

Genetic Algorithm Based Hybrid Approach to solve Multi-objective Interval Transportation Problem

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ABSTRACT

In this work, we determine the best effective solution of multi-objective interval transportation problem (MOITP) with genetic algorithm (GA) based hybrid approach. Taking the different cases involve shape parameter and aspiration level in exponential membership function, the higher degree of satisfaction can be obtained. Moreover, it provides to decision maker (DM) several different choices to acquire a protable decision in their area. Proposed work optimize more than one objective functions by taking the highest degree of satisfaction. Moreover, the objective function corresponding to the problem in this paper are non-commensurable, additionally and they coflict with one another. With realistic circumstances, a numerical instance is supplied with a statistic set to check the eectiveness of the projected GA approach.

Keywords: Interval cost, Transportation problem, GA, risk attitude parameter.

2000 Mathematics Subject Classification: 90B06, 90C60, 90C90.

1 Introduction

The one called transportation problem (TP) (Ford Jr and Fulkerson, 1956) is exceptionally well known, amongst numerous particular linear programming problems related with transportation of commodity initially stored at the different origin (supply centre) to the various destination (demand centre). The model of the TP contains a given number of supplies, demands and with one or more objectives which are to be optimized. F.L. Hitchcock (Hitchcock, 1941) first developed the TP and its model. Because of the models wide application in a decision-making process, DM always interested in the optimal solution of TP. Besides cost, delivery time, the total goods shifted or transported etc., are considered as the objectives, therefore instead of single objective transportation problem, the study and resolution of multi-objective transportation problem (MOTP) is more essential. Moreover, in day to day life situations, the parameters

related to TP should be an imprecise, this impreciseness is due to the peoples restricted understanding associated with problematic areas or inaccurate data. This inaccurate information on decision parameter represented by interval numbers or fuzzy number (Elmegeed, Salama and Elsayed, 2017).

In mathematics interval programming models deal with interval coefficients i.e. the parameters involved in TP like supply, demand, coefficient in objective function etc. some or all are in interval form. K.Kavitha and P. Pandian (Kavitha and Pandian, 2012) suggests the upper-lower method to find out the various levels in costing of the interval TP. M.R. Sa and A. Razmjoo (Safi and Razmjoo, 2013) gave the solution of fixed charge TP with an interval form parameter. A. Baidya et. al. (Baidya, Bera and Maiti, 2014) find the solution of MOITP using different methods with safety measure. Yu et. al. (Yu, Hu and Chang, 2015) formulate the MOTP with the parameter of supply, demand and in objective function expressed by interval and give the solution of it by an interactive approach.

Here, the author wants to explore the GA based solution of MOTP with interval parameters. The risk attitude parameter with three choices -0.5, 0 and 0.5 is used here to make interval cost into deterministic cost. Along with this, the exponential membership function which involves the different combination of shape parameter and aspiration level is used to provide the best and number of solutions to decision-maker. To exemplify this process, the author has illustrated a numerical example with 3 supply centers, 4 demand destination and 2 objectives whose coefficient is in the form of interval.

2 Literature Survey

S.K. Das et. al. (Das, Goswami and Alam, 1999) gives the solution of MOTP with cost objective, supply and demand are in interval form. They use the different width of an interval and convert the model into a crisp model which was solved by fuzzy programming method. Using fuzzy programming technique and different membership function, the solution of MOITP was given by D. Rani and A. G. Kumar (Rani and Kumar, 2010). The risk attitude parameter given by the DM, convert the given problem into crisp form. S. Kai et. al. (Kai, Yang, Lu and Zhi-ping, 2011) use the DMs risk attitude parameter and find the solution of an interval multi-objective assignment problem. They take three different choices of risk attitude parameter say optimistic, neutral and pessimistic and corresponding to these choices the solution also be presented. Also, using the grey situation decision-making theory, J. G. Patel and J. M. Dhodiya (Patel and Dhodiya, 2017) presented the solution of MOITP. Here, we use the GA based hybrid approach with exponential membership function and provide the number of solutions to the DM for better decesion.

3 The General Mathematical Model of MOITP

The Formulation of MOITP model is the minimization/maximization of k number of the objectives where penalty cost represented by an interval while other parameters are realistic. The mathematical model for MOITP is as follows as Mode-1 (Das et al., 1999).

