

Applications in bilinear time series models subset

Laid GASMI¹, Nabil LAICHE², and Zouaoui CHIKR EL MEZOUAR³

¹Depart. Mathematics and Computer Science
Ahmed Draia Univ., Adrar, Algeria.
gasmi1983@gmail.com, lai.gasmi@univ-adrar.dz

²Department of Mathematics
Badji Maokhtar Univ. Annabar, Algeria.
haninabil2012@gmail.com

³ Department of Mathematics, College of Science
King Khalid Univ., Abha, Saudi Arabia.
chikrtime@yahoo.fr.

ABSTRACT

This paper is an account of some discussions and concepts on bilinear time series models with coefficients under a time varying and continuous framework, these models gave a satisfying solution to practical problems in the finance, many of theoretical developments in finance have found immediate applications in finance market. In this paper we will address the estimation of a class of bilinear time series models where the coefficients of the model will be time varying and continuous functions, and our interest based for how use the observations of bilinear process to construct some methods for estimation and we complete this study with numerical illustrations.

Keywords: Bilinear models; estimation; maximum likelihood estimation; least squares approach

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1 Introduction

In recent decades, the theory of time series models took an evolutionary course by mathematicians, where we find that many economic phenomena are represented by stochastic models. There is no doubt that nonlinear time series models appeared in the statistical literature because linear models were unable to interpret or modelisation or simulation the most real-life phenomena. The impact of nonlinear time series models in the various domains is infinite, so in this context, statisticians have known the urgent need of these models and its values in development of other specialties, because many models in nature and in a variety of physical and engineering fields display some form of non-linear behavior, one such model of these models is bilinear model which was proposed by (Granger and Andersen, 1978) and (Rao, 1981), and the works of (Gabr and Rao, 1981) give some general discussions on stationarity, invertibility and methods of estimation. (Tong, 1981) studies the ergodicity of special bilinear models.

(Quinn, 1982) and (Hannan, 1982) discuss properties of some cases of these models. But the conditions of stationarity of time varying bilinear time series have been discussed by (Bibi and Oyet, 2002).

A great deal of attention has been paid to a particular class of nonlinear models namely bilinear models. Where there are a lot of papers discussing the category of these models for example see (Subba Rao and Gabr, 1984), (Bhaskara and Bhaskara, 1983), (Liu, 1990), and (Rahimpoor and Yarmohammadi, 2017).

The paper is structured as follows, we start, in **section 2**, with a brief discussions of some preliminaries in subset of bilinear time series models.

In **section 3**, we will introduce a methods of estimation for the coefficients of the model. In **section 4**, we give as an example a model modilized by bilinear model. **Section 5**, we give some numerical illustrations and simulation for the model and finally explain the results of simulation where the programming with Matlab 2009.

2 Preliminaries

Definition 2.1. We define the process $\{v_t, t \in \mathbb{Z}\}$ defined on a probability space (Ω, Γ, P) , is called bilinear process of order (p, q, r, s) and denoted by $BL(p, q, r, s)$, if and only if it satisfies the equation

$$v_t = \sum_{i=1}^p a_i v_{t-i} + \sum_{j=1}^q c_j \varepsilon_{t-j} + \sum_{i=1}^r \sum_{j=1}^s b_{ij} v_{t-i} \varepsilon_{t-j} + \varepsilon_t$$

Where $\{a_i\}_{1 \leq i \leq l_1}$, $\{c_j\}_{1 \leq j \leq l_2}$, $\{b_{ij}\}_{1 \leq i \leq l_3, 1 \leq j \leq l_4}$ are constants coefficients, the sequence $(\varepsilon_t)_{t \in \mathbb{Z}}$ is white noise process of independent random variables, not necessarily identically distributed with mean zero and variance. And often the white noise is written in the form $\varepsilon_t = \omega_t \eta_t$, where η takes Gaussian law values with zero mean and the variance equal to 1. And the sequence $(\omega_t)_{t \in \mathbb{Z}}$ of strictly positive numbers.

