations of Generalised Autoregressive Score model with its variants through Monte Carlo simulation experiment with extension to international crude oil prices using three levels of volatility persistence by varying the parameters of the models. The estimates of Akaike Information Criterion and Excess Kurtosis were used to access fitness performance. The results obtained showed that when the distributional assumption of the innovations of Generalised Autoregressive Score model with its variants is either Student-t or Skewed Student-t, specifying it as Skewed Student-t distribution fits better than the other distributions considered.

Keywords: Generalised Autoregressive Score, Innovation, Persistence, Skewed-Student-t distribution, Specification.

Mathematics Subject Classification: 62M10

1. INTRODUCTION

One of the reasons for modelling volatility is to predict future volatilities Engle and Patton (2001). In fitting a model to a financial series, the distributional assumption of the residual of a particular model may be wrongly specified, that is the residual may be assumed to be normal whereas it is non-normal. Specification refers to the gain or loss in terms of model fitness achieved when a model with a particular distributional assumption is fitted as the same model but with a different distributional assumption. The problem of misspecification on the correct probability distribution of Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model using Monte Carlo simulation approach was considered in Yaya et al. 2014 assuming Normal distribution and they concluded that AR-GARCH-GED performed better than the AR-GARCH-Normal in terms of forecast. When innovations are incorrectly specified, it can lead to spurious Integrated GARCH (IGARCH) and can also affect predicted volatilities (see Bellini and Bottolo 2007). Several works existed in literature on

the effect of wrongly specifying the distributional assumption of the innovations of GARCH models (see Andreou and Werker 2015; Klar et al. 2012; Fleming et al. 2008; Atoi 2014). However, due to inability of standard GARCH models to capture the effect of occasional jumps, Generalised Autoregressive Score (GAS) model with its variants were introduced as robust volatility models to capture the effect of occasional jumps in GARCH models (Harvey and Chakravarty 2008; Harvey 2013). The GAS variants were applied to the Nigeria All Share Index (ASI) in Yaya et al. 2016 and were recommended for prediction of financial series. The misspecification of the GAS variants have been studied in Babatunde et al (2019) yet there are limited applications on these models even though small sample properties have been studied in (see Creal and Koopman 2008; Creal et al. 2011,2013; Blasques et al. 2014) and since incorrect specification of the distributional assumption of the innovations of GARCH models affect the fitness performance of the models, there is need to investigate the distributional assumption of the innovations of GAS model with its variants and the effect of varying the parameters of the models through Monte Carlo experiment with extension to international crude oil prices. The estimates of Akaike Information Criterion (AIC) and Excess Kurtosis will be used to investigate the distributional assumption of the innovations of GAS model in this work. The choice of parameter affects the parameter space regions which are essential for the class of GAS models Blasques et al. 2014. The relevant parameter spaces for several conditional distributions were fully characterized in (Markus 2009).

This paper is aimed at investigating the specification of the distributional assumption of the innovations of GAS model with its variants through Monte Carlo experiment with extension to international crude oil prices using three levels of volatility persistence and four distributional assumptions namely; Normal probability distribution (N), Student-t probability distribution (T), Skewed Student-t probability distribution (Skew-t) and Generalised Error probability distribution (Ged). The estimates of Akaike Information Criterion and Excess Kurtosis were used to access fitness performance and the simulation results were compared to the results of the case study.

2. GENERATION OF DATA

2.1 Monte Carlo Experiment

In this paper, a Monte Carlo experiment was carried out to investigate the specification of the distributional assumptions of the GAS model and its variants by accessing the model fitness performance and the tail effect of the innovations. The data were generated through three different processes which are:

$$\text{GAS}(1,1) : \sigma_t^2 = \omega + \alpha_1 \mu_{t-1} \sigma_{t-1}^2 + (\alpha_1 + \beta_1) \sigma_{t-1}^2 \quad (1)$$

$$\text{EGAS}(1,1): \log \sigma_t^2 = \omega + \alpha_1 \mu_{t-1} \sigma_{t-1}^2 + (\alpha_1 + \beta_1) \log \sigma_{t-1}^2 \quad (2)$$