Model-1

$$\text{Min } z_k = \sum_{u=1}^m \sum_{v=1}^n [C_{Luv}^k, C_{Ruv}^k] x_{uv}, \quad C_{Luv}^k < C_{Ruv}^k, \quad k = 1, 2, 3, \dots, K \quad (3.1)$$

Subject to,

$$\sum_{v=1}^n x_{uv} = a_u, \quad u = 1, 2, \dots, m \quad (3.2)$$

$$\sum_{u=1}^m x_{uv} = b_v, \quad v = 1, 2, \dots, n \quad (3.3)$$

$$\text{and } x_{uv} \geq 0 \quad \forall u, v. \quad (3.4)$$

Where a_u is the available amount of commodity at u^{th} origin, b_v is the demand of commodity at v^{th} destination, c^{uv} is the penalty charge associated to transport unit amount of commodity from u^{th} origin to v^{th} destination for objective function and x^{uv} is the unknown amount of commodity transported from u^{th} origin to v^{th} destination. Throughout the paper, we assume that $a_u > 0$, $b_v > 0$, $c_{Luv}^k \geq 0$ and $c_{Ruv}^k \geq 0$. Moreover, the condition $\sum_{u=1}^m a_u = \sum_{v=1}^n b_v$ should be satisfied.

4 Some Preliminaries

4.1 Interval arithmetic

Many application-oriented mathematical models in operation research as LP assume that the data used to build the model is exactly known i.e. the parameters involved in TP are in realistic form or easy to convert in crisp form. However, in the real situation, most of the data are in the form of approximate value that can be done within specific limits of the value. In the LP model, the ambiguity in data is normally associated with the coefficient of the model during the formulation stage. Also, in many real-life models, it is more realistic to represent some of the coefficients in some specified range i.e. in interval form instead of fixed real numbers. In real number R , the interval arithmetic deals with closed and connected sets. Generally, interval number denoted by a lower limit and upper limit like $\tilde{I} = [I_L, I_U] = \{x/I_L \leq x \leq I_U, x \in R\}$, here, I_L and I_U denotes the lowest and highest value of the interval respectively.

4.2 Arithmetic operations

Arithmetic operations on interval numbers are well known. Given any two positive real numbers $\tilde{I} = [I_L, I_U]$ and $\tilde{J} = [J_L, J_U]$; $I_L, J_L > 0$ and any real number α , the important operations of these two interval numbers can be represented as follows (Yu et al., 2015).

1. Addition

$$\tilde{I} + \tilde{J} = [I_L + J_L, I_U + J_U]$$

2. Multiplication

$$\begin{aligned} \tilde{I} * \tilde{J} &= [\alpha, \beta]; \\ \alpha &= \min \{I_L J_L, I_L J_U, I_U J_L, I_U J_U\} \\ \beta &= \max \{I_L J_L, I_L J_U, I_U J_L, I_U J_U\} \end{aligned}$$

3. Division

$$\tilde{I}/\tilde{J} = \left[\frac{I_L, I_U}{J_L, J_U} \right] = [I_L, I_U] \left[\frac{1}{J_U}, \frac{1}{J_L} \right]; 0 \notin [J_L, J_U]$$

4. Scalar multiplication

$$\begin{aligned} \alpha \tilde{I} &= [\alpha I_L, \alpha I_U], \alpha > 0 \\ &= [\alpha I_U, \alpha I_L], \alpha < 0 \end{aligned}$$

4.3 Fuzzy membership function

With the help of exponential membership function, the data related to the given problem can be normalized. If the number L_R and U_R stand for the minimum (negative ideal solution) and maximum values (positive ideal solution) of objective z_r^ε respectively then the fuzzy membership function is written as follows (Gupta, Mehlaawat and Mittal, 2013).

$$\mu_{z_r^\varepsilon}(x) = \begin{cases} 1, & \text{if } z_r^\varepsilon \leq L_r \\ \frac{e^{S\varphi_r^\varepsilon(x)} - e^{-S}}{1 - e^{-S}}, & \text{if } L_r < z_r^\varepsilon < U_r \\ 0, & \text{if } z_r^\varepsilon \geq U_r \end{cases} \quad (4.1)$$

Where, $\varphi_r^\varepsilon(x) = \frac{z_r^\varepsilon(x) - z_r^\varepsilon(x)^{PIS}}{z_r^\varepsilon(x)^{NIS} - z_r^\varepsilon(x)^{PIS}}$, s is DMs shape parameter and $0 \leq \mu_{z_r^\varepsilon}(x) \leq 1$.

5 Risk Attitude Parameter

For a positive interval number $\tilde{x} = [x_L, x_U]$, the interval mapping function is dened as follows.

$$\phi_\varepsilon(x) = m(\tilde{x}) + (\varepsilon) d(\tilde{x}) \quad (5.1)$$

Where ε is Decision makers (DM) risk attitude parameter (Kai et al., 2011) for uncertain data of real-world problem and it is $|\varepsilon| \leq 0.5$, $m(\tilde{x}) = \frac{x_U + x_L}{2}$ is the center of $\tilde{x}$ and $d(\tilde{x}) = x_U - x_L$ is the dierence of an interval $\tilde{x}$. For effective transportation process, the DM gives risk attitude parameter. With dfferent values of risk attitude parameter, it can be divided into three categories, i.e. optimistic for $-0.5 \leq \varepsilon < 0$, most likely for $\varepsilon = 0$ and pessimistic for $0 < \varepsilon \leq 0.5$. Once the risk attitude parameter can be determined then interval value converts in crisp values by (5.1).