We consider the model of order

$$v_t = av_{t-p} + bv_{t-q}\varepsilon_{t-1} + \varepsilon_t \quad (2.1)$$

Where $(p, q) \in \mathbb{N}^* \times \mathbb{N}^*$, a and b are constantsso in case $p = q = 1$ we have the following theorem of(Pham and Tran, 1981).

Theorem 2.1. *There exists a unique strictly stationary process $(v_t)_{t \in \mathbb{Z}}$ satisfying the bilinear model $v_t = av_{t-1} + bv_{t-1}\varepsilon_{t-1} + \varepsilon_t$ under a condition $a^2 + \sigma^2 b^2 < 1$, which can be written as infinite series involving only present and the past of $(\varepsilon_t)_{t \in \mathbb{Z}}$ of the form*

$$v_t = \varepsilon_t + \sum_{j=1}^{\infty} \varepsilon_{t-j} \left[\prod_{i=1}^{j-1} (a + b\varepsilon_{t-i-1}) \right]$$

then this infinite series is almost surely convergent

Proof. We can write the model with recurrence

$$\begin{aligned} v_t &= (a + b\varepsilon_{t-1})v_{t-1} + \varepsilon_t \\ v_{t-1} &= (a + b\varepsilon_{t-2})v_{t-2} + \varepsilon_{t-1} \\ v_{t-2} &= (a + b\varepsilon_{t-3})v_{t-3} + \varepsilon_{t-2} \\ &\vdots \\ v_{t-k} &= (a + b\varepsilon_{t-k-1})v_{t-k-1} + \varepsilon_{t-k} \end{aligned}$$

So

$$v_t = \prod_{i=1}^k (a + b\varepsilon_{t-i-1})v_{t-k-1} + \sum_{j=1}^k \left\{ \prod_{i=1}^{j-1} (a + b\varepsilon_{t-i-1}) \right\} \varepsilon_{t-j} + \varepsilon_t$$

Where k tend towards ∞ we find that

$$v_t = \varepsilon_t + \sum_{j=1}^{\infty} \varepsilon_{t-j} \left[\prod_{i=1}^{j-1} (a + b\varepsilon_{t-i-1}) \right]$$

□

Now we are going to prove that this series is almost surely convergent. But in case where $p = q > 1$ and under the same condition, then the model will be

$$v_t = av_{t-p} + bv_{t-p}\varepsilon_{t-1} + \varepsilon_t \quad (2.2)$$

These models are discussed excellently by Bibi and Oyet in reference (Bibi and Oyet, 2004) where the unique strictly stationary model is

$$v_t = \varepsilon_t + \lim_{j \rightarrow +\infty} \sum_{j=1}^{\mathbf{I}(\frac{t}{p})-1} \varepsilon_{t-pj} \left\{ \prod_{i=0}^{j-1} (a + b\varepsilon_{t-ip-1}) \right\}$$

Where we denote by $\mathbf{I}(x)$ for the integer part of x .

In the case $p \neq q$, we can assume that there exists a function φ defined on a domain D of $\mathbb{R}$ such that $v_{t-q} = \varphi(t)v_{t-p}$. so in this situation our contribution based on looking a smaller interval containing the function $\varphi(t)$, then we estimate the parameters of model, the condition $a^2 + \sigma^2 b^2 \varphi^2(t) < 1$ for stationarity, then $\varphi^2(t) < \frac{1-a^2}{\sigma^2 b^2}$ so $-\sqrt{\frac{1-a^2}{\sigma^2 b^2}} < \varphi(t) < \sqrt{\frac{1-a^2}{\sigma^2 b^2}}$, if the parameter a tends to 1 then $\varphi(t)$ will converge to zero in the frame where the stationary model, we can now use the following numerical strategy to find an approximate estimate of the model parameters, in the first step we replace $\varphi(t)$ by $(\frac{1-a^2}{\sigma^2 b^2})^{\frac{1}{2}}$, so the model becomes

$$v_t = av_{t-p} + \sigma^{-1}(1-a)^{\frac{1}{2}}v_{t-p}\varepsilon_{t-1} + \varepsilon_t$$