$$\text{AEGAS}(1,1): \log \sigma_t^2 = \omega + \alpha_1 \mu_{t-1} \sigma_{t-1}^2 + \gamma_1 \tau_{t-1} + (\alpha_1 + \beta_1) \log \sigma_{t-1}^2 \quad (3)$$

where $\tau_{t-1} = \text{sign}(-z_t)(\mu_t + 1)$. 5000 time series was simulated under each generating process using (1), (2) and (3) with the probability distribution of the innovations of each generating process specified as Normal probability distribution (N), Student-t probability distribution (T), Skewed Student-t probability distribution (Skew-t) and Generalised Error probability distribution (Ged). The GAS-Normal (GAS-N), GAS-Studen-t (GAS-T), GAS-Skewed-Student-t (GAS-Skew t) and GAS-Generalized Error Distribution (GAS-GED) were estimated using (9), (10), (11) and (12) respectively. The EGAS-Normal (EGAS-N), EGAS-Student-t (EGAS-T), EGAS-Skewed-Student-t (EGAS-Skew t) and EGAS-GED were estimated using (14), (15), (16) and (17) respectively. The AEGAS-Normal (AEGAS-N), AEGAS-Student-t (AEGAS-T), AEGAS-Skewed-Student-t (AEGAS-Skew t) and AEGAS-GED were estimated using (19), (20), (21) and (22) respectively. The estimates of the parameters of return series α and β ranges from 0.02 to 0.25 and 0.65 to 0.99, respectively. The parameters α and ϕ are mostly specified in such a way that the volatility persistence is near to 1 (Bauwens et al. 2012). According to Halunga and Orme (2004), the volatility persistence of the models can be classified as either low, medium or high and can be expressed as:

Low Persistence: $\omega, \alpha_1, \beta_1, \phi_1, \gamma_1 = (0.04, 0.05, 0.65, 0.7, 0.01)$

Medium Persistence: $\omega, \alpha_1, \beta_1, \phi_1, \gamma_1 = (0.04, 0.1, 0.8, 0.9, 0.01)$

High Persistence: $\omega, \alpha_1, \beta_1, \phi_1, \gamma_1 = (0.04, 0.09, 0.9, 0.99, 0.01)$

The parameters ω and γ_1 were held constant since they do not affect volatility persistence of the models.

2.2 International Crude Oil Prices

The GAS model and its variants were fitted to the daily crude oil prices assuming four different distributions for the innovations to test the effects of specification on the innovations of the GAS models. The crude oil prices which were obtained from <http://www.eia.gov/> are the European Brent prices measured in US dollars/barrel. The data covers a period of 319 months with 6985 daily observations from May 20, 1987 to August 29, 2014.

2.3 Generalized Autoregressive Score Model (GAS)

GAS(1,1) model proposed in Harvey and Chakravarty (2008) with four different distributions assumed for the innovations and it expressed below:

$$\sigma_t^2 = w + \alpha_1 u_{t-1} \sigma_{t-1}^2 + \phi_1 \sigma_{t-1}^2 \quad (4)$$

with the distribution of the innovations given as;

$$u_t = z_t^2 - 1 \text{ for } z_t \approx N(0, 1); \quad (5)$$

$$u_t = \frac{(v+1)z_t^2}{v-2+z_t^2} - 1 \text{ for } z_t \approx T(0,1,v) \quad (6)$$

$$u_t = \frac{(v+1)z_t z_t^*}{(v-2)g_t \xi^{I_t}} - 1 \text{ for } z_t \approx SKST(0,1,\xi,v) \quad (7)$$

$$u_t = 0.5v|z_t|^v / \lambda_v^v - 1 \text{ for } z_t \approx GED(0,1,v) \quad (8)$$

where

$$z_t^* = sz_t + m, \quad I_t = \text{sign}(z_t^*) = I(z_t^* \geq 0) - I(z_t^* < 0), \quad g_t = 1 + \frac{z_t^{*2}}{(v-2)\xi^{2I_t}},$$

$$m = -\frac{\Gamma\left(\frac{v-1}{2}\right)\sqrt{v-2}}{\sqrt{\pi}\Gamma\left(\frac{v}{2}\right)}\left(\xi - \frac{1}{\xi}\right) \text{ and } s = \sqrt{\left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2}$$