5.1 Formulation of crisp multi-objective transportation model using risk attitude parameter

Let $z_k = [z_{uvk}]$ is the cost (prot and loss) matrix with respect to the objective k , where $[z_{uvk}] = [z_{uvk}^L, z_{uvk}^U]$ is representing the cost associated with k^{th} objective to transport from u^{th} origin to v^{th} destination. This $z_k = [z_{uvk}]$ is converted into crisp values $z_k^\varepsilon = [z_{uvk}^\varepsilon]$ (Kai et al., 2011) by using the following equation.

$$z_k^\varepsilon = [(z_{uvk}^\varepsilon)] = \frac{z_{uvk}^U + z_{uvk}^L}{2} + \varepsilon (z_{uvk}^U - z_{uvk}^L), \quad u = 1, 2, \dots, m, \quad v = 1, 2, \dots, n. \quad (5.2)$$

With the help of this, the MOITP is transformed into a crisp multi-objective transportation problem (CMOTP) as following Model 2.

Model-2

$$\text{Min } z_k^\varepsilon = [z_{uvk}^\varepsilon] \quad (5.3)$$

Subject to, (3.2) to (3.4) .

6 Method for the solution of Crisp Model

It is very imperative to gain the solution of any optimization problem which can reflect reliable judgment of the DMs. With a view to get desirable results that can fulfil the DMs demand, the aspiration level should be selected accordingly. The attitude and requirement of DM is changed during the decision-making process hence, the aspiration level is not decided with any consistency. With fuzzy membership functions like hyperbolic, linear, piece-wise linear, exponential and tangent, etc. one can define DMs different aspiration levels. To describe the performance of vague information, the membership function is used.

The GA is a technique for getting the solution of both constrained and unconstrained type optimization problems and it is primarily based on natural selection (Dhodiya and Tailor, 2016). The algorithm continuously modifies a population of individual solutions for optimizing the more and more effective solution for the given optimization problem. In other words, it is an optimization technique, which tries to discover such values of input so that we get the new output values or consequences. GA can be applied to optimization problems that are not well appropriate for usual optimization algorithms containing objective function is discontinuous, stochastic or uncertain, nonlinear etc. Mainly the population of the individual in each generation is modified and try to find a better and better solution. By taking the highest degree of satisfaction, the defined strategy optimizes each goal simultaneously.

We use GA with the following steps and find the solution of MOITP as follows.

1. According to risk attitude parameter, define the crisp multi-objective transportation Model-2.
2. Convert the maximum objective function of the problem into a minimum form.
3. Find out the positive ideal solution (PIS) and negative ideal solution (NIS) i.e. the maximum and minimum value for each objective function.
4. Find exponential membership value for each z_r^ε using (4.3).
5. Convert MOITP into single-objective optimization problems (SOP) as Model-3.

Model-3

$$\text{Max } \lambda = \prod_{k=1}^K \mu_k^\varepsilon(x) \quad (6.1)$$

Subject to (3.2) to (3.4) and

$$\mu_{z_k^\varepsilon}(x) = \frac{\overline{\mu_{z_k^\varepsilon}(x)}}{\overline{\mu_{z_k^\varepsilon}(x)} - \overline{\mu_{z_k^\varepsilon}(x)}}$$

Where, $\overline{\mu_{z_k^\varepsilon}(x)}$ is the DMs aspiration level for k^{th} objective function.

6. To deal with the SOP of Model-3, GA is used with various choices of the shape parameter and aspiration level. GA is applied with step (a) to (g).

(a) **Chromosome Encoding**

Any optimization problem while optimized by GA the chromosomes encoding of that problem is the first requirement. GA works with coding of parameter set rather than the actual value of parameters. In GA, each solution of the problem is represented by some finite string say chromosome and similar way every chromosome in GA represents the information about the solution. Here, we use the spanning tree-based prufer number encoding given by Gen et. al. (Gen, Li and Ida, 1999) in GA. The solution of TP involving m origins and n destinations contains $m+n+1$ basic cell in transportation tableau. These cells are linearly independent and so they can not contain any cycle, so it can be considered as tree (transportation tree) (Darus, Ibrahim and Karim, 2018). If we denote the origin center and destination center as nodes or vertices of the tree, then there are separable set of these vertices. There are $m+n$ vertices connected with edges in the transportation tree. If we consider the origins as nodes of one set and destinations as nodes of another set and drawing the edge between these nodes of these two set if there is an allocation between them, then we find a spanning tree with $m + n$ vertices and $m+n+1$ edges. We use the following step to find prufer number code given by Gen et. al. (Gen et al., 1999) to find the solution of transportation problem using GA.