Then

$$v_t = \varepsilon_t + \sum_{j=1}^{\mathbf{I}(\frac{t}{p})-1} \varepsilon_{t-pj} \left\{ \prod_{i=0}^{j-1} (a + \sigma^{-1}(1-a)^{\frac{1}{2}}\varepsilon_{t-ip-1}) \right\}$$

We consider the following bilinear time series

$$x_t = \Phi(t)v_{t-1}\varepsilon_{t-1} + \varepsilon_t \quad (2.3)$$

$(\varepsilon_t)_{t \in \mathbb{Z}}$ is a noise process sequence of independent random variables with zero mean and variance σ_t^2 , and $\{g(t)\}_{t \in \mathbb{Z}}$ is the sequence of coefficients in the form of continuous functions where

$$\Phi(t) = \int_{t_0}^t h(s) ds, \quad \forall t \in \mathbb{N}^*$$

$\{h(s), s \in [t_0, t]\}$ is a locally integrable function, and we consider that the condition $|\Phi(t)| < 1$ is verified.

There are several methods in statistical literature for estimating the parameters of stationary and ergodic bilinear time series models, but the most frequently used methods are the method of moments, and least squares method.

We suppose that ε_t is independent of past of v_t , as we can to say the initial values $v_t = 0$ and $\varepsilon_t = 0$ where $t < 1$. And let $\mathbf{H}_t$ the σ -field generated by the set $\{v_1, \dots, v_t\}$. and we consider $h(t) = \theta$, and θ_0 the true value.

So the algorithm of estimation with least squares method is based on Taylor's expansion iterative about the true value θ_0 ; see (Bibi and Oyet, 2004).

Definition 2.2. We define an ordinary least squares estimator the value solution $\hat{\theta}_n$ witch that expression is verived

$$\arg \min_{\theta \in \Delta} \{q(n, \theta)\}$$

Where Δ is the subset $\{\Phi(t), t \in \mathbb{N}^*\}$, and $q(n, \theta)$ is the penalty function defined by

$$q(n, \theta) = \frac{1}{n} \sum_{t=1}^n \varepsilon_t^2(\theta)$$

Where $\varepsilon_t(\theta) = x_t - x_{t|t-1}$, $x_{t|t-1}$ denote the orthogonal projection of x_t on the observations up to time $t - 1$ which coincide with $E(x_t | \mathbf{H}_{t-1})$, (in the sense of least squares method see ref [6], [12].

$$q(n, \theta) = q(n, \theta_0) + (\theta - \theta_0) \frac{\partial q(n, \theta_0)}{\partial \theta} + \frac{(\theta - \theta_0)^2}{2} \frac{\partial^2 q(n, \theta_0)}{\partial \theta^2} + \frac{(\theta - \theta_0)^2}{2} \left\{ \frac{\partial^2 q(n, \tilde{\theta})}{\partial \theta^2} - \frac{\partial^2 q(n, \theta_0)}{\partial \theta^2} \right\}.$$

First, we review some theorems about a stationary solutions, where $|\theta - \theta_0| \leq \xi$, $\xi > 0$. and $\tilde{\theta}$ is the indermediate value between θ and θ_0 , then another hand we have

Theorem 2.2. The model (1) accept the unique solution stationary defined in the next theorem if $\Phi^2(t)\sigma^2 < 1$ and $\Phi(t) \neq 0$.

Theorem 2.3. If the model (1) satisfies the condition

$$-\infty < E \{ \ln |\Phi(t)\varepsilon_t| \} < 0$$

Then

$$x_t = \varepsilon_t + \sum_{j=1}^{\infty} \left\{ \prod_{i=1}^j \Phi(t)\varepsilon_{t-i-1} \right\} \varepsilon_{t-j}^2.$$

Where this series converges almost surely and so defined the unique stationary solution equation.