Therefore, substituting (5), (6), (7) and (8) into (4) will give GAS-Normal, GAS-Student-t, GAS-Skewed-Student-t and GAS-GED respectively.

$$\text{GAS-Normal: } \sigma_t^2 = w + \alpha_1(z_{t-1}^2 - 1)\sigma_{t-1}^2 + \varphi_1\sigma_{t-1}^2 \quad (9)$$

$$\text{GAS-Student-t: } \sigma_t^2 = w + \alpha_1\left(\frac{(v+1)z_{t-1}^2}{v-2+z_{t-1}^2} - 1\right)\sigma_{t-1}^2 + \varphi_1\sigma_{t-1}^2 \quad (10)$$

$$\text{GAS-Skewed-Student-t: } \sigma_t^2 = w + \alpha_1\left(\frac{(v+1)z_{t-1}z_{t-1}^*}{(v-2)+g_{t-1}\xi^{I_{t-1}}} - 1\right)\sigma_{t-1}^2 + \varphi_1\sigma_{t-1}^2 \quad (11)$$

$$\text{GAS-GED: } \sigma_t^2 = w + \alpha_1\left(\frac{0.5v|z_{t-1}|^v}{\lambda_v^v} - 1\right)\sigma_{t-1}^2 + \varphi_1\sigma_{t-1}^2 \quad (12)$$

2.3 GAS Variants

The GAS variants type of the GARCH variants were considered in Harvey and Chakravarty (2008) with four probability distributions assumed for the returns. The EGAS model is given as,

$$\log \sigma_t^2 = w + \alpha_1 u_{t-1} + \varphi_1 \log \sigma_{t-1}^2 \quad (13)$$

specified without the leverage effect.

Therefore, substituting (5), (6), (7) and (8) into (13) will give EGAS-Normal, EGAS-Student-t, EGAS-Skewed-Student-t and EGAS-GED respectively.

$$\text{EGAS-Normal: } \log \sigma_t^2 = w + \alpha_1(z_{t-1}^2 - 1) + \phi_1 \log \sigma_{t-1}^2 \quad (14)$$

$$\text{EGAS-Student-t: } \log \sigma_t^2 = w + \alpha_1 \left(\frac{(v+1)z_{t-1}^2}{v-2+z_{t-1}^2} - 1 \right) + \phi_1 \log \sigma_{t-1}^2 \quad (15)$$

$$\text{EGAS-Skewed-Student-t: } \log \sigma_t^2 = w + \alpha_1 \left(\frac{(v+1)z_{t-1}z_{t-1}^*}{(v-2)+g_{t-1}\xi^{l_{t-1}}} - 1 \right) + \phi_1 \log \sigma_{t-1}^2 \quad (16)$$

$$\text{EGAS-GED: } \log \sigma_t^2 = w + \alpha_1 \left(\frac{0.5v|z_{t-1}|^v}{\lambda_v^v} - 1 \right) + \phi_1 \log \sigma_{t-1}^2 \quad (17)$$

Imposing leverage effect on (10), yields the AEGAS model,

$$\log \sigma_t^2 = w + \alpha_1 u_{t-1} + \gamma_1 l_{t-1} + \phi_1 \log \sigma_{t-1}^2 \quad (18)$$

where $l_{t-1} = \text{sign}(-z_t)(u_t + 1)$ for Normal probability distribution (N), Student-t probability distribution (T), Generalised Error probability distribution (Ged). and

$l_{t-1} = \text{sign}(-z_t^*)(u_t + 1)$ for the Skewed Student-t probability distribution (Skew-t).