Write prufer number from given spanning tree

We use the following step to write prufer number from any spanning tree:

- i. Let i be the pendant vertex with lowest numbered connected by an edge with j numbered pendant vertex in spanning tree. Then the code for GA contains number as j .
- ii. Eliminate the vertex i and edge (i, j) from the given spanning tree.
- iii. Find the next lowest numbered pendant vertex and similarly build the code by writing the vertex number right to j .
- iv. If only two vertices remaining then stop otherwise repeat step (i) to (iii).

Drawn spanning tree from given prufer number.

We use the following steps.

- i. Let P be the given prufer number, P be the set contains nodes not appear in P and considered as appropriate for concern.
- ii. Repeat the following step (i-v) until no digit left in P .
- iii.

(b) **Encoding**

The first and important task for the GA to optimize the problem is the encoding of the chromosome. To generate the solution of the MOITP chromosome must be there which represent your solution. Here, we set all 0s to all $(m + n - 1)$ genes on a chromosome, and then we randomly generate the chromosome encoding according to the problem demand.

(c) **Fitness Function Evaluation**

A fitness function or evaluation function is the numerical value associated with the given chromosome. The chromosome comes with satisfying all the problems constraints is evaluated.

(d) **Selection**

Selection is the step at which the two-parent recombine to create the o-springs with greater fitness than the parent at each subsequent generation. There are many selection processes in GA, as the tournament selection is very easy to apply with sufficient efficiency, we prefer it for our Approach. In tournament selection, parents are selected with higher fitness from k -individuals by running the tournament among them and these parents are passed on to the next generation.

(e) **Crossover**

Next important step after selection in GA was a crossover. Same like selection operator, In Crossover operator, two parents are combined to produce the new child solution with higher fitness value. Crossover is used according to the encoding of the solution. In this paper, we use a one-point crossover. It is easy and simple to apply for any encoding. Moreover, one-point crossover preserves the nature of encoding. In one-point crossover common genes are transferred between two parents so that new child with higher degree is obtained.

(f) **Mutation**

In GA, it is important that the solution at each step should be updated. By mutation operator, the GA gets back the lost genetic information. Same like crossover after mutation new offspring will generate from the selected parents with higher degree.

(g) **Termination Criteria**

Initially, we have to enter the number of iterations we want after completing an entered number of iterations the algorithm will automatically stop and display the required optimal solution. The termination condition of a Genetic Algorithm is essential in figuring out when a GA run will cease. Also, termination criteria needed to avoid time consumption and unnecessary computation. Mostly following three termination criteria are famous.

- i. When there is no further improvement in the solution. i.e. an efficient solution obtained.
- ii. When we come to our initially entered number of iterations.
- iii. When the objective values were pre-defined and we reached it. We continue to change the value of the risk attitude parameter, shape parameter and repeat the step (a) to (g) until the DMs satisfactory efficient solution achieved.

7 Numerical Example and Result Analysis

A company has three production facilities (origins), and with production capacity of 8, 19 and 17 units of a product respectively. These units are to be shipped to four warehouses B_1, B_2, B_3

and B_4 with requirement of 11, 3, 14 and 16 units respectively. The transportation cost and transportation time between companies to warehouses are given below (Patel and Dhodiya, 2017).

$$U^1 = \begin{bmatrix} [1, 2] & [1, 3] & [5, 9] & [4, 8] \\ [1, 2] & [7, 10] & [2, 6] & [3, 5] \\ [7, 9] & [7, 11] & [3, 5] & [5, 7] \end{bmatrix} \quad U^1 = \begin{bmatrix} [3, 5] & [2, 6] & [2, 4] & [1, 5] \\ [4, 6] & [7, 9] & [7, 10] & [9, 11] \\ [4, 8] & [1, 3] & [3, 6] & [1, 2] \end{bmatrix}$$

First, we convert the above interval transportation problem into crisp interval transportation problem using the risk attitude parameter technique as described above.

For, $\epsilon = -0.5$ (risk attitude parameter) above two objectives given by,

$$U_1^\epsilon = \begin{bmatrix} 1 & 1 & 5 & 4 \\ 1 & 7 & 2 & 3 \\ 7 & 7 & 3 & 5 \end{bmatrix} \quad U_2^\epsilon = \begin{bmatrix} 3 & 2 & 2 & 1 \\ 4 & 7 & 7 & 9 \\ 4 & 1 & 3 & 1 \end{bmatrix}$$