Proof. Firstly we can give relation between the condition 2 and 3 for the existence of the unique solution, we have $\Phi^2(t)\sigma^2 < 1$ implies that $\Phi^2(t)\sigma^2 < |\Phi(t)\sigma| < 1$, so then by Jensen inequality it is possible to write

$$\begin{aligned} E \{ \ln |\Phi(t)\varepsilon_t| \} &\leq \ln E \{ |\Phi(t)\varepsilon_t| \} \\ &= \ln \{ |\Phi(t)| E |\varepsilon_t| \} \end{aligned}$$

And by Hold inequality the expression it became

$$\begin{aligned} \ln \{ |\Phi(t)| E |\varepsilon_t| \} &\leq \ln \left\{ |g(t)| \sqrt{E |\varepsilon_t|^2} \right\} \\ &= \ln \{ |\Phi(t)| \sigma \} \end{aligned}$$

Where $|g(t)\sigma| < 1$ then it result that $-\infty < \ln \{ |\Phi(t)| \sigma \} < 0$. □

Theorem 2.4. *The existence of a strictly stationary process defined by stachastic equation (1), If he fulfills the condition $\sigma^2 \{ \Phi(t) \}^2 < 0$, then there exists a unique strictly stationary process $(x_t)_{t \in \mathbb{Z}}$ satisfying a above model which can be written as infinite series*

$$x_t = \varepsilon_t + \sum_{j=1}^{\infty} \varepsilon_{t-j} \left\{ \prod_{i=0}^{j-1} (\Phi(t)\varepsilon_{t-i-1}) \right\}$$

Proof. we have

$$x_t = \varepsilon_t + \sum_{j=1}^{\infty} \varepsilon_{t-j} \left\{ \prod_{i=0}^{j-1} (g(t)\varepsilon_{t-i-1}) \right\} = \varepsilon_t + \sum_{j=1}^{\infty} \varepsilon_{t-j} g^j(t) \left\{ \prod_{i=1}^j \varepsilon_{t-i} \right\}$$

Where $g(t) = \ln$ in this situation, there is a constant K positive such that $\varepsilon_{t-j}^2 \leq C$

$$\begin{aligned} E \left[\sum_{j=1}^{\infty} C \left\{ \prod_{i=0}^{j-1} (g(t)\varepsilon_{t-i-1}) \right\} \right] &= C \sum_{j=1}^{\infty} E \left[\left\{ \prod_{i=0}^{j-1} (g(t)\varepsilon_{t-i-1}) \right\} \right] \\ &\leq C \sum_{j=1}^{\infty} \sqrt{E \left[\left\{ \prod_{i=0}^{j-1} (g^2(t)\varepsilon_{t-i-1}^2) \right\} \right]} \\ &\leq C \sum_{j=1}^{\infty} \sqrt{g^{2j}(t) \left\{ \prod_{i=0}^{j-1} E(\varepsilon_{t-i-1}^2) \right\}} \\ &= C \sum_{j=1}^{\infty} \sqrt{g^{2j}(t) \left\{ \prod_{i=0}^{j-1} \sigma^2 \right\}} \end{aligned}$$

So we arrive $C \sum_{j=1}^{\infty} \sqrt{g^{2j}(t)\sigma^{2j}} = C \sum_{j=1}^{\infty} \{g(t)\sigma\}^j$, and where $|g(t)\sigma| < 1$ the the sum of series is borned by finite value

$$C \sum_{j=1}^{\infty} \{g(t)\sigma\}^j \leq C \frac{g(t)\sigma}{1 - g(t)\sigma}.$$

□

The problem that arises here is how to choose t at function $\Phi(t)$ such that the model with constant coefficients will be estimated, to estimate the categories of models with continuous coefficients with empirical moments. First we will present this approach, we assume that

$\max_{s \in [t_0, t]} |h(s)| = M$ and $\min_{s \in [t_0, t]} \{h(s)\} = m$, we find that

$$\Phi(t) = \int_{t_0}^t h(s) ds \leq \left| \int_{t_0}^t h(s) ds \right| \leq \int_{t_0}^t \max \{|h(s)|\} ds \leq M(t - t_0)$$

Due to availability of stability condition we get to the following solution for the model

$$v_t = M(t - t_0)v_{t-1}\varepsilon_{t-1} + \varepsilon_t$$

is

$$\begin{aligned} v_t &= \varepsilon_t + \sum_{j=1}^{\infty} \varepsilon_{t-j} \left\{ \prod_{i=0}^{j-1} (M(t - t_0)\varepsilon_{t-i-1}) \right\} \\ &= \varepsilon_t + \sum_{j=1}^{\infty} \varepsilon_{t-j} M^j(t - t_0)^j \left\{ \prod_{i=0}^{j-1} \varepsilon_{t-i-1} \right\} \end{aligned}$$

It is obviously to show that the infinite series is almost surely convergent.