Therefore, substituting (5), (6), (7) and (8) into (18) will give AEGAS-Normal, AEGAS-Student-t, AEGAS-Skewed-Student-t and AEGAS-GED respectively.

$$\text{AEGAS-Normal: } \log \sigma_t^2 = w + \alpha_1(z_{t-1}^2 - 1) + \gamma_1 l_{t-1} + \phi_1 \log \sigma_{t-1}^2 \quad (19)$$

$$\text{AEGAS-Student-t: } \log \sigma_t^2 = w + \alpha_1 \left(\frac{(v+1)z_{t-1}^2}{v-2+z_{t-1}^2} - 1 \right) + \gamma_1 l_{t-1} + \phi_1 \log \sigma_{t-1}^2 \quad (20)$$

$$\begin{aligned} \text{AEGAS-Skewed-Student-t: } \log \sigma_t^2 = w + \alpha_1 \left(\frac{(v+1)z_{t-1}z_{t-1}^*}{(v-2)+g_{t-1}\xi^{l_{t-1}}} - 1 \right) \\ + \gamma_1 l_{t-1} + \phi_1 \log \sigma_{t-1}^2 \end{aligned} \quad (21)$$

$$\text{AEGAS-GED: } \log \sigma_t^2 = w + \alpha_1 \left(\frac{0.5v|z_{t-1}|^v}{\lambda_v^v} - 1 \right) + \gamma_1 l_{t-1} + \phi_1 \log \sigma_{t-1}^2 \quad (22)$$

2.4 Model Specification Test

Each of the GAS models will be estimated under the four distributional assumptions of the innovations from the simulation and the case study (oil prices) and Akaike Information Criterion (AIC) and excess

kurtosis will be used to assess the fitness of the models under the different distributions of the innovations;

$$AIC = -2N^{-1}L_N\left\{(y_i); \hat{\psi}^{(N)}\right\} + 2N^{-1}\tilde{\psi} \quad (23)$$

$$k_\varepsilon = \frac{E(\varepsilon_t^4)}{[E(\varepsilon_t^2)]^2} - 3 \quad (24)$$

$$\text{where } E(\varepsilon_t^2) = \frac{w}{1 - \alpha_1 - \beta_1}$$

$$E(\varepsilon_t^4) = \frac{w^2(1 + \alpha_1 + \beta_1)(k_z + 3)}{[1 - (\alpha_1 + \beta_1)][1 - \alpha_1^2(k_z + 2) - (\alpha_1 + \beta_1)^2]} \quad \text{and } k_z \text{ is the excess kurtosis from the}$$

assumed GARCH distribution process Z_t .

3. RESULTS

3.1 Monte Carlo Experiment

The results obtained from the simulation are presented in tables 1-3.

Table 1: When the estimated model is GAS (1, 1)

Generating Process	Probability Distribution	Low Persistence $\omega, \alpha_1, \beta_1, \phi_1, \gamma_1$ $= (0.04, 0.05, 0.65, 0.7, 0.01)$		Med. Persistence $\omega, \alpha_1, \beta_1, \phi_1, \gamma_1$ $= (0.04, 0.1, 0.8, 0.9, 0.01)$		High Persistence $\omega, \alpha_1, \beta_1, \phi_1, \gamma_1$ $= (0.04, 0.09, 0.9, 0.99, 0.01)$	
		AIC	Ex. Kurtosis.	AIC	Ex. Kurtosis.	AIC	Ex. Kurtosis.
GAS-N	Normal	0.8307	0.0302	1.8724	-0.0664	3.9965	0.0018
	T	0.8308	0.0304	1.8729	-0.0665	3.9966	0.0019
	Skew t	0.8312	0.0303	1.8733	-0.0666	3.9970	0.0016
	Ged	0.8305	0.0304	1.8722	-0.0650	3.9962	0.0016
GAS-T	Normal	0.7943	3.2100	1.9364	2.3486	4.0460	1.9888
	T	0.7361	3.4328	1.8768	2.3192	3.9917	1.8695
	Skew t	0.7348	3.4035	1.8765	2.3192	3.9913	1.8695
	Ged	0.7485	3.3505	1.8834	2.3200	3.9971	1.9226
GAS-Skew t	Normal	0.7856	1.8377	1.9330	2.5106	3.7658	2.2346
	T	0.7388	1.8208	1.8663	2.3705	3.7088	2.2264
	Skew t	0.7384	1.8206	1.8664	2.3694	3.7084	2.2264
	Ged	0.7476	1.8256	1.8710	2.4188	4.1650	2.9150
GAS-Ged	Normal	0.8182	-0.0383	1.8469	0.1253	4.0841	-0.0541
	T	0.8187	-0.0383	1.8471	0.1251	4.0845	-0.0541
	Skew t	0.8184	-0.0381	1.8475	0.1251	4.0845	-0.0539
	Ged	0.8185	-0.0384	1.8470	0.1243	4.0842	-0.0551