The crisp Model corresponding to the above cost matrix is given by,

$$\text{Min } z_1^\epsilon = 1x_{11} + 1x_{12} + 5x_{13} + 4x_{14} + 1x_{21} + 7x_{22} + 2x_{23} + 3x_{24} + 7x_{31} + 7x_{32} + 3x_{33} + 5x_{34}$$

$$\text{Min } z_2^\epsilon = 3x_{11} + 2x_{12} + 2x_{13} + 1x_{14} + 4x_{21} + 7x_{22} + 7x_{23} + 9x_{24} + 4x_{31} + 1x_{32} + 3x_{33} + 1x_{34}$$

$$\text{with, } \sum_{u=1}^3 x_{u1} = 11, \sum_{u=1}^3 x_{u2} = 3, \sum_{u=1}^3 x_{u3} = 14, \sum_{u=1}^3 x_{u4} = 16, \sum_{v=1}^4 x_{1v} = 8, \sum_{v=1}^4 x_{2v} = 19, \sum_{v=1}^4 x_{3v} = 17$$

Similarly, for $\epsilon = 0$ and $\epsilon = 0.5$ the crisp model can be written. For each objective function in Model-3, Table 1 and Table 2 offers the PIS and NIS value at different risk attitude parameter and the cases of shape parameter and aspiration level respectively.

Table 1: PIS and NIS value of objective function

ϵ	Value	Z_1^ϵ	Z_2^ϵ
$\epsilon = -0.5$	PIS	110	131
	NIS	210	243
$\epsilon = 0$	PIS	148.5	172
	NIS	269.5	305.5
$\epsilon = 0.5$	PIS	187	211
	NIS	329	368

Table 2: Case with shape parameter and aspiration level

Case	Shape parameters (K_1, K_2)	Aspiration-level ($\mu_{z_1^\epsilon}, \mu_{z_2^\epsilon}$)
Case-1	(-5,-5)	0.7, 0.8
Case-2	(-4,-4)	0.75, 0.8
Case-3	(-3,-3)	0.8, 0.9
Case-4	(-2,-2)	0.85, 0.8
Case-5	(-1,-1)	0.6, 0.65

According to risk attitude parameters, the GA based solution of the given MOITP is reported

in Table 3, Table 4 and Table 5 contains the value of product operator, degree of satisfaction, membership value, objective value and solution matrix corresponding to each cases specified by the DM.

Table 3: GA based solution of MOITP at $\varepsilon = -0.5$.

Case	λ	Degree of Satisfaction	$(\mu_{Z_1^\varepsilon}, \mu_{Z_2^\varepsilon})$	Objective Value	Allocations
1	0.9905	0.9938	(0.9938, 0.9966)	(123,140)	$\begin{bmatrix} 0 & 0 & 0 & 8 \\ 11 & 3 & 0 & 5 \\ 0 & 0 & 14 & 3 \end{bmatrix}$
2	0.9803	0.9873	(0.9873, 0.9929)	(123,140)	$\begin{bmatrix} 0 & 3 & 0 & 5 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 6 & 11 \end{bmatrix}$
3	0.9611	0.9750	(0.9750, 0.9857)	(123,140)	$\begin{bmatrix} 0 & 3 & 0 & 5 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 6 & 11 \end{bmatrix}$
4	0.9275	0.9535	(0.9535, 0.9727)	(123,140)	$\begin{bmatrix} 0 & 3 & 5 & 0 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 1 & 16 \end{bmatrix}$
5	0.8744	0.9192	(0.9192, 0.9513)	(123,140)	$\begin{bmatrix} 0 & 0 & 8 & 0 \\ 11 & 0 & 6 & 2 \\ 0 & 2 & 0 & 14 \end{bmatrix}$

Table 4: GA based solution of MOITP at $\varepsilon = 0$.

Case	λ	Degree of Satisfaction	$(\mu_{Z_1^\varepsilon}, \mu_{Z_2^\varepsilon})$	Objective Value	Optimum Allocation
1	0.9786	0.9869	(0.9869, 0.9916)	(174.5,193.5)	$\begin{bmatrix} 0 & 0 & 0 & 8 \\ 11 & 0 & 0 & 8 \\ 0 & 3 & 14 & 0 \end{bmatrix}$
2	0.9581	0.9746	(0.9746, 0.9830)	(174.5,193.5)	$\begin{bmatrix} 0 & 3 & 0 & 5 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 6 & 11 \end{bmatrix}$

3	0.9216	0.9526	(0.9526, 0.9670)	(174.5,193.5)	$\begin{bmatrix} 0 & 3 & 0 & 5 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 6 & 11 \end{bmatrix}$
4	0.8615	0.9160	(0.9160, 0.9405)	(174.5,193.5)	$\begin{bmatrix} 0 & 3 & 0 & 5 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 6 & 11 \end{bmatrix}$
5	0.7730	0.8605	(0.8605, 0.8983)	(174.5,193.5)	$\begin{bmatrix} 0 & 3 & 0 & 5 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 6 & 11 \end{bmatrix}$