3 Maximum likelihood estimation

Model 01

To make an application, we consider the following model with time varying coefficients ; see (Bibi, 2003).

$$v_t = a(t)v_{t-p} + b(t)v_{t-p}\varepsilon_{t-1} + \varepsilon_t$$

Where

$$a(t) = \begin{cases} a_1, & \text{where } t = 2k + 1 \\ a_2, & \text{where } t = 2k, \end{cases}$$

the same thing for

$$b(t) = \begin{cases} b_1, & \text{where } t = 2k + 1 \\ b_2, & \text{where } t = 2k, \end{cases}$$

where $k \in \mathbb{N}$.

There are already established methods in the literature for estimating the parameters of stationary and ergodic bilinear time series models, the most frequently used methods are the method of moments and least squares, in this situation we apply the maximum likelihood method. And suppose that we have the observations $v_1, v_2, \dots, v_N$ for the model $(v_t)_{t \in \mathbb{Z}}$. So under a condition of stability of the model $\{\max(|a_1|, |a_2|)\}^2 + \sigma^2 \{\max(|b_1|, |b_2|)\}^2$, we can obtain the joint density function $F(v_1, v_2, \dots, v_N, \Omega)$, Ω is subset of the coefficients of the model where

$\Omega = (a_1, a_2, b_1, b_2)$, this function is defined as follows

$$\begin{aligned} F(v_1, v_2, \dots, v_N, \Omega) &= \prod_{t=1}^N \frac{1}{\sigma\sqrt{2\pi}} \exp \left\{ \frac{-\varepsilon_t^2}{2\sigma^2} \right\} \\ &= \prod_{t=1}^N \frac{1}{\sigma\sqrt{2\pi}} \exp \left\{ \frac{-(v_t - a(t)v_{t-p} + b(t)v_{t-p}\varepsilon_{t-1})^2}{2\sigma^2} \right\} \\ &= \prod_{t=1}^N \frac{1}{\sigma\sqrt{2\pi}} \exp \left\{ -\frac{(v_{t|t-1} - v_t)^2}{2\sigma^2} \right\} \end{aligned}$$

Where $\varepsilon_t = v_t - v_{t|t-1}$, the predictor $v_{t|t-1}$, for estimating the coefficients we want to get a solution Ω which maximize the logarithm likelihood function Φ , where

$$\Phi(v_i, \Omega)_{i=1, \dots, n} = \ln F(v_1, v_2, \dots, v_N, \Omega)$$

Then

$$\begin{aligned} \Phi(v_i, \Omega)_{i=1, \dots, N} &= \ln \left[\prod_{i=1}^N \frac{1}{\sigma\sqrt{2\pi}} \exp \left\{ -\frac{(v_{t|t-1} - v_t)^2}{2\sigma^2} \right\} \right] \\ &= -N \ln \sigma - \frac{N}{2} \ln 2\pi - \frac{1}{2\sigma^2} \sum_{t=1}^N (v_{t|t-1} - v_t)^2 \end{aligned}$$

We will find the derivatives to estimate the parameters

And then when we obtain the partial derivatives of $\Phi(v_i, \Omega)_{i=1, \dots, N}$, so we will go to estimate the coefficients of the model. Let now $\Omega = (a_1, a_2, b_1, b_2) = (\theta_1, \theta_2, \theta_3, \theta_4)$, here T indicates the transpose of a matrix

$$\mathbf{L}(v_i, \Omega)_{i=1, \dots, N} = \left(\frac{\partial \Phi(v_i, \Omega)}{\partial \theta_1}, \frac{\partial \Phi(v_i, \Omega)}{\partial \theta_2}, \frac{\partial \Phi(v_i, \Omega)}{\partial \theta_3}, \frac{\partial \Phi(v_i, \Omega)}{\partial \theta_4} \right)^T$$