GAS (1,1) is defined as Generalized Autoregressive Score of order 1

Table 1 presents the results when the generating process is GAS under the four distributional assumptions and the model with the least AIC is judged to fit best. When Normal and Student-t distributions were assumed, GAS-GED and GAS-Skewed-Student-t fit best respectively at all level of persistence. When Skewed-Student-t was assumed, GAS-Skewed-Student-t fits best at both low and high persistence while GAS-Student-t fits best at medium persistence. When GED was assumed, GAS-Normal fits best at all levels persistence.

Table 2: When the estimated model is EGAS (1,1)

Generating Process	Probability Distribution	Low Persistence $\omega, \alpha_1, \beta_1, \phi_1, \gamma_1$ = (0.04, 0.05, 0.65, 0.7, 0.01)		Med. Persistence $\omega, \alpha_1, \beta_1, \phi_1, \gamma_1$ = (0.04, 0.1, 0.8, 0.9, 0.01)		High Persistence $\omega, \alpha_1, \beta_1, \phi_1, \gamma_1$ = (0.04, 0.09, 0.9, 0.99, 0.01)	
		AIC	Ex. Kurtosis.	AIC	Ex. Kurtosis.	AIC	Ex. Kurtosis.
EGAS-N	Normal	2.9712	-0.1426	3.1911	-0.0595	6.8848	-0.0203
	T	2.9717	-0.1425	3.1912	-0.0599	6.8848	-0.0206
	Skew t	2.9713	-0.1425	3.1916	-0.0595	6.8851	-0.0209
	Ged	2.9712	-0.1423	3.1908	-0.0598	6.8844	-0.0201
EGAS-T	Normal	2.8826	1.9115	3.2822	2.8712	6.9947	3.2835
	T	2.8381	1.9696	3.2264	2.9566	6.9103	3.1252
	Skew t	2.8377	1.9703	3.2261	2.9576	6.9100	3.1259
	Ged	2.8444	1.9473	3.2363	2.2189	6.9213	3.2735
EGAS-Skew t	Normal	3.0026	2.2405	3.2698	2.2046	6.7668	2.4131
	T	2.9420	2.2061	3.2086	2.2293	6.6843	2.0485
	Skew t	2.9418	2.2051	3.2082	2.2283	6.6842	2.0485
	Ged	2.9508	2.2189	3.2163	2.2050	6.6942	0.0681
EGAS-Ged	Normal	3.0134	-0.0205	3.1941	0.15743	7.1089	0.0973
	T	3.0140	-0.0256	3.1940	0.1662	7.1087	0.0941
	Skew t	3.0143	-0.0207	3.1944	0.1658	7.1091	0.0944
	Ged	3.0139	-0.0205	3.1934	0.1636	7.1089	1.9654

EGAS (1,1) is defined as Exponential Generalized Autoregressive Score of order 1

Table 2 presents the results when the generating process is EGAS under the four distributional assumptions and the model with the least AIC is judged to fit best. When Normal distribution was assumed, EGAS-Normal fits best at low persistence, whereas EGAS-GED fits best at both medium and high persistence. When Student-t and Skewed-Student-t were assumed, EGAS-Skewed-Student-t fits best at all level of persistence. When GED was assumed, GAS-Normal, GAS-GED and GAS-Student-t fit best at low, medium and high persistence respectively.