 Table 5: GA based solution of MOITP at $\varepsilon = 0.5$

Case	λ	Degree of Satisfaction	$(\mu_{Z_1^\varepsilon}, \mu_{Z_2^\varepsilon})$	Objective Value	Optimum Allocation
1	0.9657	0.9800	(0.9800, 0.9854)	(226, 247)	$\begin{bmatrix} 0 & 3 & 0 & 5 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 6 & 11 \end{bmatrix}$
2	0.9357	0.9627	(0.9627, 0.9720)	(226, 247)	$\begin{bmatrix} 0 & 3 & 0 & 5 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 1 & 16 \end{bmatrix}$
3	0.8846	0.9330	(0.9330, 0.9482)	(226, 247)	$\begin{bmatrix} 0 & 0 & 8 & 0 \\ 11 & 0 & 0 & 8 \\ 0 & 3 & 6 & 11 \end{bmatrix}$
4	0.8048	0.8854	(0.8854, 0.9089)	(226, 247)	$\begin{bmatrix} 0 & 3 & 0 & 5 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 6 & 11 \end{bmatrix}$
5	0.6999	0.7307	(0.7307, 0.9578)	(241, 222)	$\begin{bmatrix} 0 & 3 & 0 & 5 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 1 & 16 \end{bmatrix}$

Above Table 3, Table 4 and Table 5 gives the value of objective function, degree of satisfaction, product operator and allocations corresponding to different cases involving in Table 2. It is also shows that the change in shape parameter, influence the degree of satisfaction level for each objective function. The above results are according to the DM's preference, he/she can add more cases if he/she is not satisfied with the obtained solution.

8 Convergence Criteria of GA for MOITP

To find the convergence criteria of GA for $(-3, -3)$ shape parameter and $(0.8, 0.9)$ aspiration level at different risk attitude parameters $\epsilon = -0.5$, $\epsilon = 0$ and $\epsilon = 0.5$, we have used different number of population and different number of iteration which are shown with value of degree of satisfaction in the Figure 1.

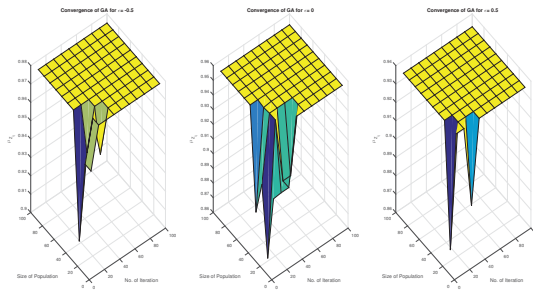


Figure 1: Convergence of GA at $(-3, -3)$ shape parameter and $(0.8, 0.9)$ aspiration level.

Figure 1 indicate that the developed algorithm converges after 10 iteration and 20 or more than 20 population for $\epsilon = -0.5$. Similarly, the developed algorithm converges after 50 iterations with 10 or more than 10 populations for $\epsilon = 0$ and 30 iterations and 10 or more than 10 populations for $\epsilon = 0.5$. Figure 2, Figure 3 and Figure 4 provided alternative choices of the solutions to DM as per his/her necessity. For other shape parameter and aspiration level, the convergence criteria of GA approximately remain same.