And we present a matrix of second order partial derivatives

$$\mathbf{O}(v_i, \Omega)_{i=1, \dots, N} = \left(\frac{\partial^2 \Phi(v_i, \Omega)}{\partial \theta_i \partial \theta_j} \right), \forall i, j \in \{1, \dots, 4\}$$

And with the use of Taylor's expression to find or approximate the estimated value $\hat{\Omega}$, so first of all suppose the initial value of this program can also tell the true value Ω^0 , then we look for the solution of the following equation where we can find the value estimate

$$\mathbf{L}(v_i, \Omega)_{i=1, \dots, N} + \mathbf{O}(v_i, \Omega)_{i=1, \dots, N}(\hat{\Omega} - \Omega) = 0$$

So this equation can be rewritten as

$$\hat{\Omega} = \Omega - \mathbf{O}^{-1}(v_i, \Omega)_{i=1, \dots, N} \mathbf{L}(v_i, \Omega)_{i=1, \dots, N}$$

Model 02 We assume the following model with constant coefficients of order $BL(0, 0, 2, 1)$.

$$x_t = bx_{t-2}\varepsilon_{t-1} + \varepsilon_t$$

And let the same model but with continuous coefficients

$$x_t = \int_0^t \lambda e^{-\lambda s} ds x_{t-2} \varepsilon_{t-1} + \varepsilon_t$$

Then

$$x_t = (1 - e^{-\lambda t}) x_{t-2} \varepsilon_{t-1} + \varepsilon_t$$

So we can write x_t

$$x_t = \varepsilon_t + \sum_{j=1}^{\infty} (1 - e^{-\lambda t})^j \left\{ \prod_{i=1}^{j-1} \varepsilon_{t-i} \right\} \varepsilon_{t-j}^2.$$

In this situation

$$\begin{aligned} x_{t|t-1}(\lambda) &= \sum_{j=1}^{\infty} (1 - e^{-\lambda t})^j \left\{ \prod_{i=1}^{j-1} \varepsilon_{t-i} \right\} \varepsilon_{t-j}^2 \\ &= \sum_{j=1}^{\infty} C_k^j e^{-\lambda t(j-k)} \left\{ \prod_{i=1}^{j-1} \varepsilon_{t-i} \right\} \varepsilon_{t-j}^2 \end{aligned}$$

4 Modelization by bilinear models

We can take the present example of a bilinear model of order $(1, 0, 1, 1)$

$$v_t = a v_{t-1} + b \varepsilon_{t-1} v_{t-1} + \varepsilon_t$$

We consider the following model

$$v_t = a \ln |v_{t-k}| + b \varepsilon_{t-1} \ln |v_{t-k}| + \varepsilon_t$$

So in this situation there is a function defined with time where $\ln |v_{t-k}| = \varphi(t) v_{t-p}$, then the model become

$$v_t = a \varphi(t) v_{t-p} + b \varphi(t) \varepsilon_{t-1} v_{t-p} + \varepsilon_t$$

The nature of this model is classified under the category of variant bilinear models over time, and under a condition $a^2 \varphi^2(t) + \sigma^2 b^2 \varphi^2(t) < 1$

Theorem 4.1. *Let $\{\varepsilon_t, t \in \mathbb{Z}\}$ be a sequence of independent identically distributed random variables dedined on a probability space (Ω, Γ, P) such that $E\varepsilon_t = 0$ and $E\varepsilon_t^2 = \sigma^2 < \infty$. The existence of a strictly stationary process $\{v_t, t \in \mathbb{Z}\}$ of the model $v_t = a \varphi(t) v_{t-1} + b \varphi(t) \varepsilon_{t-1} v_{t-1} + \varepsilon_t$ under a condition $a^2 \varphi^2(t) + \sigma^2 b^2 \varphi^2(t) < 1$.*

$$v_t = \varepsilon_t + \sum_{j=1}^{\infty} \left\{ \prod_{i=1}^{j-1} (a \varphi(t-i) + b \varphi(t-i) \varepsilon_{t-i}) \right\} \varepsilon_{t-j}$$

and

$$v_t = \varepsilon_t + \sum_{j=1}^{\infty} \left\{ \prod_{i=1}^{j-1} \varphi(t-i) \prod_{i=1}^{j-1} (a + b \varepsilon_{t-i}) \right\} \varepsilon_{t-j}$$