Table 3: When the estimated model is AEGAS (1,1)

Generating Process	Probability Distribution	Low Persistence $\omega, \alpha_1, \beta_1, \phi_1, \gamma_1$ $= (0.04, 0.05, 0.65, 0.7, 0.01)$		Med. Persistence $\omega, \alpha_1, \beta_1, \phi_1, \gamma_1$ $= (0.04, 0.1, 0.8, 0.9, 0.01)$		High Persistence $\omega, \alpha_1, \beta_1, \phi_1, \gamma_1$ $= (0.04, 0.09, 0.9, 0.99, 0.01)$	
		AIC	Ex. Kurtosis.	AIC	Ex. Kurtosis.	AIC	Ex. Kurtosis.
AEGAS-N	Normal	2.9740	0.0469	3.2348	0.0288	7.1088	-0.0355
	T	2.9743	0.0470	3.2349	0.0303	7.1093	-0.0355
	Skew t	2.9744	0.0468	3.2353	0.0308	7.1099	-0.0348
	Ged	2.9739	0.0480	3.2345	0.0296	7.1090	-0.0362
AEGAS-T	Normal	3.0065	3.0046	3.2338	2.0086	6.7503	2.0809
	T	2.9408	3.1205	3.1834	2.1655	6.6899	2.0538
	Skew t	2.9405	3.1180	3.1830	2.1661	6.6897	2.0544
	Ged	2.9502	3.1683	3.1911	2.0491	6.6989	2.0612
AEGAS-Skew t	Normal	2.9722	3.2488	3.1983	1.5378	6.9671	1.4054
	T	2.9005	3.2852	3.1538	1.4843	6.9220	1.3410
	Skew t	2.9003	3.2830	3.1534	1.4843	6.9217	1.3410
	Ged	2.9124	3.2267	3.1556	1.4871	6.9235	1.3532
AEGAS-Ged	Normal	2.9745	-0.0236	3.2928	-0.0084	6.9215	-0.0284
	T	2.9749	-0.0236	3.2928	-0.0086	6.9220	-0.0284
	Skew t	2.9753	-0.0236	3.2931	-0.0088	6.9222	0.0286
	Ged	2.9749	-0.0236	3.2924	-0.0084	6.9219	-0.0285

AEGAS (1,1) is defined as Asymmetric Exponential Generalized Autoregressive Score of order 1

Table 3 presents the results when the generating process is AEGAS under the four distributional assumptions and the model with the least AIC is judged to fit best. When Normal distribution was assumed, GAS-GED fits best at both low and medium persistence, whereas GAS-Normal fits best at high persistence. When Student-t and Skewed-Student-t were assumed, GAS-Skewed-Student-t fits best at all level of persistence. When GED was assumed, GAS-Normal fits best at both low and high persistence while GAS-GED fits best at medium persistence.

3.2 International Crude Oil Prices

The plot of the oil prices and the estimates of AIC and excess kurtosis are presented in Figure 1 and Table 4 respectively.