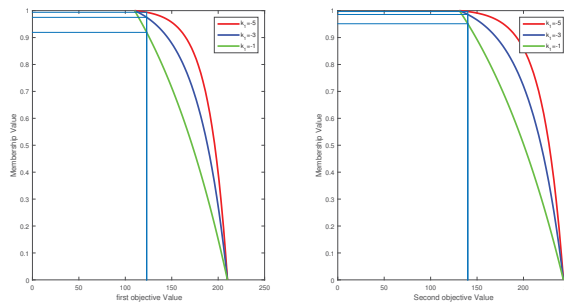


Figure 2: The degree of satisfaction for $\epsilon = -0.5$

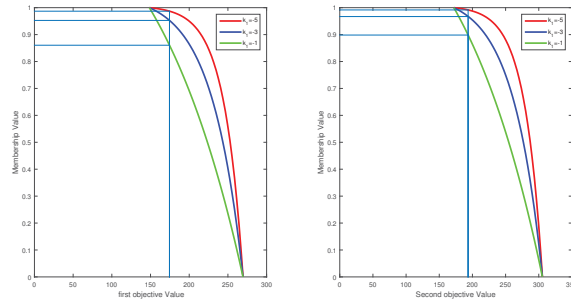


Figure 3: The degree of satisfaction for $\epsilon = 0$

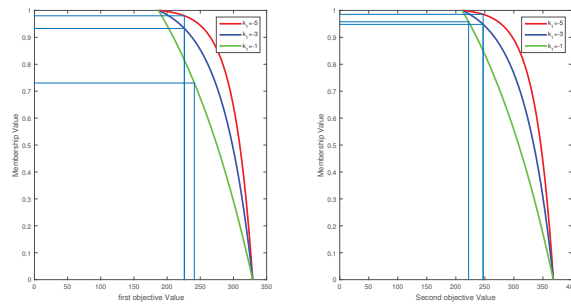


Figure 4: The degree of satisfaction for $\epsilon = 0.5$

Figure 5, Figure 6 and Figure 7 indicates, the efficient solutions of first and second objective as (123, 140) at $\epsilon = -0.5$, (174.5, 193.5) at $\epsilon = 0$ and (226, 247) at $\epsilon = 0.5$ for $(-5, -5)$, $(-4, -4)$, $(-3, -3)$ and $(-2, -2)$ shape parameters and its corresponding (0.7, 0.8), (0.75, 0.8), (0.8, 0.9) and (0.85, 0.8) aspiration level respectively.

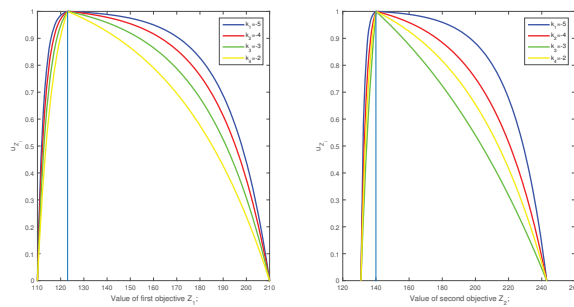


Figure 5: The distribution of objective functions for $\epsilon = -0.5$

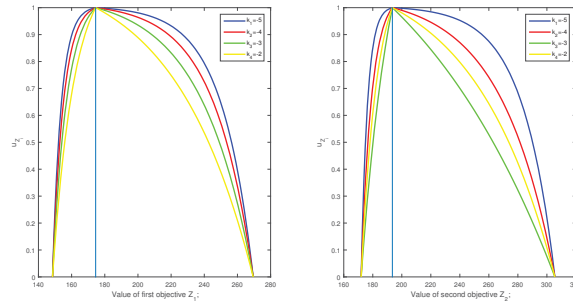
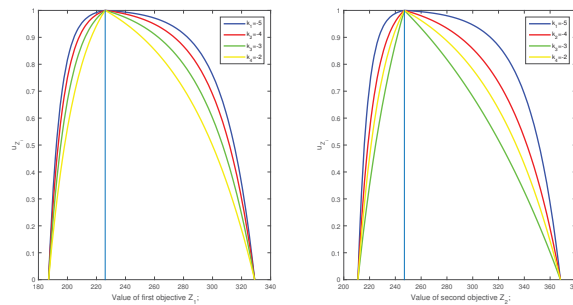

 Figure 6: The distribution of objective functions for $\epsilon = 0$

 Figure 7: The distribution of objective functions for $\epsilon = 0.5$

Figure 5 shows the degree of satisfaction levels of objectives as (0.9938, 0.9966), (0.9873, 0.9929), (0.9750, 0.9857), (0.9535, 0.9727) and (0.9192, 0.9513) at $\epsilon = -0.5$ for (-5, -5), (-4, -4), (-3, -3), (-2, -2) and (-1, -1), shape parameters and its corresponding (0.7, 0.8), (0.75, 0.8), (0.8, 0.9) and (0.85, 0.8) and (0.6, 0.65) aspiration level respectively. Similar Figure 7 and Figure 8 shows the degree of satisfaction levels of objectives as (0.9865, 9916), (0.9746, 0.9830), (0.9526, 0.9670), (0.9160, 0.9405) and (0.8605, 0.8983) at $\epsilon = 0$ for (-5, -5), (-4, -4), (-3, -3), (-2, -2) and (-1, -1), shape parameters and its corresponding (0.7, 0.8), (0.75, 0.8), (0.8, 0.9) and (0.85, 0.8) and (0.6, 0.65) aspiration level and (0.9800, 0.9854), (0.9627, 0.9720), (0.9330, 0.9482), (0.8854, 0.9089) and (0.7307, 0.9578) for (-5, -5), (-4, -4), (-3, -3), (-2, -2) and (-1, -1), shape parameters and its corresponding (0.7, 0.8), (0.75, 0.8), (0.8, 0.9) and (0.85, 0.8) and (0.6, 0.65) aspiration level respectively.

9 Comparison

To further review of developed hybrid approach, here is the advantage of this hybrid approach on following approaches.

Table 6: Solution with different approaches

Deepika Rani (Rani and Kumar, 2010) (Linear membership function)	Deepika Rani (Rani and Kumar, 2010) (Exponential membership function)	Deepika Rani (Rani and Kumar, 2010) (Hyperbolic membership function)	S. K. Das, A. Goswami and S. Alam (Das et al., 1999)	Jignasha Patel and Jayesh Dhodiya (Patel and Dhodiya, 2017)
[172.2,222.55] [206.1,252.75]	[171.50,221.63] [207.54,254.36]	[172.2,222.55] [206.1,252.75]	[119.14,214.42] [180.64,241.1]	[146,241] [133,222]

Solution of MOITP using GA based hybrid approach

Following are the solutions with GA based hybrid approach corresponding to cases of Table 2.