And we have $\varphi^2(t) < \frac{1}{a^2 + \sigma^2 b^2} = C$ we suppose that $a^2 + \sigma^2 b^2 > 1$ so

$$\forall t : -\left(\frac{1}{a^2 + \sigma^2 b^2}\right)^{\frac{1}{2}} < \varphi(t-i) < \left(\frac{1}{a^2 + \sigma^2 b^2}\right)^{\frac{1}{2}}$$

Where

$$-\left(\frac{1}{a^2 + \sigma^2 b^2}\right)^{\frac{n}{2}} < \prod_{i=1}^n \varphi(t-i) < \left(\frac{1}{a^2 + \sigma^2 b^2}\right)^{\frac{n}{2}}$$

So $\forall \eta > 0, \exists n \in \mathbb{N} : \left| \prod_{i=1}^n \varphi(t-i) \right| < \eta$.

5 Illustration numerical of estimation

In this section we estimate the parameters of model in case $p = 1$.

$$v_t = a(t)v_{t-p} + b(t)v_{t-p}\varepsilon_{t-1} + \varepsilon_t$$

ns : Number of simulations

N : Number of observations

σ^2 : The variance This section is devoted to a comment on the study by simulations of the

(a_1, a_2, b_1, b_2)	(0.1500, 0.3000, 0.4500, 0.5500)
$(\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2)$	(0.1530, 0.3226, 0.3697, 0.5342)
$var(\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2)$	(0.0652, 0.0623, 0.0666, 0.0511)
$mse(\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2)$	(0.0225, 0.0900, 0.2025, 0.0931)
$ku(\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2)$	(4.4303, 3.2602, 3.2602, 3.0159)

Table 1: $N = 100, ns = 250, \sigma^2 = 1$

(a_1, a_2, b_1, b_2)	(0.1500, 0.3000, 0.4500, 0.5500)
$(\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2)$	(0.1517, 0.3240, 0.3818, 0.5345)
$var(\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2)$	(0.0257, 0.0249, 0.0236, 0.0154)
$mse(\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2)$	(0.0225, 0.0900, 0.2025, 0.0977)
$ku(\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2)$	(4.3094, 4.3094, 2.8587, 0.1023)

Table 2: $N = 300, ns = 250, \sigma^2 = 1$

(a_1, a_2, b_1, b_2)	(0.1500, 0.3000, 0.4500, 0.5500)
$(\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2)$	(0.1613, 0.3238, 0.3797, 0.5487)
$var(\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2)$	(0.0025, 0.0021, 0.0024, 0.0214)
$mse(\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2)$	(0.0029, 0.0091, 0.0190, 0.0254)
$ku(\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2)$	(3.0155, 3.0155, 2.8052, 3.0125)

Table 3: $N = 3000, ns = 250, \sigma^2 = 1$

theoretical properties of the estimators, in the case of this model we observe in the tables the

asymptotic behaviour realities. So in the general case we observe that the convergence criteria have been verified.

The results obtained in the tables present the following conclusions, as expected the increase of the sample size N substantially improves the results in terms of the precision of the estimators.

indeed, the calculated estimators which improves the accuracy of the estimation, Also Kurtosis tends towards the value of references 3 but there are some disturbances in the table are varied with the change of the number of simulations.

We also observe that the increase of the sample size ensures the tendency of mse and var towards 0.

Conclusion

We find in this application that the asymptotic behaviour of the estimators has been verified, such that there are cases of size of the sample and many of the simulations where the estimators will be identical with the true values. Then in the estimation by the method of likelihood is very effective for the time-varying coefficients of the bilinear models.

In another way also the convergence criteria were checked (approximation mse and var to zero and ku converges to 3). Where the data show that the method works in the right direction.

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