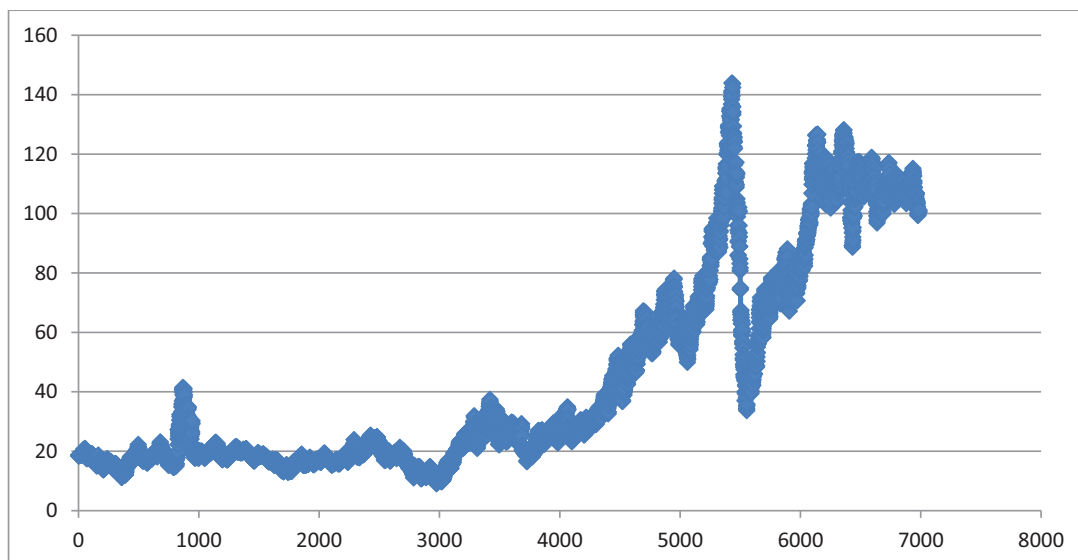


Figure 1: Plot of Oil Prices

Table 4: Specification tests for Models for Crude Oil prices

Estimated Model	Probability Distribution	AIC	Ex.Kurtosis
GAS	Normal	-6.6887	1.9414
	T	-6.7414	2.4315
	Skewed- t	-6.7420	2.4438
	Ged	-6.7392	2.1768
EGAS	Normal	-6.6873	1.9509
	T	-6.7427	2.3643
	Skewed- t	-6.7433	2.3781
	Ged	-6.7361	2.1424
AEGAS	Normal	-6.6874	1.9403
	T	-6.7432	2.4034
	Skewed- t	-6.7438	2.4193
	Ged	-6.7405	2.1204

In Table 4, the GAS, EGAS and AEGAS models were fitted to the international crude oil prices with four probability distributions assumed for the returns. The estimates of the AICs showed that specifying the innovations of the GAS, EGAS and AEGAS models as Skew- t distributions fits better than the other assumed distributions since it has the least AIC under each estimated model.

4. DISCUSSION AND CONCLUSION

In this paper, we have investigated the specification of the distributional assumptions of the innovations of GAS, EGAS and AEGAS models through Monte Carlo approach with extension to international crude oil prices. Four distributional assumptions were considered namely; Normal, Student- t , Skewed- t and Generalised error distribution with the estimates of Akaike Information Criterion and Kurtosis used to access model performance.

Based on the estimates of AIC for both the Monte Carlo simulations and the international crude oil, our results showed that when the generating process was GAS, EGAS and AEGAS with Student-t and Skew-t distributions assumed for the innovations, specifying the innovations as Skew-t distribution fits better than the other distributional assumptions at low, medium and high persistence. That is, specifying these models as GAS-Skew-t, EGAS-Skew-t and AEGAS-Skew-t fits better. However, when the generating process was GAS, EGAS and AEGAS with Normal distribution assumed for the innovations, specifying the innovations as Generalised error distribution fits better at low, medium and high persistence except for specifying the innovations of EGAS and AEGAS as Normal at low and high persistence respectively. When the generating process is AR-GARCH with Normal distribution assumed the AR-GARCH-GED fits better and is expected to realize better forecasts than AR-GARCH-Normal model Engle and Patton (2001).

Based on the estimates of the excess kurtosis, our results showed that when the generating processes for the GAS model and its variants assumed Normal and Generalised error distributions, the estimated innovations are close to normal whereas when the generating processes for the GAS model and its variants assumed Student-t and Skewed-Student-t, the estimated innovations deviates from normality. This shows that the probability distribution assumed has tail effect on the innovations and we recommend Skewed-Student-t distribution to be assumed over Student-t distribution for a better fit whereas GED should be assumed over Normal distribution where there is absence of tail effect.

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