Table 7: Solution by GA based hybrid approach.

Case	Risk attitude parameter (ϵ)	Objective value	Solution
Case-1	-0.5	$Z_1 = 123 \ Z_2 = 140$	$\begin{bmatrix} 0 & 0 & 0 & 8 \\ 11 & 3 & 0 & 5 \\ 0 & 0 & 14 & 3 \end{bmatrix}$
	0	$Z_1 = 174.5 \ Z_2 = 193.5$	$\begin{bmatrix} 0 & 0 & 0 & 8 \\ 11 & 0 & 0 & 8 \\ 0 & 3 & 14 & 0 \end{bmatrix}$
	0.5	$Z_1 = 226 \ Z_2 = 247$	$\begin{bmatrix} 0 & 3 & 0 & 5 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 6 & 11 \end{bmatrix}$
Case-2	-0.5	$Z_1 = 123 \ Z_2 = 140$	$\begin{bmatrix} 0 & 3 & 0 & 5 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 6 & 11 \end{bmatrix}$
	0	$Z_1 = 174.5 \ Z_2 = 193.5$	$\begin{bmatrix} 0 & 3 & 0 & 5 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 6 & 11 \end{bmatrix}$
	0.5	$Z_1 = 226 \ Z_2 = 247$	$\begin{bmatrix} 0 & 3 & 0 & 5 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 1 & 16 \end{bmatrix}$

Case	Risk attitude parameter (ϵ)	Objective value	Solution
Case-3	-0.5	$Z_1 = 123 \ Z_2 = 140$	$\begin{bmatrix} 1 & 1 & 5 & 4 \\ 1 & 7 & 2 & 3 \\ 7 & 7 & 3 & 5 \end{bmatrix}$
	0	$Z_1 = 174.5 \ Z_2 = 193.5$	$\begin{bmatrix} 0 & 3 & 0 & 5 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 6 & 11 \end{bmatrix}$
	0.5	$Z_1 = 226 \ Z_2 = 247$	$\begin{bmatrix} 0 & 0 & 8 & 0 \\ 11 & 0 & 0 & 8 \\ 0 & 3 & 6 & 11 \end{bmatrix}$
Case-4	-0.5	$Z_1 = 123 \ Z_2 = 140$	$\begin{bmatrix} 0 & 3 & 5 & 0 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 1 & 16 \end{bmatrix}$
	0	$Z_1 = 174.5 \ Z_2 = 193.5$	$\begin{bmatrix} 0 & 3 & 0 & 5 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 6 & 11 \end{bmatrix}$
	0.5	$Z_1 = 226 \ Z_2 = 247$	$\begin{bmatrix} 0 & 3 & 0 & 5 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 6 & 11 \end{bmatrix}$
Case-5	-0.5	$Z_1 = 123 \ Z_2 = 140$	$\begin{bmatrix} 0 & 0 & 8 & 0 \\ 11 & 0 & 6 & 2 \\ 0 & 2 & 0 & 14 \end{bmatrix}$
	0	$Z_1 = 174.5 \ Z_2 = 193.5$	$\begin{bmatrix} 0 & 3 & 0 & 5 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 6 & 11 \end{bmatrix}$
	0.5	$Z_1 = 226 \ Z_2 = 247$	$\begin{bmatrix} 0 & 3 & 0 & 5 \\ 11 & 0 & 8 & 0 \\ 0 & 0 & 1 & 16 \end{bmatrix}$

Table 6 represents, the individual solution to MOITP with different approaches and Table 7 represents that GA based hybrid approach with risk attitude parameter theory. can effectively handle uncertainty in the objective function of MOITP and provides the set of solutions for MOITP. GA based hybrid approach with risk attitude provides a situation wise, efficient solution with sensitivity analysis to DM and many alternatives for DM to take profits and best decision in the decision making process.

10 Conclusion

Due to some unexpected factor the parameters in TP is in form of interval and its solution is possible with the genetic algorithm based hybrid approach. The approach can effectively solve the MOITP and provided the batter choice and numbers of solution to the DM for taking better decision in their area. The exponential membership function uses the different combination of

shape parameter and aspiration level which helps to obtain the different solutions. Moreover, if the DM is not satisfying with the obtained solution, he/she can add a greater number of cases with different shape parameter and aspiration level.

11 Future Scope

Here, the problem was solved by considering the objectives are in form of interval but not supply and demand. The problem can be extended to the case where the supply and demand are in form of interval. Also, by adding the conveyance the problem may extend to solid interval multi-objective transportation problem.

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